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Modified Alternative to Ridge Estimator

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Abstract
The Ordinary Least Square (LS) estimator remains Best Linear Unbiased Estimator (BLUE) when all the assumptions surrounding it stay intact, but at an iota of violation of the assumptions, it becomes inefficient and unstable. Multicollinearity is one of the causes of the violation of the assumptions. The ridge regression estimator given by (Hoerl & Kannard (1970)) is consistently attractive shrinkage method to reduce the effects of multicollinearity. In ridge estimation, the estimation of ridge parameter (k) is vital. However, there is still no way to compute its optimal value. In this article, we suggest a new method of finding the regression parameters which completely avoids the computation of "k". New method compared to ridge estimators at different choices ridge parameters by simulation techniques in terms of Mean Square Error (MSE). Monte-Carlo simulation technique indicated that the proposed estimator performs better than LS estimators as well as Hoerl and Kennard's ridge estimator at 149 different choice of ridge parameters reviewed in this article.

Keywords: Multicollinearity, Ridge estimator, Mean square error, Simulation.

1 Introduction

The LS estimator is the most popularly used estimator to estimate the parameters in a regression model. In multiple regression analysis situations where the explanatory variables are highly inter correlated, referred to as the problem of multicollinearity. Presence of multicollinearity has posed a great threat to the LS estimator in linear regression analysis. Several erudite scholars have proposed several estimators to combat the problem of multicollinearity in the literature. (Hoerl & Kannard (1970)) introduced the ridge regression (RR) estimator as an alternative to the LS estimator. The ridge regression estimator is obtained by simply adding an equal amount $k > 0$ to each diagonal element of the correlation matrix in the LS estimator, such that the reduction in the variance term of the slope parameter is greater than the increase in the squared bias of it. At this stage, the main interest lies in finding a value of the ridge parameter, say "k". A lot of ways of estimating the ridge parameter "k" have been proposed. Most recently, (Mermi et al. (2024)) considered 366 estimators for estimating ridge or shrinkage parameter k for regression models. To mention a few, 149 ridge parameters considered in this article are listed in the **Appendix**. However, there is no explicit formula for this ridge parameter.The conventional wisdom is that no single method would be uniformly better than all the others.

Recently, to estimate regression parameters (Dorugade (2016)) introduces two estimators without estimating the ridge parameter "k" unlike other existing estimators. Using simulation study they have shown that these estimators have desirable robustness features against the multicollinearity, heteroscedasticity and outlier situations. On the similar line in this paper a new estimator obtained by modifying estimator given by (Dorugade (2016)) to approximate the regression parameters and then compared it with RR estimators at different well known existing ridge parameters. The other parts of this article are as follows: In Section 2, we define model and parameter estimation methods with their bias and MSE. In Section 3, we have proposed a new estimator and discuss its some properties. In Section 4, performances of the proposed estimators evaluated and compared to other estimators which are considered under this study. Finally, article ends with some concluding remarks.

2 Model and Estimators

Consider, widely used linear regression model

$$Y = X\beta + \varepsilon, \tag{1}$$

where, ***X*** is an (n$\times$p) data matrix of predictors which are non stochastic, Y is a (n$\times$1) response vector, β is a (p$\times$1) vector of unknown regression coefficients, and ϵ be vector of random disturbances, such that $E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon') = \sigma^2 I_n$. σ^2 is the unknown variance parameter. I_n is identity matrix of order "n". For the sake of convenience, we assume that the matrix X and response variable Y are standardized in such a way that $X^{'}X$ is a non-singular correlation matrix and $X^{'}Y$ is the correlation between X and Y.

Let $\wedge$ and T be the matrices of eigen values and eigen vectors of $X^{'}X$, respectively, satisfying $T^{'}X^{'}XT = \wedge$ = diagonal (λ_1,λ_2,...,λ_p), where λ_i being the ith eigen value of $X^{'}X$ and $T^{'}T = TT^{'} = I_p$. We obtain the equivalent model

$$Y = Z\alpha + \varepsilon, \tag{2}$$

where Z = XT , it implies that $Z^{'}Z = \wedge$, and $\alpha = T^{'}\beta$ (see(Montgomery etal. (2021)))
Then LS estimator of α is given by

$$\hat{\alpha}_{LS} = (Z^{'}Z)^{-1}Z^{'}Y = \wedge^{-1}Z^{'}Y. \tag{3}$$

Therefore, LS estimator of β is given by

$$\hat{\beta}_{LS} = T\hat{\alpha}_{LS}.$$

2.1 Ridge Regression Estimator (RR)

RR estimator suggested by (Hoerl & Kannard (1970)) is one of the widely used biased regression methods, among all the biased regression methods such as, principal component regression (PCR), partial least squares regression (PLSR), etc., as it yields more stable estimates to the regression coefficients under severe multicollinearity when the number of predictors less than the number of observations.
It is given as:

$$\hat{\alpha}_{RR} = \left[I - k\left(\wedge + kI\right)^{-1}\right]\,\hat{\alpha}_{LS} \tag{4}$$

Therefore, RR estimator of β is given by

$$\hat{\beta}_{RR} = T\hat{\alpha}_{RR}$$

and MSE of $\hat{\alpha}_{RR}$is

$$MSE(\hat{\alpha}_{RR}) = \hat{\sigma}^2\sum_{i=1}^{p}\lambda_i/(\lambda_i + k)^2 + k^2\sum_{i=1}^{p}\hat{\alpha}_i{}^2/(\lambda_i + k)^2 \tag{5}$$

We observe that, when $k = 0$ in (5), MSE of LS estimator of α is recovered. Hence

$$MSE(\hat{\alpha}_{LS}) = \hat{\sigma}^2\sum_{i=1}^{p}1/\lambda_i \tag{6}$$

(Hoerl et al. (1975)) suggested that, the value of "k" is chosen small enough, for which the mean squared error of ridge estimator, is less than the mean squared error of LS estimator. Some of the well known methods for choosing ridge parameters used to compute $\hat{\alpha}_{RR}$ are listed in **Appendix**.

2.2 Alternative Estimator

(Dorugade (2016)) gives two different estimators of α respectively denoted by $\hat{\alpha}_{A1}$ and $\hat{\alpha}_{A2}$ given as:

$$\hat{\alpha}_{iA1} = \frac{(n-1)\,\hat{\sigma}}{S_i} \quad i = 1, 2, ..., p \tag{7}$$

$$\hat{\alpha}_{iA2} = \frac{\hat{\sigma}}{\sqrt{S_i}} \quad i = 1, 2, ..., p \tag{8}$$

where, $\hat{\sigma}^2$ is the LS estimator of σ^2 i.e. $\hat{\sigma}^2 = \frac{Y'Y - \hat{\alpha}'_{LS} Z'Y}{n-p-1}$ and

$S_i = \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2 \quad i = 1, 2, ..., p$

Therefore, proposed alternative estimators of β are respectively given by $\hat{\beta}_{iA1} = T\hat{\alpha}_{iA1}$ and $\hat{\beta}_{iA2} = T\hat{\alpha}_{iA2}$. MSE of $\hat{\alpha}_{A1}$ and $\hat{\alpha}_{A2}$ are

$$MSE\left(\hat{\alpha}_{A1}\right) = (n-1)^2 \hat{\sigma}^4 \sum_{i=1}^{p} \left[\frac{1}{\lambda_i (S_{1i})^2}\right] + \sum_{i=1}^{p} (B_{1i} \hat{\alpha}_i)^2 i = 1, 2, ..., p \tag{9}$$

$$MSE\left(\hat{\alpha}_{A2}\right) = \hat{\sigma}^4 \sum_{i=1}^{p} \left[\frac{S_i}{\lambda_i (S_{1i})^2}\right] + \sum_{i=1}^{p} (B_{2i} \hat{\alpha}_i)^2 i = 1, 2, ..., p \tag{10}$$

where, , $B_2 = \left(\frac{\hat{\sigma}\sqrt{S}}{S_1} T - I_{p\times 1}\right)$

and $S_{1i} = \sum_{j=1}^{n} y_j \left(x_{ij} - \bar{x}_i\right)$ for $i = 1, 2, ..., p \quad j = 1, 2, ..., n$

where, $\hat{\alpha}_i$ is the ith element of $\hat{\alpha}_{LS}$, $i = 1, 2, ..., p$

3 Proposed Estimator

To resolve the problem of choosing optimal ridge parameter in ridge regression (Dorugade (2016)), introduces two estimators of α denoted by $\hat{\alpha}_{A1}$ and $\hat{\alpha}_{A2}$ respectively given in (7) and (8) alternative to ridge estimator by avoiding computation of ridge parameter k. These estimators are based on suitable estimator of σ. It is well known that correlation coefficients between the regressors as well as eigen values of $\left(X'X\right)$ are plays an important role in detection of multicollinearity among the various regressor variables. Also, most of the researchers suggested ridge parameters based on these eigen values. By motivating these applications of correlation coefficients and eigen values, in this article we suggested modification in $\hat{\alpha}_{A2}$ to introduce a new estimator of α based on correlation coefficients (r_i) between response and regressor variables, and eigen values (λ_i) of $\left(X'X\right)$.

