The Aligarh Journal of Statistics
Vol. 45 (2025), pp. 113-124 ISSN 0971-0388

Optimizing Transportation Problem Through Linear Constraints with Optimality

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AMS Subject Classification: 90B06

Received: 03/01/2024 Accepted: 24/03/2025

Abstract

A transportation problem fundamentally pertains to achieving the most efficient way to satisfy the demand of destinations using the resources available from other sources. We contemplate the fundamental concept of the m ×n transportation problem unraveled by linear algebra having m+n-1 linear equations. The intent of the presentation is to showcase a specialized method for acquiring a basic primal solution through m+n-1 linear equations. This proposed method is named the linear method for optimization of the transportation problem. As well, we explore the new optimality for accomplishing the optimal solution for the transportation problem. This algorithm explains simpler, streamlined procedures by obtaining the optimal solution for transportation problems, whether maximizing and/or minimizing objective functions. It includes numerical examples to aid in understanding and implementing the algorithm.

Keywords:Transportation problem, Minimization and maximization problems, Basic primal solution, Linear method, Reduced cost matrix, Dummified cost matrix, Optimality, Fractional prospective quantity (Q_{ij}).

1 Introduction

The pressure on organizations to discover healthier customs to create and deliver value to customers grows stronger in today's highly competitive market. When and how to ship the product to customers in the specified quantities. They tend to face more difficulty in delivering goods or products in a cost-effective manner. To rectify this challenge, transportation models provide a strong framework to optimize the variables. They ensure certainty to efficiently move raw materials and finished goods and to timely deliver them.
One of the most intriguing linear programming problems involving product or service distribution is the transportation problem. Production and transportation both play critical roles in balancing a manufacturing company's overall supply chain and goodwill. When an industry produces a finished product, that product must reach its intended market in the optimum time. The consumers of the finished product may not necessarily be located in the same vicinity as the industry. Consequently, transportation becomes critical to ensure that the consumers or end users have access to the goods and services meant for them. The transportation problem, as the name suggests, is concerned with the logistical movement of various resources and finished goods from one location to another. The most important factor in deciding the quantity, cost, and transportation routes is the optimum allocation. The sources (factories) are assured to supply goods to various destinations (warehouses) based on their demands. Every source strives to reduce transportation costs in delivering goods. This is how a transportation problem arises. The trans-

portation model is useful for selecting routes with the lowest travel costs. The model also facilitates the destinations to satisfy their demands in an optimum manner.
The transportation problem was first formulated mathematically by F. L. Hitchcock Hitchcock (1941) in 1941 and later, in 1947, discussed in detail by the Nobel Laureate T. C. Koopmans Koopmans (1949). As a linear programming (LP) problem, it was first formulated by G. B. Dantzig in 1951 Dantzig (1951), who applied the simplex method to this special LP problem. In 1954, Charnes and Cooper Charnes & Cooper (1954) developed a stepping-stone method that explained the implementation of the simplex method for the problem. The transportation problem has been detailed by several researchers Sudhakar, Arunsankar & Karpagam (2012); Quddoos, Javaid & Khalid (2012) till now. Klingman and Russell, in 1975 Klingman & Russell (1975), solved it further and discussed their method in detail to motivate other researchers to develop a new method. They proposed a general method of solving constraints in an equivalent standard transportation model having only one additional origin and destination. In 2017, Reena Patel et al. Patel, Patel & Bhathawala (2017) discussed the optimality of the transportation problem with a degree of freedom (m-1) for optimality. Many manufacturers have used the optimization techniques most frequently in linear programming problems to elicit the solutions to real-world problems in recent years. It is critical to develop new approaches that will allow the model to fit into the real world as much as possible.
We are familiar with the m×n transportation problem, which has m+n constraints that must be met by the mn unknowns x_{ij}. The reduced row echelon form is then executed. It is shown that one equation can be omitted and the remaining m+n-1 equations are solved in terms of the transported units x_{ij}s. So, we are approaching m+n-1 allocations while solving the problem. In this paper, we introduce the linear method for answering transportation problems with m+n-1 linear constraints for attaining x_{ij} and ensuring that the solution is optimal by utilizing some numbers that aid the algorithm. For a better understanding of the methodology, we also provide illustrated numerical examples.

