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Construction of Some Specified Confounded Asymmetrical Factorial Designs

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Abstract

Asymmetric confounded factorial designs are important and needed in practice due to the fact that many times the levels of factors are different and availability of homogeneous blocks is also a problem in sectors like agriculture. There are various methods available to construct blocks for confounded asymmetric factorial designs; however the methods are intricate. One of the methods is developed by Das and Giri, in which asymmetrical designs are converted to symmetrical factorial design first. In this article a simple technique for the construction of blocks in confounded asymmetric factorial designs is proposed. The constructed designs will also be balanced. This technique is specifically useful for the construction of $s \times 2^n$ design with blocks of size $s \times 2^{n-1}(s = 3, 4)$ and 2×3^n design with blocks of size $2 \times 3^n(n = 2, 3)$. Further, the method to identify the confounded effect in asymmetrical factorial design is also discussed.

Keywords: Asymmetric factorial designs, Confounding, Balanced designs.

1 Introduction

In many situations, the experimenter wants to investigate the effects of multiple factors on a response variable with efficiency, where the variables may not have equal levels in various fields such as agriculture, medical and industrial. In such cases asymmetric factorial experiments provide a flexible and realistic framework for exploring multifactorial relationships offering a balance between scientific rigor and practical feasibility. When the numbers of treatment combinations are large, then there is a need to allocate the treatment combinations in more than one block due to some constraints such as homogeneity or availability of experimental material. In such a situation, unimportant interaction effects can be confounded. For example, suppose that the effect of three fertilizers,F_1, F_2, and F_3 on the yield of sugarcane is to be obtained. Fertilizer F_1 has 3 levels and the fertilizers F_2 and F_3 each have 2 levels. Then this experiment is described as $3 \times 2 \times 2(i.e. 3 \times 2^2)$ asymmetric factorial experiment.

For asymmetrical factorial design, confounding is somewhat different from the symmetrical factorial designs. It is difficult to achieve balance while constructing blocks for confounded asymmetrical factorial designs. Balancedness is an important property for construction of designs. There are several methods available for construction of confounded asymmetrical factorial designs. The most popular methods are developed by Yates (1935;1937), Kishen and Srivastav (1959), Nair and Rao (1948), Das (1960) and others. Still a general technique for construction of blocks for confounded asymmetrical factorial design does not exist.

Yates (1937) introduced the method for the construction of asymmetrical factorial designs and also the analysis of these designs. Yates has given the method for the construction of confounded designs with factors having two and three levels. Das (1960) has developed a method for the construction of balanced confounded design. These designs are linked with the appropriate fractions of symmetrical factorial designs. This method requires more than one replication to balance the design.

Banerjee (1977) has given a method to confound asymmetrical factorial designs by linking them with the

symmetrical factorial design of n factors where each factor has two levels (i. e. 2^n). This method uses fractional replications, scheme of association for replacement of the combinations of the factors of symmetrical factorial design having the levels of the factor of asymmetrical factorial design. This method is helpful to obtain the design even when the numbers of levels with one or more factors are the product of prime numbers. Hinkelmann and Kemthorne (2005) has given a method for construction of the blocks in asymmetrical factorial confounded designs using Kronecker product. Das and Ghosh (2018) have suggested another method for the construction and also for the analysis of asymmetrical factorial experiments of type 6×2^2 in blocks of size 12. For constructing the design of the type 6×2^2 there have taken three choices for the confounded interactions and five different cases are used for a choice. Out of those five cases there are mainly two different cases for each choice with respect to loss of information. Adepoju and Ipinyomi (2016) have also given the method for construction of asymmetrical fractional factorial design. Wanyoike and John (2021) have developed straight forward procedure to construct balanced asymmetrical factorial designs in which main effects and higher order interactions are estimated with high efficiencies. Godolphin (2019) has studied two level factorial designs arranged in the block of size 2^q. He has introduced construction approach related to graph theory, enabling the estimation main effects and selected two factor interactions.
It is also important to identify the confounded effect in asymmetrical confounded designs. Kane (1994) has developed a method to identify the confounded effect in 2^n symmetric factorial design for confounding in two or more than two blocks. Furthermore Ghosh and Bagui (1998) have suggested one method to identify the confounded effect in symmetrical design. In this article, the authors have suggested a method to identify the confounded effect in asymmetrical confounded design for some specified designs.
Section 2 represents a simple technique for the construction of asymmetrical factorial experiments of the type 3×2^n with blocks of size $3 \times 2^{n-1}$ and 2×3^n design with blocks of size $2 \times 3^{n-1}$. In Section 3, a method is proposed to identify the confounded effects in asymmetrical design of the particular type. Section 4 illustrates performance of the designs constructed using methods proposed in section 2. Also a demonstration is given using actual problem of how to identify confounded effects. Section 5 comprises conclusions.