Let $r_i = r(Y, Z_i) \quad i = 1, 2, ..., p$ is the vector of order $1 \times p$ of correlation coefficients between response and regressor variables and λ_i being the ith eigen value of $\left(X'X\right)$.

New estimator called as modified alternative estimator denoted by $\hat{\alpha}_{MA}$ and given by

$$\hat{\alpha}_{MA} = r_i \lambda_i \frac{(n-1)\hat{\sigma}}{S_i} \quad i = 1, 2, ..., p \tag{11}$$

where, $\hat{\sigma}^2$ is the LS estimator of σ^2 i.e. $\hat{\sigma}^2 = \frac{Y'Y - \hat{\alpha}'_{LS} Z'Y}{n-p-1}$ and

$S_i = \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2 \quad i = 1, 2, ..., p$

Therefore, a proposed modified alternative estimator of β is given by $\hat{\beta}_{MA} = T\hat{\alpha}_{MA}$.

New estimator can expressed as the function of LS estimator given as

$$\hat{\alpha}_{MA} = r_i \lambda_i \frac{(n-1)\hat{\sigma}}{S_i} \quad i = 1, 2, ..., p$$

$$= r_i\lambda_i \frac{(n-1)\,\hat{\sigma}}{S_{1i}}\,\hat{\beta}_{iLS}$$

or $\hat{\alpha}_{MA} = r_i\lambda_i \frac{(n-1)\,\hat{\sigma}}{S_{1i}} T\,\hat{\alpha}_i \qquad i = 1, 2, ..., p$

where, $\hat{\alpha}_i$ is the ith element of $\hat{\alpha}_{LS}$, $i = 1, 2, ..., p$.

3.1 Bias, Variance and MSE of $\hat{\alpha}_{MA}$

Bias of $\hat{\alpha}_{MA}$:

$$Bias\,(\hat{\alpha}_{MA}) = E[\hat{\alpha}_{MA}] - \alpha$$

$$= B\alpha \text{ where, } B = \left(r\Lambda \frac{(n-1)\,\hat{\sigma}}{S_1} T - I_{p\times 1}\right)$$

Variance of $\hat{\alpha}_{MA}$:

$$Var\,(\hat{\alpha}_{MA}) = E[(\hat{\alpha}_{MA} - E(\hat{\alpha}_{MA}))\,(\hat{\alpha}_{MA} - E(\hat{\alpha}_{MA}))']$$

$$= C\sigma^2\Lambda^{-1}\,C' \text{ where, } C = \left(r\Lambda \frac{(n-1)\,\hat{\sigma}}{S_1} T\right)$$

where $\sigma^2\wedge^{-1} = \sigma^2(Z'Z)^{-1} = V(\hat{\alpha}_{LS}) = MSE(\hat{\alpha}_{LS})$

MSE of $\hat{\alpha}_{MA}$:

$$MSE\,(\hat{\alpha}_{MA}) = V(\hat{\alpha}_{MA}) + [Bias\,(\hat{\alpha}_{MA})\,]^2$$

$$= C\,\sigma^2\Lambda^{-1}\,C' + B\alpha\alpha' B$$

or

$$MSE\,(\hat{\alpha}_{MA}) = (n-1)^2\hat{\sigma}^4 \sum_{i=1}^{p} \lambda_i \left(\frac{r_i}{S_{1i}}\right)^2 + \sum_{i=1}^{p} (B_i\hat{\alpha}_i)^2 \tag{12}$$

where, $\hat{\alpha}_i$ is the ith element of $\hat{\alpha}_{LS}$, $i = 1, 2, ..., p$

3.2 The Performance of the $\hat{\alpha}_{MA}$

This section compares the performance of the $\hat{\alpha}_{MA}$with the, $\hat{\alpha}_{LS}$ and $\hat{\alpha}_{RR}$ using smaller MSE criteria.

3.2.1 Comparison Between the $\hat{\alpha}_{MA}$ and $\hat{\alpha}_{LS}$

Using (6) and (12) we investigate the following difference:

$$MSE\,(\hat{\alpha}_{LS}) - MSE\,(\hat{\alpha}_{MA}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} - (n-1)^2\hat{\sigma}^4 \sum_{i=1}^{p} \lambda_i \left(\frac{r_i}{S_{1i}}\right)^2 - \sum_{i=1}^{p} (B_i\hat{\alpha}_i)^2$$

Therefore, $MSE\,(\hat{\alpha}_{LS}) \;\geq\; MSE\,(\hat{\alpha}_{MA})$ if and only if

$$(S_{1i}\hat{\sigma})^2 \geq \;\lambda_i\,\left[\lambda_i(r_i(n-1)\hat{\sigma}^2)^2 + (S_{1i}B_i\hat{\alpha}_i)^2\right]$$

3.2.2 Comparison Between the $\hat{\alpha}_{MA}$ and $\hat{\alpha}_{RR}$

In order to compare $\hat{\alpha}_{MA}$ with $\hat{\alpha}_{RR}$ in the MSE sense, using (5) and (12) we investigate the following difference:

$$MSE\,(\hat{\alpha}_{RR}) - MSE\,(\hat{\alpha}_{MA}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i+k)^2} + k^2 \sum_{i=1}^{p} \frac{\hat{\alpha}_i{}^2}{(\lambda_i+k)^2} - (n-1)^2\hat{\sigma}^4 \sum_{i=1}^{p} \lambda_i \left(\frac{r_i}{S_{1i}}\right)^2 - \sum_{i=1}^{p} (B_i\hat{\alpha}_i)^2$$

Therefore, $MSE\,(\hat{\alpha}_{RR}) \;\geq\; MSE\,(\hat{\alpha}_{MA})$ if and only if

$$S_{1i}^2\left[\lambda_i\hat{\sigma}^2 + (k\hat{\alpha}_i)^2\right] \geq \;(\lambda_i+k)^2\;\left[\lambda_i(r_i(n-1)\hat{\sigma}^2)^2 + (S_{1i}B_i\hat{\alpha}_i)^2\right]$$

4 Evaluation of the Performances of the Proposed and Review Estimators

Consequently, parameter estimation methods are based on LS estimators do not assure the desirable results in the presence of multicollinearity. The Monte Carlo experiment of this research was conducted using R-statistical programming language to evaluate the performance of the reviewed and proposed estimator compare to LS estimators. The following factors were used to conduct the simulation study with the aid of R-statistical programming codes.

4.1 ***Sample Size (n)***

To understand the effect of small and large samples on the performance of all estimators under consideration, the number of observations "n" took the values: 20, 50, 100, 400, and 1000.