2 Mathematical Model of Transportation Problem

Let us consider the standard transportation problem with m sources S_i (with supplies a_i, i= 1, 2, 3... m) and n destinations D_j (with demands b_j, j= 1,2,3... n).
c_{ij} = Transportation cost for transporting the load from the source S_i to destination D_j
x_{ij}= The number of load units moving from S_i to D_j.
Mathematically the problem can be stated as minimize

$$Minimize\mathbb{Z} = \sum_{i=1}^{i=m}\sum_{j=1}^{j=n} C_{\text{ij}}x_{\text{ij}}$$

Subject to constraints:

$$\begin{aligned} &\sum_{j=1}^{n} x_{\text{ij}} = \ a_i \ \ for \ \ i = 1,2,3........, m \ \ (Supply \ \ constraints) \\ &\sum_{i=1}^{m} x_{\text{ij}} = \ b_i \ \ for \ \ j = 1,2,3,......, n \ \ (Demand \ \ constrains) \qquad (A) \\ &x_{\text{ij}} > 0 \text{ for all } i \ \& \ j \end{aligned}$$

The balanced or standard transportation problem is one that meets the aforementioned requirement in Table 1 i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Only methods for resolving balanced or standard transportation issues have been devised.

The transportation problem is mathematically expressed in the system (A) above. We use the linear programming technique with m+n equality constraints, which the mn unknowns x_{ij} must meet. To obtain the restrictions involving, as stated in system (B), we solve system (A) from its equivalent reduced row echelon form. **?**(Hardley, n.d.).

$$
\begin{array}{llllll}
x_{11}+x_{12}+...+x_{1n} & & & & =a_1 \\
& x_{21}+x_{22}+..+x_{2n} & & & =a_2 \\
& & \ddots & & \vdots \\
& & x_{m1}+x_{m2}+...+x_{mn} & & =a_m \\
x_{12}+ & x_{22}+... & +x_{m2} & & =b_2 \\
x_{13}+ & x_{23}+... & +x_{m3} & & =b_3 \\
\ddots & & & & \vdots \\
x_{1n}+ & x_{2n}+... & +x_{mn} & & =b_m
\end{array} \qquad \text{(B)}
$$

2.1 Unbalanced Transportation Problem

Sometimes the transportation problem is not balanced in real-world scenarios; instead, it is unbalanced. When supply surpasses demand or demand surpasses supply, this circumstance arises. An issue is referred to as an unbalanced transportation problem if the total of all measures does not match the total of all requirements that is $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$. The following is the formulation of the two examples of the unbalanced transportation problem.

Case I: When supply $>$ demand (Excessive production), then problem is modeled as follows:
The Transportation problem may be stated as follows,
Minimize$\mathbb{Z}$=$\sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{\text{ij}}\, x_{\text{ij}}$
Subject to constrain$\sum_{j=1}^{n} x_{\text{ij}} < \ a_i$ for i= 1, 2, 3........, m (Supply constraints)
$\sum_{i=1}^{m} x_{\text{ij}} = \ b_i$ for j = 1,2,3, ..., n (Demand constrains) $x_{\text{ij}} \geqslant 0\ \forall i\ \&\ j$

Case II: When demand $>$ supply (Inadequate production), then problem is modeled as follows:
The Transportation problem may be stated as follows,
Minimize$\mathbb{Z}$=$\sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{\text{ij}}\, x_{\text{ij}}$
Subject to constrain$\sum_{j=1}^{n} x_{\text{ij}} = \ a_i$ for i= 1, 2, 3........, m (Supply constraints)
$\sum_{i=1}^{m} x_{\text{ij}} < \ b_i$ for j = 1,2,3, ..., n (Demand constrains) $x_{\text{ij}} \geqslant 0\ \forall i\ \&\ j$

3 Methodology

In this methodology, the focus is on the linear method. The linear method is considered to be one of the most effective approaches for solving minimization and maximization problems.

3.1 Linear Method For Minimization Problem

We start the methodology with the m+n-1 linear equations of transportation problem to obtain the transported units x_{ij} for locating into the transportation matrix. This methodology is applicable to both balanced and unbalanced transportation problems.

1. Draw the transportation problem and verify that the problem is balanced; if not, then switch to balance by adding dummy rows or columns.