2 Materials And Methods

2.1 Construction of Confounded Asymmetrical Factorial Design of the $s \times 2^n$ in the Block of Size $s \times 2^{n-1} (s = 3, 4)$

To construct a confounded asymmetric factorial design of the type $s \times 2^n$ where one factor is having s (3 or 4) levels and remaining n factors are having 2 levels each the method is given below. Let F be a factor with s levels, express s as p^m where p should be prime.To construct the replicates having two blocks follow the method stepwise.

- **Step 1:** Find all possible treatment combinations.
- **Step 2:** Consider defining equation of the effect to be confounded and obtain the first replication with two blocks. The block having treatment combination (0, 0, ..., 0) is called as the principal block.
- **Step 3:** To obtain second replication, interchange treatment combinations with highest levels (> 2) of factor F of principal block and non-principal block of first replication. Remaining treatment combinations will be same in both the blocks.
- **Step 4:** To obtain third replication consider second replication obtained in step 3; interchange the $s - 2$ penultimate treatment combinations of factor F.

2.2 Construction of Confounded Asymmetrical Factorial Design of the Type 2×3^n in the Blocks of Size $2 \times 3^{n-1}(n = 2, 3)$

Let F be a factor with 2 levels and remaining (2 or 3) factors are having 3 levels each. To construct the replicates having two blocks each, for the confounded asymmetric factorial experiment of the type 2×3^n in the blocks of size $2 \times 3^{n-1}$ follow the method stepwise.

- **Step 1:** Find all possible treatment combinations.
- **Step 2:** Consider defining equation of the effect to be confounded and obtain first replication with two blocks. The block having treatment combination (0, 0, ..., 0) is called as the principal block.
- **Step 3:** To obtain second replication, consider treatment combinations with higher order of factor F of principal block and non-principal blocks. Replace these treatment combinations from B_1 to B_2, B_2 to B_3 and B_3 to B_1. Remaining treatment combinations will be same in all the blocks.

3 Identification Of Confounded Interaction In Asymmetrical Factorial Design

3.1 Identification of Confounded Interaction in Asymmetrical Factorial Design of the Type $s \times 2^n$ in the Block of Size $s \times 2^{n-1}; (2 < s \leq 4)$

In the case of $s \times 2^n$ factorial design confounded in the block of size $s \times 2^{n-1}$, there will be only two blocks per replication. Hence, to identify confounded interactions, follow the method stepwise:

- **Step 1:** Identify principal block and non-principal block.
- **Step 2:** Find the treatment combinations from the non-principal block which contain a single letter having power one or two.
- **Step 3:** Combine these treatment combinations multiplicatively to give the confounded interaction.

3.2 Identification of Confounded Interaction in Asymmetrical Factorial Design of the 2×3^n in the Block of Size $2 \times 3^{(n-1)}$ ($n = 2, 3$)

In the case of 2×3^n factorial design confounded in the block of size $2 \times 3^{(n-1)}$, there will be only three blocks per replication. Hence, to identify confounded interactions, follow the method stepwise:

- **Step 1:** Find the principal block and the non-principal block.
- **Step 2:** Find those treatment combinations which contain a single letter in the non-principal block that have power one or two (e.g., the treatment combinations 010, 002, $\ldots$) from any one of the non-principal block treatment combinations.
- **Step 3:** Combine these treatment combinations multiplicatively to get the confounded interaction.
- **Step 4:** Now check the condition,

$$\sum_{i=1}^{n} a_i X_i = 0 \pmod 3 \tag{1}$$

For the corresponding identified confounded interaction $\prod X_i^{a_i}$ in the principal block, where $\alpha_i \in$ GF(3).

- **Step 5:** In the remaining two blocks (non-principal blocks),

$$\sum_{i=1}^{n} a_i X_i = j \pmod 3 \quad \text{where } j = 1, 2 \tag{2}$$

Must hold for the corresponding confounded interaction $\prod X_i^{a_i}$

4 Illustrations

4.1 Construction of 3×2^2 Asymmetrical Factorial Design in Blocks of Size $3 \times 2^{2-1}$

Consider factor F having three levels 0, 1, and 2, and two more factors A and B, each having two levels (0 and 1). To obtain replications for this 3×2^2 factorial experiment, in which the highest-order interaction FAB is confounded, the method given in Section 2.1 is applied stepwise as follows:

Step 1: All the possible treatment combinations for this asymmetrical design are:

$$000 \quad 001 \quad 010 \quad 011 \quad 100 \quad 101 \quad 110 \quad 111 \quad 200 \quad 201 \quad 210 \quad 211$$

Step 2: Since interaction FAB is to be confounded, the defining equation is considered as:

$$X_1 + X_2 + X_3 = \begin{cases} 0 \\ 1 \end{cases} \pmod{(2)}$$

The first replication using the defining equation is constructed as follows:

Table 1 : Replication I for 3×2^2 factorial experiment confounding FAB

Block 1 (=0 mod 2)	**Block 2 (=1 mod 2)**
000	001
011	010
101	100
110	111
200	201
211	210

Step 3: Consider levels (> 2), that is, the 3^{rd} level denoted by 2. Therefore in the replication obtained in step 2, consider the treatment combinations 200 and 211 from Block 1 and interchange them with treatment combinations 201 and 210 from Block 2, where the factor F is at the highest level. The remaining treatment combinations in both blocks remain the same as in Replication I. The new replication obtained is listed below:

Table 2 : Replication II for 3×2^2 factorial experiment confounding FAB

Block 1	**Block 2**
000	001
011	010
101	100
110	111
201	200
210	211

Step 4: For the third replication, we consider the penultimate level of factor F. The treatment combinations 101 and 110 are in Block 1, and 100 and 111 are in second block. Replace these treatment combinations from Block 1 to Block 2 and vice versa. The remaining treatment combinations in both the blocks are same as Replication II. Now we get Replication III

Table 3 : Replication III for 3×2^2 factorial experiment confounding FAB

Block 1	**Block 2**
000	001
011	010
100	101
111	110
201	200
210	211

Thus the block content of the confounded 3×2^2 asymmetrical factorial design in 3×2 plots.

Table 4 : The confounded 3×2^2 asymmetrical factorial design in 3×2 plots.

Replication I		**Replication II**		**Replication III**	
Block 1	**Block 2**	**Block 1**	**Block 2**	**Block 1**	**Block 2**
000	001	000	001	000	001
011	010	011	010	011	010
101	100	101	100	100	101
110	111	110	111	111	110
200	201	201	200	201	200
211	210	210	211	210	211

In the above design, in Replication I, (000, i.e., (1)) is present in Block 1, so B_1 is the principal block and B_2 is the non-principal block. The non-principal block contains single letters with a power of one or two: (001, i.e., b), (010, i.e., a), and (100, i.e., f). Their multiplication is fab, therefore the confounded interaction is (fab), and the effect of FAB is confounded in all replications.

4.2 Construction of Blocks in 4×2^2 Asymmetrical Factorial Design in 4×2 Plots

Consider factor F having four levels 0, 1, 2 and 3 and two more factors A and B, each having two levels: 0 and 1. To obtain replications for this 4×2^2 factorial experiment, in which the highest-order interaction FAB is to be confounded, the method given in Section 2.1 is applied stepwise as follows:

Step 1: All the possible treatment combinations for this asymmetrical design are:

$$000 \quad 001 \quad 010 \quad 011 \quad 100 \quad 101 \quad 110 \quad 111 \quad 200 \quad 201 \quad 210 \quad 211 \quad 300 \quad 301 \quad 310 \quad 311$$

Step 2: Since interaction FAB is to be confounded, the defining equation is considered as:

$$X_1 + X_2 + X_3 = \begin{cases} 0 \\ 1 \end{cases} \qquad (\text{mod } (2))$$

The first replication using the defining equation is constructed as follows:

Table 5 : Replication I for 4×2^2 factorial experiment confounding FAB

Block 1 (=0 mod 2)	**Block 2 (=1 mod 2)**
000	001
011	010
101	100
110	111
200	201
211	210
301	300
310	311

Step 3: In this problem, the factor F is having 4 levels. Consider the levels greater than 2, i.e., the 3^{rd} and 4^{th} levels, denoted by 2 and 3. Therefore, in the replication obtained in Step 2, consider the treatment combinations 301, 310, 200, and 211 from Block 1 and interchange them with treatment combinations 300, 311, 201, and 210 from Block 2, where the factor F is at the highest and second-highest levels. The remaining treatment combinations in both blocks are the same as in Replication I. The new replication obtained is listed below:

Table 6 : Replication II for 4×2^2 factorial experiment confounding FAB

Block 1	**Block 2**
000	001
011	010
101	100
110	111
201	200
210	211
300	301
311	310

Step 4: From the replication obtained in Step 3, consider the treatment combinations 300, 311, 101, and 110 from Block 1 and interchange them with treatment combinations 301, 310, 100, and 111 from Block 2, where the factor F is at the highest and second-last highest levels. The remaining treatment combinations in both blocks are the same as in Replication I. The new replication obtained is listed below:

Table 7 : Replication III for 4×2^2 factorial experiment confounding FAB

Block 1	**Block 2**
000	001
011	010
100	101
111	110
201	200
210	211
301	300
310	311

Thus the block contents of the confounded 4×2^2 asymmetrical factorial design in 4×2 plots is as shown in Table 8.