4.2 ***Strength of Correlation among Explanatory Variables (ρ)***

In this study, to investigate the effects of different degrees of multicollinearity on the estimators, we consider different correlations, ρ = 0.*90*, 0.95, 0.99, 0.999 and 0.9999. Following, (McDonald & Galarneau (1975)) the explanatory variables are generated by

$$x_{ij} = (1-\rho^2)^{1/2}u_{ij} + \rho\, u_{ip}, \quad i = 1,2,...,n \quad j = 1,2,...,p.$$

Where, u_{ij} are independent standard normal pseudo-random numbers and ρ is specified so that the theoretical correlation between any two explanatory variables is given by ρ^2. Likewise, the response variable was generated using the equation $Y = X\beta + \varepsilon$.

4.3 The number of explanatory variables (p)

Explanatory variables, p = 2, 4 and 7 were used to obtain the best estimators with respect to MSE.

4.4 Variance of the error term (σ^2)

The variance of the error terms is taken as σ^2 = 1, 8, 30, 100 and 200.

4.5 ***Criteria for Measuring Goodness of an Estimator***

We use the MSE to measure the goodness of an estimator. Ridge estimators are proposed to have smaller MSE compared to LS estimators. The MSE and the predictive ability of the estimators were evaluated by computing

$$AMSE(\hat{\beta}) = \frac{1}{2000}\sum_{i=1}^{p}\sum_{s=1}^{2000}(\hat{\beta}_{is} - \beta_i)^2.$$

where, $\hat{\beta}_{is}$denote the estimator of the ith parameter in the sth replication and β_i, $i = 1,2,...,p$ are the true parameter values. 'β' parameter vectors are chosen arbitrarily with/or without intercepts. Values of "f$_m$" reported, indicates the frequency with which each estimator had the lowest $AMSE(\hat{\beta})$. We consider the method that leads to the maximum "f$_m$" to the best from the AMSE point of view.

4.6 Replications

The experiment is repeated 2000 times and computed "f$_m$" values for all estimators. The simulation is carried out for data exhibits with multicollinearity.

Part A:

Here, to investigate the performance of the suggested estimator $\hat{\beta}_{MA}$ compare to $\hat{\beta}_{LS}$, $\hat{\beta}_{A1}$, $\hat{\beta}_{A2}$ and $\hat{\beta}_{RR}$ estimators computed using different ridge parameters listed in **Appendix.** We consider, ρ = 0.99; p = 4; n = 20,50,100; σ^2 = 1, 8, 30,100 and 200. Two thousand simulations are run and AMSE of estimators were computed. For each simulation, the dependent variables are computed by the specified protocol. Values of "f$_m$" computed and reported in Table-1.

From Table-1, it is found that there is no explicit ridge parameter that performs well in every situation. The ridge estimators act differently in various sample sizes, dimensions and error variances. However, lead to which is the best estimator in each of different situation and which one of the best among all estimators we briefly summarized the results from Table-1 and reported in Table-2. In the following we listed findings of the results reported in Table-1.

1. In general, LS give better performances at low error variances and increasing sample sizes.

2. By increasing both the error variance and sample size performance of $\hat{\beta}_{A2}$ is improved.

3. Proposed estimator $\hat{\beta}_{MA}$ gives better performance not only for increasing the error variance but also for sample sizes. However, for each sample size at low error variances $\hat{\beta}_{MA}$ gives less performance compare to others.

4. Including $\hat{\beta}_{A1}$, $\hat{\beta}_{RR}$ at different choice of 'k' gives the mixed performance. There is no any particular 'k' for which $\hat{\beta}_{RR}$ gives continuously better performance than others.

Table 1 "f$_m$" values of LS and other estimators $(p = 4, \quad \rho = 0.99)$

Estimator	k	$n = 20$					50					100				
		$\sigma^2 = 1$	8	30	100	200	1	8	30	100	200	1	8	30	100	200
$\hat{\beta}_{MA}$	-	0	208	534	386	294	0	14	718	658	440	0	0	302	712	506
$\hat{\beta}_{A1}$	-	0	10	48	2	8	0	0	72	4	2	0	0	28	18	2
$\hat{\beta}_{A2}$	-	0	0	32	282	168	0	0	0	250	348	0	0	0	114	386
$\hat{\beta}_{RR}$	k_1	6	2	0	2	2	0	2	0	0	0	0	2	4	2	0
	k_2	26	2	0	2	10	34	10	2	2	2	16	28	4	0	0
	k_3	8	6	2	0	0	12	8	0	0	0	10	4	6	0	0
	k_4	0	16	6	2	0	0	4	8	2	0	0	0	2	2	2
	k_5	0	2	10	8	4	0	0	12	2	8	0	0	14	6	14
	k_6	0	2	0	0	0	2	0	0	0	0	4	4	0	0	0
	k_7	4	0	0	0	0	0	2	2	0	0	0	0	0	0	0
	k_8	2	10	16	10	2	0	16	40	4	4	0	0	28	18	16
	k_9	0	0	2	0	0	0	0	8	0	2	2	6	12	6	0
	k_{10}	6	0	0	0	0	4	4	2	0	0	2	10	6	2	2
	k_{11}	118	0	0	0	0	278	16	0	0	0	456	32	0	0	0
	k_{12}	68	12	0	0	0	156	34	2	2	0	72	28	2	0	4
	k_{13}	12	8	8	6	0	28	8	12	0	2	54	8	12	6	6
	k_{14}	0	16	10	8	16	0	0	14	12	20	0	0	18	24	10
	k_{15}	116	2	0	0	0	28	6	0	0	0	0	6	4	0	0
	k_{16}	4	6	2	0	0	0	4	0	0	0	0	36	0	0	0
	k_{17}	84	22	0	0	0	60	64	6	0	0	18	92	22	6	0
	k_{18}	6	4	4	0	0	0	6	4	0	0	4	12	0	0	2
	k_{19}	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
	k_{20}	18	0	0	0	0	18	0	0	0	0	18	0	0	0	0
	k_{21}	56	24	22	18	6	32	42	22	12	12	44	40	24	18	12
	k_{22}	30	20	8	2	2	24	18	22	8	2	30	50	22	16	4
	k_{23}	44	18	6	8	2	52	36	10	12	2	18	26	18	8	6
	k_{24}	0	6	4	0	0	0	2	4	0	0	0	4	0	0	0
	k_{25}	14	8	24	4	0	14	6	6	0	0	4	8	4	0	0
	k_{26}	8	8	6	0	0	6	32	6	0	0	4	30	6	0	0
	k_{27}	8	0	0	0	0	6	0	0	0	0	2	0	0	0	0
	k_{28}	0	0	0	4	24	0	0	0	0	10	0	0	0	0	0
	k_{29}	0	0	0	6	8	0	0	0	2	4	0	0	0	0	0
	k_{30}	0	0	0	6	8	0	0	0	0	4	0	0	0	0	2
	k_{31}	0	18	16	0	0	0	12	22	4	0	0	6	32	14	6
	k_{32}	0	26	26	6	12	0	4	14	6	20	0	2	6	6	20
	k_{33}	0	14	12	8	2	0	8	28	2	2	0	2	26	20	12

Continued....