2. Write constraints accordingly to the problem by the system–(B).

3. Choose maxi (a_i, b_j), i= 1,2...m and j = 2,3...n.

   Case-1:If $b_j > a_i$, i= 1,2...m and j = 2,3...n then consider maximum b_j to compare with a_1 and allot the corresponding cell (1, j) with a_1. That will exhaust one corresponding equation and reduce the other corresponding equation by $b_j - a_1$.

   (a) From the reduced system–(B), find x_{ij} = mini{ b_j - a_1, b_j, a_i} and allot the corresponding cell (i, j) with the appropriate quantity. Again, this will reduce the system–(B).

   (b) Continue this process with the reduced system–(B) by considering the appropriate minimum quantity to allot to the corresponding cell till the demand and supply are exhausted.

   Case-II:If $a_i > b_j$, i= 1,2...m and j = 2,3...n then consider maximum a_i to compare with minimum b_j(j = 2,3...n) and allot the corresponding cell (i, j) with b_j That will exhaust one corresponding equation and reduce the other corresponding equation by a_i-b_j of system–(B).

   (a) From the reduced system–(B), find x_{ij} = mini{ a_i - b_j, b_j, a_i } and allot the corresponding cell (i, j) with the appropriate quantity. Again, this will reduce the system–(B).

   (b) Continue this process with the reduced system–(B) by considering the appropriate minimum quantity to allot to the corresponding cell till the demand and supply are exhausted.

4. Calculate the transportation cost by multiplying the allocated quantities with corresponding transportation costs and summing up.

5. If the degeneracy is discovered. i.e., numbers of allocations are less than m+n-1 to resolve the degeneracy at the primal basic solution; we continue by allocating a small quantity close to zero to one or more (if needed) unoccupied cells to get m+n-1 occupied cells. The cell containing this extremely small quantity is considered to be an occupied cell and is also denoted by (Δ) or (ϵ). This small quantity will not affect the total cost and supply and demand values. It is better to allocate (Δ) to unoccupied cells that have the lowest transportation cost in a minimization problem. The quantity of (Δ) is considered so small that is transferred to an occupied cell and that is does not change the quantity of allocation.

3.2 Linear Method for Maximization Problem

This section presents the linear method to solve the maximization-type transportation problem, which is also easy to apply to both types of balanced and unbalanced transportation problems.

1. Draw the transportation problem and verify that the problem is balanced; if not, then switch into balanced by adding dummy rows or columns

2. Start with converting maximization problem into a minimization problem by subtracting all the elements from the highest element in the given transportation table. The modified transporting minimization problem can be solved in the usual manner.

3. Perform step 2 and 3 of the linear method for minimization transportation problem as discussed above.

4. Calculate the transportation cost by multiplying the allocated quantities with corresponding transportation costs and summing up considering the original cost matrix.

5. If the degeneracy is discovered. I.e., numbers of allocations are less than m+n-1 to resolve the degeneracy at the primal basic solution; we continue by allocating a small quantity close to zero to one or more (if needed) unoccupied cells to get m+n-1 occupied cell. The cell containing this

extremely small quantity is considered to be an occupied cell and is also denoted by (Δ) or (ϵ). This small quantity will not affect the total cost and supply and demand values. It is better to allocate (Δ) to unoccupied cells that have the lowest transportation cost in a maximization problem. The quantity of (Δ) is considered so small that is transferred to an occupied cell and that is does not change the quantity of allocation.

3.3 Formation of Reduced Cost Matrix

1. Subtract each of the elements of every row from the largest entry of the row of the transportation table and place them on the right top of corresponding element.
2. Apply the same operation on each of the columns and place them on the right bottom of the corresponding element.
3. Form a revise transportation table whose elements are the summation of right top and right bottom elements attained in Steps 1 and 2.

3.4 Dummified Cost Matrix

A dummified transportation cost matrix is to transform the cost matrix with dummified cost. Generally, in the transportation problem $\forall\ C_{ij} \geq 0$. In these circumstances, we need to dummify the transportation matrix to make the cost non-zero as well as the cost will be raised uniformly, and this process will be applied to the entire problem.