Table 8 : The confounded 4×2^2 asymmetrical factorial design in 4×2 plots.

Replication I		**Replication II**		**Replication III**	
Block 1	**Block 2**	**Block 1**	**Block 2**	**Block 1**	**Block 2**
000	001	000	001	000	001
011	010	011	010	011	010
101	100	101	100	100	101
110	111	110	111	111	110
200	201	201	200	201	200
211	210	210	211	210	211
301	300	300	301	301	300
310	311	311	310	310	311

4.3 Construction of Blocks in 2×3^2 Asymmetrical Factorial Design in 2×3 Plots

Consider factor F having two levels: 0 and 1, and two factors A and B, each having three levels: 0, 1, and 2. To obtain replications for this 2×3^2 factorial experiment, in which the highest-order interaction FAB is confounded, the method given in Section 2.2 is applied stepwise as follows:

Step 1: All the possible treatment combinations for this asymmetrical design are:

000 001 002 010 011 012 020 021 022 100 101 102 110 111 112 120 121 122

Step 2: Now consider the highest-order interaction FAB is confounded. The First Replication is obtained by using the defining equation as:

$$X_1 + X_2 + X_3 = \begin{cases} 0 \\ 1 \\ 2 \end{cases} \pmod{3}$$

The first replication using the defining equation is constructed as follows:

Table 9 : Replication I for 2×3^2 factorial experiment confounding FAB

Block 1 (= 0 mod 3) (B_1)	**Block 2 (= 1 mod 3) (B_2)**	**Block 3 (= 2 mod 3) (B_3)**
000	001	002
012	010	011
021	022	020
102	100	101
111	112	110
120	121	122

Step 3: For the second replication, we replace the treatment combinations containing highest level of the factor F from block 1 (B_1) to Block 2 (B_2), B_2 to B_3, and B_3 to B_1, and the remaining treatment combinations are same as in Replication I. The new replication obtained is as shown in Table 10.

Table 10 : Replication II for 2×3^2 factorial experiment confounding FA^2B^2

Block 1 (B_1)	**Block 2 (B_2)**	**Block 3 (B_3)**
000	001	002
012	010	011
021	022	020
101	102	100
110	111	112
122	120	121

Thus Block Contents of the confounded as shown in Table 11.

Table 11 : The confounded 2×3^2 asymmetrical factorial design in 2×3 plots

Replication I			**Replication II**		
B_1	B_2	B_3	B_1	B_2	B_3
000	001	002	000	001	002
012	010	011	012	010	011
021	022	020	021	022	020
102	100	101	101	102	100
111	112	110	110	111	112
120	121	122	122	120	121

In Table 11, Replication I; B_1 is the principal block and B_2, B_3 are non-principal blocks. In block B_3, single letters with a power of one or two are (020 i.e. a^2) and (002 i.e. b^2) their multiplication is a^2b^2, but equation (1) is not satisfied.

Now consider block B_2 and find all the single letters which have a power of one or two. Such letters are b, a, and f thus combining these treatment combinations interaction effect FAB is obtained. Also this satisfies equation (1) $X_1 + X_2 + X_3 = 0 \mod (3)$ in the principal block, and equation (2) $X_1 + X_2 + X_3 = j \mod (3), j = 2, 3$ in the non-principal blocks. Thus in replication I, effect of FAB is confounded. In Replication II, block B_1 is the principal block and B_2, B_3 are non-principal blocks. In B_3, single letters with a power of one or two are (020 i.e. a^2), (002 i.e. b^2), and (100 i.e. f). their multiplication is fa^2b^2. Also, this satisfies the equation (1) $X_1 + X_2^2 + X_3^2 = 0$ for the principal block. Thus in Replication II, FA^2B^2 is confounded.

5 Conclusion

In this research article, we have proposed a simple method for the construction of blocks in confounded asymmetrical factorial design of the particular type. The advantage of this over existing methods of asymmetrical factorial designs is that there is neither need to convert the asymmetrical to symmetrical design nor the need to use fractional design. Another advantage of the proposed method is that these will produce balanced designs which are in line with Das and Giri's design. In this paper, methods to identify the confounded effects in each replication for asymmetrical factorial designs are also proposed.

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