	k_{34}	208	14	0	0	0	224	18	2	0	0	26	26	0	0	0
	k_{35}	18	2	0	0	0	4	4	0	0	0	10	8	2	0	0
	k_{36}	0	0	0	20	24	0	0	0	0	30	0	0	0	4	24
	k_{37}	0	0	16	16	14	0	0	4	20	24	0	0	0	16	18
	k_{38}	0	0	12	36	22	0	0	2	30	30	0	0	0	14	36
	k_{39}	0	54	10	6	4	0	34	8	6	4	0	28	42	12	2
	k_{40}	0	24	18	16	8	0	26	12	12	22	0	20	14	8	8
	k_{41}	4	52	16	10	0	0	36	22	6	4	0	26	58	20	8
	k_{42}	118	0	0	0	0	278	16	0	0	0	456	32	0	0	0
	k_{43}	0	42	16	0	0	0	32	20	0	0	0	20	30	0	0
	k_{44}	0	8	26	12	2	0	2	8	10	4	0	0	0	6	0
	k_{45}	0	26	10	14	0	2	16	16	2	2	0	22	38	12	2
	k_{46}	8	22	2	0	0	4	44	0	0	0	0	64	4	0	0
	k_{47}	0	0	0	4	20	0	0	0	0	2	0	0	0	0	6
	k_{48}	0	0	6	22	60	0	0	0	8	12	0	0	0	0	0
	k_{49}	0	0	0	0	10	0	0	0	0	8	0	0	0	0	0
	k_{50}	18	6	2	0	0	12	16	4	0	0	16	12	4	0	0
	k_{51}	0	0	14	4	2	0	0	10	22	10	0	0	2	2	8
	k_{52}	0	8	42	16	6	0	0	12	22	36	0	0	0	12	24
	k_{53}	56	18	12	6	10	32	20	6	12	12	36	16	6	24	10
	k_{54}	16	10	6	2	2	8	6	8	2	2	4	20	12	6	4
	k_{55}	28	12	0	0	0	4	10	8	4	2	16	26	14	4	2
	k_{56}	8	0	0	0	0	4	4	0	0	0	6	2	0	0	0
	k_{57}	12	16	8	0	0	12	4	10	0	0	8	16	0	0	0
	k_{58}	8	6	8	4	4	10	8	2	2	2	10	8	10	4	2
	k_{59}	6	2	4	0	0	6	2	14	4	2	6	6	4	6	6
	k_{60}	14	0	2	0	0	8	4	2	0	0	8	10	0	4	0
	k_{61}	58	20	14	4	4	36	34	10	2	2	24	26	12	6	2
	k_{62}	8	6	0	0	0	14	12	4	2	0	2	4	6	0	0
	k_{63}	0	2	0	0	0	2	0	0	0	0	2	0	0	0	0
	k_{64}	14	6	0	0	0	8	8	4	0	0	10	14	10	0	0
	k_{65}	0	0	0	0	0	0	0	0	0	0	2	4	0	0	0
	k_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	k_{67}	26	6	0	0	0	12	10	0	0	0	2	6	0	0	0
	k_{68}	12	6	0	0	0	4	10	0	0	0	6	10	8	2	2
	k_{69}	10	0	0	0	0	8	6	0	2	0	4	12	2	0	4
	k_{70}	8	2	0	0	0	22	0	0	0	0	8	2	0	0	0
	k_{71}	0	0	6	0	0	0	0	2	0	0	0	0	4	0	0
	k_{72}	0	2	4	4	0	0	0	6	0	0	0	0	4	2	0
	k_{73}	0	2	4	0	0	0	0	0	0	0	0	0	8	2	2
	k_{74}	72	106	26	0	2	22	134	26	4	0	4	156	30	6	2
	k_{75}	0	0	16	32	16	0	0	0	22	14	0	0	0	12	22
	k_{76}	0	2	14	10	4	0	0	8	4	8	0	0	4	6	16
	k_{77}	18	12	0	0	0	0	22	2	0	0	0	40	2	0	0
	k_{78}	8	8	6	6	2	6	8	2	2	2	4	12	12	2	4
	k_{79}	6	8	4	4	4	0	4	10	2	4	2	8	16	10	2
	k_{80}	6	34	26	12	8	0	14	38	32	10	0	20	68	20	36
	k_{81}	34	24	4	6	2	22	30	4	0	0	8	50	18	8	0
	k_{82}	12	6	4	0	2	2	8	8	4	2	12	20	12	12	2
	k_{83}	0	28	28	10	6	2	10	44	34	16	4	10	66	30	28
	k_{84}	28	14	0	0	0	16	34	4	0	0	14	46	4	4	0

Continued. . . .

	k_{85}	0	14	24	4	2	0	10	50	30	16	2	14	76	18	24
	k_{86}	40	18	10	2	0	14	24	18	2	4	12	52	22	20	6
	k_{87}	18	66	48	16	12	0	54	80	40	26	0	72	132	44	50
	k_{88}	16	4	0	0	0	8	2	0	0	0	8	0	0	0	0
	k_{89}	2	0	0	0	0	4	0	0	0	0	6	0	0	0	0
	k_{90}	8	0	0	0	0	4	0	0	0	0	0	2	0	0	0
	k_{91}	0	0	0	208	426	0	0	0	40	246	0	0	0	6	50
	k_{92}	0	0	0	2	4	0	0	0	0	0	0	0	0	0	0
	k_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	k_{94}	0	2	10	8	4	0	0	12	2	8	0	0	14	6	14
	k_{95}	2	22	12	4	6	0	44	20	4	4	0	48	26	8	2
	k_{96}	4	86	12	2	0	0	78	32	6	0	0	64	34	16	6
	k_{97}	8	20	2	0	0	4	44	0	0	0	0	62	4	0	0
	k_{98}	0	4	22	24	16	0	8	4	20	18	0	2	8	6	14
	k_{99}	0	8	32	22	14	0	4	30	20	20	0	0	16	20	24
	k_{100}	6	34	40	12	10	0	26	18	20	10	2	12	22	14	12
	k_{101}	0	20	26	8	10	0	16	20	24	8	0	4	16	22	32
	k_{102}	0	28	20	12	8	0	4	26	18	12	0	2	14	28	16
	k_{103}	6	14	0	0	0	0	32	6	0	0	0	48	2	0	0
	k_{104}	34	18	8	0	0	30	16	4	0	0	36	26	2	0	0
	k_{105}	2	32	32	6	0	0	52	50	20	4	0	36	46	12	2
	k_{106}	0	26	24	14	14	0	32	12	20	6	2	28	42	20	10
	k_{107}	0	12	34	14	12	0	26	8	12	10	0	14	30	34	18
	k_{108}	0	40	4	4	0	0	42	22	0	0	0	48	34	0	0
	k_{109}	26	4	4	0	0	12	18	8	0	0	12	10	0	0	0
	k_{110}	46	48	18	4	0	60	64	6	0	0	80	60	20	2	0
	k_{111}	40	8	4	0	0	30	14	8	0	0	18	20	0	0	0
	k_{112}	0	0	2	6	2	0	0	0	0	2	0	0	0	0	0
	k_{113}	0	0	2	10	16	0	0	0	6	10	0	0	0	0	2
	k_{114}	0	0	4	10	24	0	0	0	0	10	0	0	0	0	0
	k_{115}	118	40	16	2	0	72	112	14	0	0	86	114	28	0	0
	k_{116}	102	16	0	0	0	148	8	2	0	0	148	10	2	0	0
	k_{117}	0	28	14	0	0	0	34	52	4	2	0	34	44	4	0
	k_{118}	26	8	2	0	0	14	18	0	0	0	72	32	4	0	0
	k_{119}	0	0	0	6	18	0	0	0	6	4	0	0	0	0	2
	k_{120}	0	0	4	50	80	0	0	0	12	30	0	0	0	4	12
	k_{121}	0	0	12	18	16	0	0	0	24	22	0	0	0	20	26
	k_{122}	0	26	26	6	2	0	10	10	14	4	0	2	60	36	22
	k_{123}	0	0	66	16	6	0	0	22	46	12	0	0	6	26	20
	k_{124}	0	34	18	30	44	0	14	2	30	16	0	20	12	20	12
	k_{125}	0	0	0	0	2	0	0	0	2	2	0	0	0	2	0
	k_{126}	0	10	20	28	12	0	20	12	16	14	0	6	26	10	10
	k_{127}	0	0	4	8	14	0	0	0	4	28	0	0	0	0	10

Continued. . . .