The dummified transportation cost matrix can be obtained with the help of the relationship,

$$C'_{ij}=\begin{cases} C_{ij}+m+n & C_{ij}\neq 0 \\ C_{ij} & C_{ij}=0 \end{cases}$$

where C'_{ij} = Dummy non-zero cost of transportation from source i to destination j.
m = numbers of rows and n = numbers of columns

3.5 Degenerate Transportation Problem

A basic feasible solution in which the total number of non-negative allocations is less than m+n-1 is called a degenerate basic feasible solution. Degeneracy must also be removed for the solution achieved to be optimal. Thus, in the transportation problem Degeneracy occurred in two different stages:

1) At the stage of initial basic feasible solution

2) While checking the optimality When we gain the degeneracy in first stage, we can remove the degeneracy from the solution by employing the algorithm described as above.

In the second stage, we remove the degeneracy by performing the optimality test in the dummified matrix. Accordingly, we put the (Δ) on the highest /lowest cost of the dummified matrix for minimization/maximization or the original cost matrix having minimum / maximum cost for minimization/ maximization transportation problem.

3.6 Optimality Test

We use multiplication and division of real numbers to achieve the optimal cost to transportation problem. As we know that 1 is the identity for multiplication, thus we apply this simple concept in optimality for finding the numbers corresponding to the given number of rows and columns of transportation problem.

This algorithm necessitates the inclusion of some sets of numbers $\lambda_i's$ $(\lambda_1, \lambda_2, ..., \lambda_m)$ and $\mu_j's$ $(\mu_1, \mu_2, ..., \mu_n)$ which help to determine the improved (optimal) allocations. The procedure to accomplish the improved (optimal) solution of the transportation problem is stated as follows:

1. (a) Consider the basic primal solution as we derived by the LCT Method or any other basic primal solution method then concentrate on the costs of the m+n-1 occupied cells individually.
   (b) Begin with converting the original m×n matrix into the reduced cost matrix then again transforming this reduced cost matrix into a dummified cost matrix with the actual allocations. The dummified cost matrix problem can be solved.
2. Assign λ_i and μ_j against the right and bottom side corners of the corresponding rows and columns of the transportation table respectively.
3. Locate 1 at one of them λ_i or μ_j where the corresponding row or column has the most individual allocations.
4. Next, considering the occupied cells, compute the other values of $\lambda_i's$ and $\mu_j's$ using the formula $C_{ij} = \lambda_i \times \mu_j$.
5. Compute Fractional Prospective Quantity via employing $Q_{ij} = \frac{C_{ij}}{\lambda_i \times \mu_j}$ for each unoccupied cell.
6. If all Fractional Prospective Quantities Q_{ij} are lessgrater than 1($\leq$ 1), it indicates the optimal solution. If this is not the case assign 'θ' to the cell with the highest positive value of Q_{ij}.
7. Starting with the selected cell, trace a closed loop and assign (+) and (–) sign alternately.
8. Recognize the maximum possible increment in the value of θ which is minimum amongst the (-) signed allocated values.
9. Now adding and subtracting the minimum allocated value where there is (+) and (-) signs respectively.
10. Repeat the mentioned procedure until the optimum solution will be reached.
11. To calculate the total transportation cost as the sum of the product of cost and the corresponding allotted value of supply or demand, we must consider the original matrix i.e., Total cost= $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{\text{ij}} x_{\text{ij}}$.

4 NUMERICAL EXAMPLES

1. Consider the following cost-minimizing transportation problem.

	W_1	W_2	W_3	Supply
S_1	25	45	10	200
S_2	30	65	15	100
S_3	15	60	55	400
Demand	200	100	300	

Step 1 indicates that the issue is the unbalanced transportation problem; therefore, in order to make it a balanced problem, we must add a dummy column.

According to the problem, write the equation in terms of system (B), then elect for maxi (a_i , b_j), i=1,2,..m and j=2,3,..n

	W_1	W_2	W_3	W_4	Supply
S_1	25	45	10	0	200
S_2	30	65	15	0	100
S_3	15	60	55	0	400
Demand	200	100	300	100	

$$x_{11}+x_{12}+x_{13}+x_{14}= 200$$
$$x_{21}+x_{22}+x_{23}+x_{24}= 100$$
$$x_{31}+x_{32}+x_{33}+x_{34}= 400$$
$$x_{12}+x_{22}+x_{32}= 100$$
$$x_{13}+x_{23}+x_{33}=300$$
$$x_{14}+x_{24}+x_{34}= 100$$

We are doing Case-II: a_i ¿ b_j , i=1,2,..m and j=2,3,..n because it requires a different approach than Case-I. Now, compare the max a_3 to the mini b_2 and allot b_2 to the allied cell (i,j).