	k_{128}	0	16	14	24	8	0	12	20	14	4	0	8	24	26	16
	k_{129}	0	0	6	42	22	0	0	0	12	30	0	0	0	0	2
	k_{130}	0	30	30	12	0	0	46	16	32	16	0	34	40	40	14
	k_{131}	0	0	56	16	12	0	0	2	46	34	0	0	0	20	46
	k_{132}	0	18	16	0	0	0	12	22	4	0	0	6	32	14	6
	k_{133}	0	0	0	6	12	0	0	0	0	0	0	0	0	0	2
	k_{134}	0	38	10	8	2	0	42	22	30	4	0	20	30	42	14
	k_{135}	0	0	12	36	46	0	0	0	38	66	0	0	0	8	54
	k_{136}	0	34	22	4	2	0	22	20	14	2	0	16	30	14	4
	k_{137}	0	0	18	42	24	0	0	0	20	54	0	0	0	12	40
	k_{138}	0	36	32	10	10	0	52	22	20	16	0	8	32	40	22
	k_{139}	0	2	46	20	14	0	0	6	52	14	0	0	2	56	28
	k_{140}	0	26	20	10	4	0	6	10	8	0	0	10	16	10	6
	k_{141}	0	0	8	18	18	0	0	0	14	18	0	0	0	14	12
	k_{142}	0	22	16	8	2	0	16	10	14	4	0	16	16	24	18
	k_{143}	0	0	14	24	20	0	0	0	26	34	0	0	0	14	22
	k_{144}	0	6	14	30	22	0	4	8	24	30	0	6	2	24	16
	k_{145}	2	0	0	142	208	0	0	0	14	42	0	2	0	2	30
	k_{146}	0	20	22	0	8	0	30	18	12	4	0	8	18	10	8
	k_{147}	64	38	12	8	18	44	36	14	8	0	54	38	14	14	16
	k_{148}	0	54	0	0	0	0	42	24	0	0	0	30	24	16	6
	k_{149}	46	2	0	0	0	58	16	0	0	0	58	14	4	0	0
$\hat{\beta}_{LS}$	-	142	2	0	0	0	282	6	0	0	0	478	26	0	0	0

From Table-1 we observed that for fixed value of '$\rho = 0.99$' and p=4 changes the values of n and σ^2. There are 15 set of (ρ, p, n,σ^2) values. These are arranged as (0.99, 4, 20, 1), (0.99, 4, 20, 8),…, (0.99, 4, 100, 200) and numbered as 1, 2,…, 15 respectively.

In Table-2 we have listed first five preferably estimators according to values of 'f_m' in decreasing order over all 15 set. Then count 'b' as the number of times each estimator appeared at first preference only, over all 15 sets. Highest the value of 'b' the corresponding estimator is the best among all the estimators over all 15 sets.

Table 2 Preference list of estimators ($\rho = 0.99,\ p = 4$)

$n = 20$					50				
$\sigma^2 = 1$	8	30	100	200	1	8	30	100	200
$\hat{\beta}_{RR}(k_{34})$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{RR}(k_{91})$	$\hat{\beta}_{LS}$	$\hat{\beta}_{RR}(k_{74})$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$
$\hat{\beta}_{LS}$	$\hat{\beta}_{RR}(k_{74})$	$\hat{\beta}_{RR}(k_{123})$	$\hat{\beta}_{A2}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{RR}(k_{42})$	$\hat{\beta}_{RR}(k_{115})$	$\hat{\beta}_{RR}(k_{87})$	$\hat{\beta}_{A2}$	$\hat{\beta}_{A2}$
$\hat{\beta}_{RR}(k_{115})$	$\hat{\beta}_{RR}(k_{96})$	$\hat{\beta}_{RR}(k_{131})$	$\hat{\beta}_{RR}(k_{91})$	$\hat{\beta}_{RR}(k_{145})$	$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{96})$	$\hat{\beta}_{A1}$	$\hat{\beta}_{RR}(k_{139})$	$\hat{\beta}_{RR}(k_{91})$
$\hat{\beta}_{RR}(k_{42})$	$\hat{\beta}_{RR}(k_{87})$	$\hat{\beta}_{RR}(k_{87})$	$\hat{\beta}_{RR}(k_{145})$	$\hat{\beta}_{A2}$	$\hat{\beta}_{RR}(k_{34})$	$\hat{\beta}_{RR}(k_{110})$	$\hat{\beta}_{RR}(k_{117})$	$\hat{\beta}_{RR}(k_{131})$	$\hat{\beta}_{RR}(k_{135})$
$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{148})$	$\hat{\beta}_{A1}$	$\hat{\beta}_{RR}(k_{120})$	$\hat{\beta}_{RR}(k_{120})$	$\hat{\beta}_{RR}(k_{12})$	$\hat{\beta}_{RR}(k_{17})$	$\hat{\beta}_{RR}(k_{105})$	$\hat{\beta}_{RR}(k_{123})$	$\hat{\beta}_{RR}(k_{137})$

$n = 100$				
$\sigma^2 = 1$	8	30	100	200
$\hat{\beta}_{LS}$	$\hat{\beta}_{RR}(k_{74})$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$
$\hat{\beta}_{RR}(k_{45})$	$\hat{\beta}_{RR}(k_{115})$	$\hat{\beta}_{RR}(k_{87})$	$\hat{\beta}_{A2}$	$\hat{\beta}_{A2}$
$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{17})$	$\hat{\beta}_{RR}(k_{85})$	$\hat{\beta}_{RR}(k_{139})$	$\hat{\beta}_{RR}(k_{135})$
$\hat{\beta}_{RR}(k_{116})$	$\hat{\beta}_{RR}(k_{87})$	$\hat{\beta}_{RR}(k_{80})$	$\hat{\beta}_{RR}(k_{87})$	$\hat{\beta}_{RR}(k_{91})$
$\hat{\beta}_{RR}(k_{115})$	$\hat{\beta}_{RR}(k_{96})$	$\hat{\beta}_{RR}(k_{83})$	$\hat{\beta}_{RR}(k_{134})$	$\hat{\beta}_{RR}(k_{87})$

From Table -2 we observed that our suggested estimator $\hat{\beta}_{MA}$ is appearing 9 times out of 15 at the first preference hence having maximum value of 'b' as compare to other estimators. Finally, we conclude that $\hat{\beta}_{MA}$ produce better results compare to other estimators which are considered under this study.

Part B:

In the same vein, to compute and represent the results under various degree of multicollinearity i.e., at low, moderate and high degree of multicollinearity, we consider ρ = 0.90, 0.95, 0.99, 0.999 and 0.9999. Here input values are p (= 2, 4, 7), n (= 20, 50, 100, 400, 1000) and σ^2(= 1, 8, 30, 100, 200). These input values are ordered according to the increase of values. For each selected ρ, for fixed value of 'ρ' changes the values of p, n and σ^2. There are 75 set of (ρ, p, n,σ^2) values. These are arranged as (0.90, 2, 20, 1), (0.90, 2, 20, 8),. . . , (0.90,7,1000,200) and numbered as 1, 2,. . . , 75 respectively. For arbitrarily chosen parameter vectors 'β', values of "f$_m$" are computed for all the 75 possible sets. According to values of 'f$_m$' in decreasing order over all 75 sets we short listed the best preferably 5 estimators for each choice of 'ρ'. Then count the value of 'b' for each estimator appears at first preference over all 75 sets and these results are reported in Table-3.

Table 3 Preference list of estimators

$\rho = 0.90$	0.95	0.99	0.999	0.9999
$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$	$\hat{\beta}_{MA}$
$\hat{\beta}_{RR}(k_{42})$	$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{42})$	$\hat{\beta}_{RR}(k_{107})$	$\hat{\beta}_{RR}(k_{87})$
$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{87})$
$\hat{\beta}_{RR}(k_{10})$	$\hat{\beta}_{RR}(k_{11})$	$\hat{\beta}_{RR}(k_{10})$	$\hat{\beta}_{RR}(k_{106})$	$\hat{\beta}_{RR}(k_{106})$
$\hat{\beta}_{RR}(k_{121})$	$\hat{\beta}_{RR}(k_{74})$	$\hat{\beta}_{RR}(k_{80})$	$\hat{\beta}_{RR}(k_{123})$	$\hat{\beta}_{RR}(k_{123})$

From Table-3, we observe that our suggested estimator $\hat{\beta}_{MA}$ is not sensitive to the level of multicollinearity. At each level of multicollinearity it is at the first preference among the preference list of five estimators.

Finally, although there are many strategies for choosing an optimal value for "k", there is no consensus regarding the best or most general way to choose "k". Hence, suggested estimator in this article is better alternative to ridge estimator to cope with the problem of multicollinearity in linear regression.

5 Conclusion

Suggested improved alternative estimator $\hat{\beta}_{MA}$ is totally free from unknown ridge parameter "k", which obviously reduces the computational part of "k" and results in simplest method of estimation of β. Finally, in the presence of multicollinearity the suggested estimator performed better in terms of MSE as compared to other estimators which are considered under this study. We believe that the findings of this paper will be useful for the practitioners.

6 Acknowledgement

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7 Conflicts of Interest

Authors declare that they have no conflicts of interest.

Appendix: List of ridge parameters considered

1. $k_1 = \hat{\sigma}^2 / \hat{\alpha}^2_{\max}$ (Hoerl & Kannard (1970))

2. $k_2 = \hat{\sigma}^2 \big/ \max(\hat{\alpha}_i)$ (Hoerl & Kannard (1970))
3. $k_3 = \hat{\sigma}^2 \big/ \sum_{i=1}^{p} \hat{\alpha}_i{}^2$ (Hoerl & Kannard (1970))
4. Find k_4 s.t.,$\sum_{i=1}^{p} \frac{\hat{\alpha}_i^2}{\hat{\sigma}^2\left(\frac{1}{k_4}+\frac{1}{\lambda_i}\right)} = p\hat{\sigma}^2\left(\frac{n-p}{n-p-2}\right)$ (Sclove (1973))
5. $k_5 = \arg\min_{u\geq 0} \frac{1}{n}\sum e_i(u)^2 / \left\{1-h_{ii}(u)\right\}^2$ ((Allen (1974))
   $e_i(k)$the residual of the ith observation in the fitted model with ridge parameter$k$,
   $$H(k) = [h_{ij}(k)] = X(X'X+kI)^{-1}X'$$
6. $k_6 = p\hat{\sigma}^2 \big/ \sum_{i=1}^{p} \hat{\alpha}_i{}^2$ (Hoerl et al. (1975))
7. Find k_7 s.t., $compute\ k_{(i)} = \frac{p\hat{\sigma}^2}{\left\{\alpha(k_{(i-1)})'\alpha(k_{(i-1)})\right\}}$ (Hoerl & Kannard (1976))
   $$until\ \ k_{(i)} = \frac{k_{(i)}-k_{(i-1)}}{k_{(i-1)}} \leq \delta where,\ \delta = 20tr\left[\left(X'X\right)^{-1}\Big/p\right]^{-1.3} and\ \ k_{(-1)} = 0$$
8. $k_8 = p\hat{\sigma}^2 \big/ \sum_{i=1}^{p} \lambda_i \hat{\alpha}_i{}^2$ (Lawless & Wang (1976))
9. $k_9 = \ \hat{\sigma}^2 \sum_{i=1}^{p} (\lambda_i\, \hat{\alpha}_i)^2 \Big/ \left[\sum_{i=1}^{p}\left(\lambda_i\, \hat{\alpha}_i{}^2\right)\right]^2$ (Hocking et al. (1976))
10. $k_{10} = (p-2)\,\hat{\sigma}^2 \big/ \sum_{i=1}^{p} \hat{\alpha}_i{}^2$ (Thisted (1977))
11. Find $k_{11} = k$ s.t., $Minimzes \left|\sum_{i=1}^{p} \lambda_i\left(k\hat{\alpha}_i{}^2(k)-\hat{\sigma}^2\right)/\left(\lambda_i+k\right)^3\right|$
    (Dempster et al. (1977))
12. Find k_{12} s.t., $\sum_{i=1}^{p}\left[\hat{\alpha}_i^2 \Big/ \left(\hat{\sigma}^2\left(\frac{1}{k_{12}}+\frac{1}{\lambda_i}\right)\right)\right] = p$ (Dempster et al. (1977))
13. $k_{13} = \arg\min_{u\geq 0} n\sum_{i=1}^{p} e_i(u)^2 \Big/ \left[\sum_{i=1}^{p}\left\{1-h_{ii}(u)\right\}\right]^2$ (Golub et al. (1979))
14. $k_{14} = 1/\hat{\alpha}^2_{\max}$ (Schaefer et al. (1984))
15. $k_{15} = 1 \Big/ \hat{\beta}'\,\hat{\beta}$ (Schaefer et al. (1984))
16. $k_{16} = (p+1) \Big/ \hat{\beta}'\,\hat{\beta}$ (Schaefer et al. (1984))
17. $k_{17} = p\hat{\sigma}^2 \big/ \sum_{i=1}^{p}\left\{\ \hat{\alpha}_i{}^2 \big/ \left[1+\left(\ 1+\lambda_i\left(\hat{\alpha}_i{}^2/\hat{\sigma}^2\right)\right)^{\ 1/2}\right]\right\}$ (Nomura (1988))
18. $k_{18} = (r_1-2)\,\hat{\sigma}^2 / \hat{\alpha}'\,\hat{\alpha}$ where, r$_1$ the rank of X (Brown (1994))
19. $k_{19} = (r_1-2)\,\hat{\sigma}^2 tr(X'X) \Big/ (r_1 \hat{y}'\,\hat{y})$ (Brown (1994))
20. Find $k_{20} = k$ s.t., $k = \begin{cases} \frac{p\hat{\sigma}^2}{\left(\hat{\beta}-J\right)'\left(\hat{\beta}-J\right)-\hat{\sigma}^2 tr(X'X)^{-1}} & if\ \left(\hat{\beta}-J\right)'\left(\hat{\beta}-J\right) > \hat{\sigma}^2 tr(X'X)^{-1} \\ \frac{p\hat{\sigma}^2}{\left(\hat{\beta}-J\right)'\left(\hat{\beta}-J\right)} & otherwise \end{cases}$
    $where,\ \ J = \left[\sum_{i=1}^{p} \hat{\beta}_i \Big/ p\right] I_{p\times 1}$ (Crouse et al. (1995))
21. $k_{21} = \left(\sum_{i=1}^{p} \hat{\sigma}^2/\hat{\alpha}_i^2\right)/p$ (Kibria (2003))
22. $k_{22} = Median\ \left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)$ (Kibria (2003))

23. $k_{23} = \hat{\sigma}^2 \Big/ \left(\prod_{i=1}^{p} \hat{\alpha}_i^2\right)^{1/p}$ (Kibria (2003))

24. $k_{24} = (\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}^2{}_{\max})$ (Khalaf & Shukur (2005))

25. $k_{25} = \max\left[(\lambda_i\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i{}^2)\right]$ (Alkhamisi,Khalaf & Shukur (2006))

26. $k_{26} = mean\left[(\lambda_i\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i{}^2)\right]$ (Alkhamisi,Khalaf & Shukur (2006))

27. $k_{27} = median\left[(\lambda_i\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i{}^2)\right]$ (Alkhamisi,Khalaf & Shukur (2006))

28. $k_{28} = \max\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right)$ (Alkhamisi & Shukur (2007))

29. $k_{29} = A.M.\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right)$ (Alkhamisi & Shukur (2007))

30. $k_{30} = Median\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right)$ (Alkhamisi & Shukur (2007))

31. $k_{31} = k_6 + (1/\lambda_{\max})$ (Alkhamisi & Shukur (2007))

32. $k_{32} = k_8 + (1/\lambda_{\max})$ (Alkhamisi & Shukur (2007))

33. $k_{33} = k_1 + (1/\lambda_{\max})$ (Alkhamisi & Shukur (2007))

34. $k_{34} = p\hat{\sigma}^2 / \sum_{i=1}^{p}\left\{\ \hat{\alpha}_i{}^2 / \left[\ \left[\left(\hat{\alpha}_i{}^4\lambda_i^2/4\hat{\sigma}^2\right) + \left(6\hat{\alpha}_i{}^4\lambda_i/\hat{\sigma}^2\right)\right]^{\ 1/2} - \left(\hat{\alpha}_i{}^2\lambda_i/2\hat{\sigma}^2\right)\right]\ \right\}$
(Batah et al. (2008))