Hence, we allot the cell (3,2) with mini b_2, i.e., x_{32}=100, x_{12} = x_{22}=0
As a result, one equivalent equation will be exhausted, and the other corresponding equation will be reduced by a_3- b_2.

$$x_{11}+ \qquad x_{13}+x_{14}= 200$$
$$x_{21}+ \qquad x_{23}+x_{24}= 100$$
$$x_{31}+ \qquad x_{33}+x_{34}= 400$$
$$x_{13}+x_{23}+x_{33}=300$$
$$x_{14}+x_{24}+x_{34}= 100$$

Obtain x_{ij}= mini (a_3 - b_2, b_3, a_i) from the reduced system and assign the associated cell with the right value. This will further deplete the system. So, x_{ij} = mini (a_2, b_4)of the cell (2,4) with mini a_2, by x_{24} = 100,x_{14} = x_{34} = x_{21} = x_{23} = 0

$$x_{11}+ \qquad x_{13}= 200$$
$$x_{31}+ \qquad x_{33}= 300$$
$$x_{13}+x_{33}=300$$

Carry out this process with the reduced system. Another time, x_{ij} = mini (a_1,b_2) of the cell (1,3) with mini a_1, x_{13}=200, x_{11}=0. Carry on with this method using the simplified system.

$$x_{31}+x_{33}= 300$$
$$x_{33}=300$$

Simplifying the Eq. now yield x_{31}= 200, x_{33}= 100 All the $x'_{ij}s$ values are now listed in the table for each related cell after all the equations have been solved, as shown below:

The total transportation cost is 16,500.

Here we observe that the problem is degenerate. So, we add Δ for the m+n-1 allocations to make it a non-degenerate problem.

	W_1	W_2	W_3	W_4	Supply
S_1			x_{13}= 200		200
S_2				x_{24}= 100	100
S_3	x_{31}= 200	x_{32}= 100	x_{33}= 100		400
Demand	200	100	300	100	

	W_1	W_2	W_3	W_4	Supply
S_1	25	45	10 $\boxed{200}$	0	200
S_2	30	65	15 $\boxed{\Delta}$	0	100
S_3	15 $\boxed{200}$	60 $\boxed{100}$	55 $\boxed{100}$	0 $\boxed{100}$	400
Demand	200	100	300	100	

Later, apply the optimality test; here we applied the formation reduced cost matrix; afterwards, we proceeded with the dummified cost matrix and applied the optimality to that matrix.

Reduced Cost Matrix

25^{20}_{5}	45^{0}_{20}	10^{35}_{45}	0^{45}_{0}
30^{35}_{0}	65^{0}_{0}	23^{50}_{40}	0^{65}_{0}
15^{45}_{15}	60^{0}_{5}	55^{5}_{0}	0^{60}_{0}

Table 1: Reduced Cost Matrix-1

25	20	80	45
35	0	90	65
60	5	5	60

Table 2: Reduced Cost Matrix-2

Dummified Cost Matrix

32	27	87	52
42	7	97	72
67	12	12	67

Table 3: Dummified Cost Matrix

To compute the fractional prospective quantity Q_{ij}, we used the dummified cost matrix table and found the number of sets λ_i or μ_j in steps 1 to 5.

Q_{11}=3.06, Q_{12}= 0.32, Q_{14}= 0.83, Q_{21} =3.60, Q_{22}=0.07,Q_{34} =1116.6

Determine the Q_{ij}'s, identify the largest positive value, and then reprise steps 6 to 9 until the optimal solution is reached. The optimal table with optimal allocations in the original cost matrix is displayed.

	W_1	W_2	W_3	W_4	Supply	λ_i
S_1	32	27	87 [200]	52	200	λ_1=1044
D_2	42	7	97 [Δ]	72 [100]	100	λ_2= 1164
D_3	67 [200]	12 [100]	12 [100]	67	400	λ_3= 1
Demand	200	100	300	100		
μ_j	μ_1=8/10 =0.01	μ_2=1/12 =0.08	μ_3=1/12 =0.08	μ_4=72/1164 =0.06		

	W_1	W_2	W_3	W_4	Supply
S_1	25	45	10 [200]	0	200
S_2	30	65	15 [100]	0	100
S_3	15 [200]	60 [100]	55	0 [100]	400
Demand	200	100	300	100	

At this point, all the Fractional Prospective Quantities for all unoccupied cells $Q_{ij} \leq 1$. Hence, we achieve the optimal transportation cost.