35. $k_{35} = G.M.\left[(\lambda_i\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i{}^2)\right]$ (Muniz & Kibria (2009))

36. $k_{36} = \max\left(\sqrt{\hat{\alpha}_i^2/\hat{\sigma}^2}\right)$ (Muniz & Kibria (2009))

37. $k_{37} = Median\left(\sqrt{\hat{\alpha}_i^2/\hat{\sigma}^2}\right)$ (Muniz & Kibria (2009))

38. $k_{38} = G.M.\left(\sqrt{\hat{\alpha}_i^2/\hat{\sigma}^2}\right)$ (Muniz & Kibria (2009))

39. $k_{39} = G.M.\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)$ (Muniz & Kibria (2009))

40. $k_{40} = \max\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)$ (Muniz & Kibria (2009))

41. $k_{41} = Median\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)$ (Muniz & Kibria (2009))

42. $k_{42} = \max\left\{0, \left[\left(p\hat{\sigma}^2 \Big/ \hat{\alpha}'\hat{\alpha}\right) - (1/n\,(VIF_i)_{\max})\right]\right\}$ (Dorugade & Kashid (2010))
where $VIF_j = \frac{1}{1-R_j{}^2}$ $j = 1, 2, ..., p$ is variance inflation factor of jth regressor and$R_j{}^2$ is the squared multiple correlation coefficient resulting from the regression of X_j against all other explanatory variables.

43. $k_{43} = \left\{\hat{\sigma}^2\lambda_{\max}\sum_{i=1}^{p}\left(\lambda_i\,\hat{\alpha}_i{}^2\right) + \left[\sum_{i=1}^{p}\left(\lambda_i\,\hat{\alpha}_i{}^2\right)\right]^2\right\} \Big/ \lambda_{\max}\sum_{i=1}^{p}\left(\lambda_i\,\hat{\alpha}_i{}^2\right)$ (Al-Hassan (2010))

44. $k_{44} = k_1 + (1/mean(\lambda_i))$ (Alkhamisi (2010))

45. $k_{45} = \max\left(\sqrt{\left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}_i{}^2)\right]}\right)$ (Muniz et al. (2012))

46. $k_{46} = G.M.\left(\sqrt{\left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}_i{}^2)\right]}\right)$ (Muniz et al. (2012))

47. $k_{47} = \max\left[1 \Big/ \left(\sqrt{\left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}_i{}^2)\right]}\right)\right]$ (Muniz et al. (2012))

48. $k_{48} = G.M.\left[1 \Big/ \left(\sqrt{\left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}_i{}^2)\right]}\right)\right]$ (Muniz et al. (2012))

49. $k_{49} = Median\left[1 \Big/ \left(\sqrt{\left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}_i{}^2)\right]}\right)\right]$ (Muniz et al. (2012))

50. $k_{50} = p\hat{\sigma}^2 \Big/ \hat{\alpha}'\hat{\alpha}\ where,\ \hat{\sigma}^2 = RSS/n-p+2$ (Khalaf (2012))
and RSSdenote the residual sum of squares

51. $k_{51} = \left[p/\sum_{i=1}^{p}|\hat{\alpha}_i|\right]^2$ (Alkhamisi (2012))

52. $k_{52} = \left\{(n-p)(p+2)/\left[(n+2)\left(\sum_{i=1}^{p}|\hat{\alpha}_i|\right)\right]\right\}^2$ (Alkhamisi (2012))

53. $k_{53} = A.M.\ \left(2\hat{\sigma}^2/\lambda_{\max}\hat{\alpha}_i^2\right)$ (Dorugade (2014b))

54. $k_{54} = Median\ \left(2\hat{\sigma}^2/\lambda_{\max}\hat{\alpha}_i^2\right)$ (Dorugade (2014b))

55. $k_{55} = G.M.\ \left(2\hat{\sigma}^2/\lambda_{\max}\hat{\alpha}_i^2\right)$ (Dorugade (2014b))

56. $k_{56} = H.M.\ \left(2\hat{\sigma}^2/\lambda_{\max}\hat{\alpha}_i^2\right)$ (Dorugade (2014b))

57. $k_{57} = \hat{\sigma}^2/\min\left[\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right) + (1/\lambda_i)\right]$ (Dorugade (2014a)

58. $k_{58} = A.M.\ (k_1 \& k_6)$ (Khalaf & Iguernane (2014))

59. $k_{59} = G.M.\ (k_1 \& k_6)$ (Khalaf & Iguernane (2014))

60. $k_{60} = H.M.\ (k_1 \& k_6)$ (Khalaf & Iguernane (2014))

61. $k_{61} = \left\{\begin{array}{ll} sum(k_1 \& k_6) & if\quad k_1 + k_8 < 1 \\ sum(k_1 \& k_6)/2 & if\quad k_1 + k_8 > 1 \end{array}\right\}$ (Khalaf & Iguernane (2014))

62. $k_{62} = \sqrt{5}p\hat{\sigma}^2/\lambda_{\max}\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Karaibrahimoglu et al. (2016))

63. $k_{63} = p\hat{\sigma}^2/\sqrt{\lambda_{\max}}\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Karaibrahimoglu et al. (2016))

64. $k_{64} = 2p\hat{\sigma}^2/\sum_{i=1}^{p}\left(\lambda_i{}^{1/4}\right)\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Karaibrahimoglu et al. (2016))

65. $k_{65} = 2p\hat{\sigma}^2 \Big/ \sqrt{\sum_{i=1}^{p}(\lambda_i)}\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Karaibrahimoglu et al. (2016))

66. $k_{66} = p^2\hat{\sigma}^2/\lambda_{\max}^2\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Asar et al. (2014))

67. $k_{67} = p^3\hat{\sigma}^2/\lambda_{\max}^3\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Asar et al. (2014))

68. $k_{68} = p\hat{\sigma}^2 \Big/ \lambda_{\max}^{1/3}\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Asar et al. (2014))

69. $k_{69} = p\hat{\sigma}^2 \Big/ \left(\sum_{i=1}^{p}\sqrt{\lambda_i}\right)^{1/3}\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Asar et al. (2014))

70. $k_{70} = 2p\hat{\sigma}^2/\sqrt{\lambda_{\max}}\sum_{i=1}^{p}\hat{\alpha}_i^2$ (Asar et al. (2014))

71. $k_{71} = A.M.(r_i)\qquad where\ r_i = correlation(Y, X_i)\quad i = 1, 2, ..., p$ (Dorugade (2015))

72. $k_{72} = Median\,(r_i)$ (Dorugade (2015))

73. $k_{73} = H.M.(r_i)$ (Dorugade (2015))

74. $k_{74} = \hat{\sigma}$ (Dorugade (2016))

75. $k_{75} = \max(\lambda_i)$ (Alkhamisi & MacNeill (2015))

76. $k_{76} = A.M.(\lambda_i)$ (Alkhamisi & MacNeill (2015))

77. $k_{77} = Median\,(\lambda_i)$ (Alkhamisi & MacNeill (2015))

78. $k_{78} = qk_1 + (1-q)k_6 \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

79. $k_{79} = qk_1 + (1-q)k_9 \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

80. $k_{80} = qk_1 + (1-q)k_{51} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

81. $k_{81} = qk_1 + (1-q)k_{77} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

82. $k_{82} = qk_6 + (1-q)k_9 \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

83. $k_{83} = qk_6 + (1-q)k_{51} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

84. $k_{84} = qk_6 + (1-q)k_{77} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

85. $k_{85} = qk_9 + (1-q)k_{51} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

86. $k_{86} = qk_9 + (1-q)k_{77} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

87. $k_{87} = qk_{51} + (1-q)k_{77} \quad where\ 0 < q < 1$ (Alkhamisi & MacNeill (2015))

88. $k_{88} = \max(w_i) \ \ where, \ \ w_i = (\lambda_{med}\hat{\sigma}^2)\big/((n-p-1)\hat{\sigma}^2 + \lambda_{med}\hat{\alpha}_i^2)$
(Uzuke et al. (2017))

89. $k_{89} = Median\,(w_i)$ (Uzuke et al. (2017))

90. $k_{90} = G.M.(w_i)$ (Uzuke et al. (2017))

91. $k_{91} = \max(1/w_i)$ (Uzuke et al. (2017))

92. $k_{92} = Median\,(1/w_i)$ (Uzuke et al. (2017))

93. $k_{93} = G.M.(1/w_i)$ (Uzuke et al. (2017))

94. Find $k_{94} = k_{(j)}\ s.t.\ \ k_{(j)} = \arg\min_{k \geq 0} f(k, k_{(j-1)})$ (Wong & Chiu (2015)) $where, \quad f(k, k_{(j-1)}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\lambda_i^2 \hat{\alpha}_i^2}{(\lambda_i+k)^2(\lambda_i + k_{(j-1)})^2}$

$start\ \ at\ \ k_{(0)} = 0$with the convention Iteration stops when $\left|k_{(j)} - k_{(j\ -1)}\right| \leq 10^{-6}\ \ or\ \ j = 2000$

95. $k_{95} = sqrt(k_1)$ (Lukman & Ayinde (2017))

96. $k_{96} = sqrt(k_6)$ (Lukman & Ayinde (2017))

97. $k_{97} = sqrt(k_{35})$ (Lukman & Ayinde (2017))

98. $k_{98} = \left(\sum_{i=1}^{p} \hat{\sigma}^2/\lambda_i\hat{\alpha}_i^2\right)/p$ (Lukman & Ayinde (2017))

99. $k_{99} = sqrt(k_{98})$ (Lukman & Ayinde (2017))

100. $k_{100} = \hat{\sigma}^2 \Big/ \left(\prod_{i=1}^{p} \lambda_i \hat{\alpha}_i^2\right)^{\frac{1}{p}}$ (Lukman & Ayinde (2017))

101. $k_{101} = sqrt(k_{100})$ (Lukman & Ayinde (2017))

102. $k_{102} = Median\ \left[\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right]$ (Lukman & Ayinde (2017))

103. $k_{103} = Median\,(sqrt(ww_i)) \quad where, \quad ww_i = (\lambda_i \hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_i \hat{\alpha}_i^2)$ (Lukman & Ayinde (2017))

104. $k_{104} = \frac{1}{p}\sum_{i=1}^{p} (\lambda_i \hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_i \hat{\alpha}_i^2)$ (Lukman & Ayinde (2017))

105. $k_{105} = sqrt(k_{104})$ (Lukman & Ayinde (2017))

106. $k_{106} = p\sum_{i=1}^{p} (\lambda_i \hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_i \hat{\alpha}_i^2)$ (Lukman & Ayinde (2017))

107. $k_{107} = sqrt(k_{106})$ (Lukman & Ayinde (2017))

108. $k_{108} = sqrt\,(k_{24})$ (Lukman & Ayinde (2017))

109. $k_{109} = Median\,(sqrt(vv_i)) \quad where, \quad vv_i = (\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_{\max}\hat{\alpha}_i^2)$ (Lukman & Ayinde (2017))

110. $k_{110} = \max\ \left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_{\max}\hat{\alpha}_i{}^2)\right]$ (Lukman & Ayinde (2017))

111. $k_{111} = G.M.\ \left[(\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_{\max}\hat{\alpha}_i{}^2)\right]$ (Lukman & Ayinde (2017))

112. $k_{112} = \max\left[1/\left((\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_{\max}\hat{\alpha}_i{}^2)\right)\right]$ (Lukman & Ayinde (2017))

113. $k_{113} = G.M.\left[1/\left((\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_{\max}\hat{\alpha}_i{}^2)\right)\right]$ (Lukman & Ayinde (2017))

114. $k_{114} = Median\left[1/\left((\lambda_{\max}\hat{\sigma}^2)/((n-p-1)\hat{\sigma}^2\ + \lambda_{\max}\hat{\alpha}_i{}^2)\right)\right]$
(Lukman & Ayinde (2017))

115. $k_{115} = A.M.\ \left\{sqrt\left[\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right]\right\}$ (Asar & Genc (2015))

116. $k_{116} = G.M.\ \left\{sqrt\left[\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right]\right\}$ (Asar & Genc (2015))

117. $k_{117} = \max\ \left\{sqrt\left[\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right]\right\}$ (Asar & Genc (2015))

118. $k_{118} = H.M.\ \left\{sqrt\left[\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right]\right\}$ (Asar & Genc (2015))

119. $k_{119} = A.M.\ \left\{sqrt\left[1/\left(\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right)\right]\right\}$ (Asar & Genc (2015))

120. $k_{120} = H.M.\ \left\{sqrt\left[1/\left(\hat{\sigma}^2/(\lambda_i\hat{\alpha}_i^2)\right)\right]\right\}$ (Asar & Genc (2015))

121. $k_{121} = (1/k_{33}) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

122. $k_{122} = \sqrt{k_{33}} + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

123. $k_{123} = \left(1/\sqrt{k_{33}}\right) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

124. $k_{124} = \max\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

125. $k_{125} = \max\left[1/\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

126. $k_{126} = \max\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

127. $k_{127} = \max\left[1\Big/\sqrt{\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)}\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

128. $k_{128} = k_{21} + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

129. $k_{129} = \left[1/median\left(1/\left(\hat{\sigma}^2/\lambda_i\hat{\alpha}_i^2\right)\right)\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

130. $k_{130} = \left[\sqrt{median\left(1/\left(\hat{\sigma}^2/\lambda_i\hat{\alpha}_i^2\right)\right)}\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

131. $k_{131} = \left[1 \Big/ \sqrt{median\left(1/\left(\hat{\sigma}^2/\lambda_i\hat{\alpha}_i^2\right)\right)}\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

132. $k_{132} = k_6 + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

133. $k_{133} = (1/k_{103}) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

134. $k_{134} = \sqrt{k_{103}} + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

135. $k_{135} = \left(1 \big/ \sqrt{k_{103}}\right) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

136. $k_{136} = k_{23} + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

137. $k_{137} = [1/A.M.(vv_i)] + (1/\lambda_{\max}) \qquad i = 1, 2, ..., p$ (Lukman & Ayinde (2017))

138. $k_{138} = \sqrt{A.M.(vv_i)} + (1/\lambda_{\max}) \qquad i = 1, 2, ..., p$ (Lukman & Ayinde (2017))

139. $k_{139} = \left[1 \Big/ \sqrt{A.M.(vv_i)}\right] + (1/\lambda_{\max}) \qquad i = 1, 2, ..., p$ (Lukman & Ayinde (2017))

140. $k_{140} = Median\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

141. $k_{141} = Median\left[1/\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

142. $k_{142} = Median\left(\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right) + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

143. $k_{143} = Median\left[1 \Big/ \sqrt{\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)}\right] + (1/\lambda_{\max})$ (Lukman & Ayinde (2017))

144. $k_{144} = k_6\left[\max(\hat{\alpha}_i)/\min(\lambda_i)\right]$ (Iguernane (2016))

145. $k_{145} = k_8\left[\max(\hat{\alpha}_i)/\min(\lambda_i)\right]$ (Iguernane (2016)))

146. $k_{146} = \sqrt{median\sqrt{\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)}}$ (oktas & Sevinc (2016))

147. $k_{147} = \hat{\sigma}^2 \big/ (median\,(\hat{\alpha}_i))^2$ (oktas & Sevinc (2016))

148. $k_{148} = k_6 + \left(1 \Big/ \lambda_{\max}\hat{\alpha}'\hat{\alpha}\right)$ (Bhat (2016))

149. $k_{149} = k_6 + (\lambda_{\min}/2\lambda_{\max})$ (Bhat (2016))

where, $\hat{\alpha}_i$ is the ith element of $\hat{\alpha}_{LS}$, $i = 1, 2, ..., p$ and $\hat{\sigma}^2$ is the LS estimator of σ^2i.e. $\hat{\sigma}^2 = \frac{Y'Y - \hat{\alpha}'_{LS}Z'Y}{n-p-1}$.

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