The optimal transportation cost is 200*10 + 100*15 + 200*15 + 100*60 = 12,500.

2. Determine the minimizing transportation cost.

	W_1	W_2	W_3	W_4	Supply
S_1	19	16	15	3	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
Demand	6	6	8	23	

Through the method of Linear, we acquire a basic primal solution as under:

	W_1	W_2	W_3	W_4	Supply
S_1				x_{14}=11	11
S_2		x_{22}=6	$x_{23} = 7$		13
S_3	x_{31}=6		x_{33}=1	x_{34}=12	19
Demand	6	6	8	23	

Resolving the equations, we got the values x_{11}=x_{12}=x_{13}=x_{21}=x_{24}=x_{23}=0 and other values are allocated in the above table.

The total transportation cost is 941.
Now, we perform the optimality test as below:

At this point, all the Fractional Prospective Quantities for all unoccupied cells, $Q_{ij} \leq 1$. Hence, we achieve the optimal cost of the problem.

The optimal transportation cost = 11*3 + 1*17 + 12*23 + 5*32 + 6*27 + 8*18 = 792.

	W_1	W_2	W_3	W_4	Supply
S_1	19	16	15	3 [11]	11
S_2	17 [1]	18	14	23 [12]	13
S_3	32 [5]	27 [6]	18 [8]	41	19
Demand	6	6	8	23	

3. Find the maximization transportation cost.

	A_1	A_2	A_3	A_4	Supply
D_1	6	6	11	15	80
D_2	4	6	10	12	120
D_3	6	4	7	6	150
D_4	4	10	14	14	70
D_5	8	8	7	9	90
Demand	100	200	120	80	

Here the problem is the unbalanced transportation problem so it is essential to add dummy column for making it balanced problem. Afterwards, we apply the Linear method for maximization to generate a basic primal solution as below.

	A_1	A_2	A_3	A_4	A_5	Supply
D_1		x_{12}=80				80
D_2		x_{22}=70	x_{23}=50			120
D_3	x_{31}=100	x_{32}=50				150
D_4				x_{44}=60	x_{44}=10	70
D_5			x_{53}=70	x_{54}=20		90
Demand	100	200	120	80	10	

Resolving all the equations we have x_{11}=x_{13}=x_{14}=x_{15}=x_{21}=x_{24}= x_{25}=x_{33}=x_{34}=x_{35}=x_{41}=x_{42}= x_{43}=x_{51}=x_{52}=x_{55}=0 and other values are allocated in the above table.

The total transportation cost is 3,710.

Here, we apply the optimality test for the optimal solution as follows:

At this point, all the Fractional Prospective Quantities for all unoccupied cells $Q_{ij} \leq 1$. Hence, we achieve the optimal values.

The total profit is 80*15 + 120*10 + 100*6 + 40*4 + 70*10 + 90*8 = 4,580.

	A_1	A_2	A_3	A_4	A_5	Supply
D_1	6	6	11	15 [11]	0	80
D_2	4	6	10 [120]	12	0	120
D_3	6 [100]	4 [40]	7	6	0 [10]	150
D_4	4	10 [70]	14	14	0	70
D_5	8	8 [90]	7	9	0	90
Demand	100	200	120	80	10	

5 Conclusion

As a result, this prior study explored the initial basic feasible solution via m+n-1 linear constraints, establishing a new way to attain the basic primal solution that differs from the previously stated methods. Following that, we used the multiplicative identity to determine the number of sets required to use the optimality algorithm to satisfy the optimal criteria for the optimal transportation mix. The goal of this study is to explain the technique for acquiring the optimal solution to balance the unbalanced transportation challenges. First, the paper discusses how to find the non-degenerate optimal solution. Besides non-degenerate, this technique assures that the degenerate optimal solution can be improved in an accomplishment of its optimality. The study discusses the minimization and maximization of transportation problems to help the reader fully comprehend the technique. As a result, decision-makers can understand and apply the algorithm to determine the basic primal solution and the optimal solution to real-world transportation problems.

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