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Estimation of Parameters in Sarima Model: Bayesian Approach

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Abstract

A popular univariate time-series forecasting model for short-term fluctuations in air traffic flow is the SARIMA model, and its parameters are often estimated using the method of maximum likelihood. This paper focused on estimation of its parameters using the Bayesian approach. The data used is the number of domestic air passengers travelling through Air India and Spice Jet airlines. The forecasts from the Bayesian parameter estimation performed better than those using maximum likelihood estimation.

Keywords: Air India, Spice Jet, Air traffic flow, ARIMA model, Bayesian approach.

1 Introduction

One of the most advanced methods for time series forecasting is the Autoregressive Integrated Moving Averages (ARIMA) method. The most popular model for forecasting seasonal time series is SARIMA. The ARIMA model extended with seasonality parameters is the SARIMA model (see Chen et al. (2009), Mounika & Bhatracharyulu (2023)). Its parameters are often estimated using the maximum likelihood approach. The SARIMA $(p, d, q)(P, D, Q)_s$ model is in the form:

$$\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D Y_t = \theta(B)\Theta(B^s)\epsilon_t$$

where:

- Y_t represents the value of the time series at time t, $t = 1, 2, \ldots, T$,
- B denotes the backward shift operator,
- d represents regular differences,
- D denotes seasonal differences,
- ϵ_t is white noise with mean zero and variance σ^2,
- $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$,
- $\Phi(B) = 1 - \Phi_1 B - \cdots - \Phi_P B^P$,
- $\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q$,
- $\Theta(B) = 1 + \Theta_1 B + \cdots + \Theta_Q B^Q$.

Box and Jenkins proposed Bayesian parameter estimation for the ARIMA model as an alternative to the maximum likelihood estimation. This paper focuses on the estimation of parameters in the SARIMA model using the Bayesian approach (see Albassam et al. (2022), Abel et al. (2010), Bunn (1975), Ghosh et al (2007), Koop et al. (1995), Liang (2005), Zhu et al (2018)).
The civil aviation sector is a consistently growing and vital sector in the economic progress of a nation. It plays an important role in carrying people or goods from one point to another, particularly when travelling over short distances. As one of the main service sectors, aviation contributes significantly to the economic growth of both developed and developing countries (see Mounika & Bhatracharyulu (2023)).

The traffic flow data of the number of domestic passengers who traveled with Air India and Spice Jet is fitted with an appropriate seasonal ARIMA (SARIMA) model in this study. Twenty percent of the data is used as a testing set, while the remaining eighty percent is used as a training set. To estimate model parameters, two methods are used: the Bayesian technique and maximum likelihood estimation. A comparison of the forecasts from the two methodologies is given, along with an analysis of the additional information obtained from the Bayesian method.

2 Model Building for Air India

The number of domestic air passengers who traveled in each month through Air India during the period 2009 to 2019 is considered for model building and forecasting. The size of the dataset is 132 months (11 years). The average number of passengers traveling in a month is 425,842.3 with a standard deviation of 121,197.7 and a coefficient of variation in the data of 28.5%. The range of passengers traveled is (182,356, 680,680). The data is negatively skewed, i.e., Sk $= -0.19$, and platykurtic with $\gamma_2 = -1.004$. The stability in the data was tested using CDVI = 8.9.
Data Plot: The figure below represents the data plot for the data points.

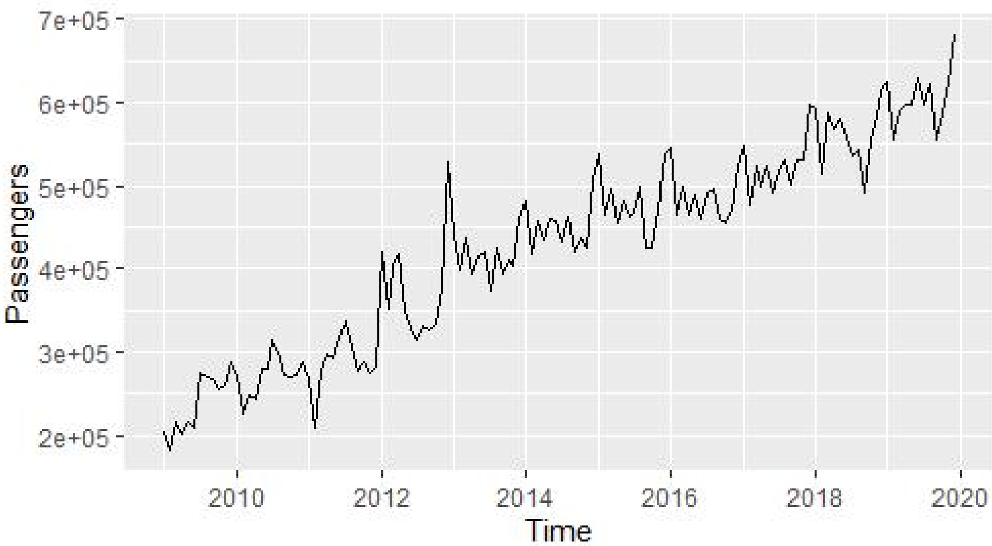

Figure 1: Data Plot of the Number of Domestic Air Passengers.

The stationarity in the data was tested through the ADF test and it was found that the data is stationary (DF = -4.0471 with lag = 4, p = 0.01).
Holt's Winter Model: The best fitted Holt's-Winter model is:

$$F_{t+m} = L_t + B_t m + C_{t-c+m}$$

where

$$L_t = (0.6633)(Y_t - C_{t-c}) + (1 - 0.6633)(L_{t-1} + B_{t-1})$$
$$B_t = (1 \times 10^{-4})(L_t - L_{t-1}) + (1 - 1 \times 10^{-4})B_{t-1}$$
$$C_t = (1 \times 10^{-4})(Y_t - L_t) + (1 - 1 \times 10^{-4})C_{t-c}.$$

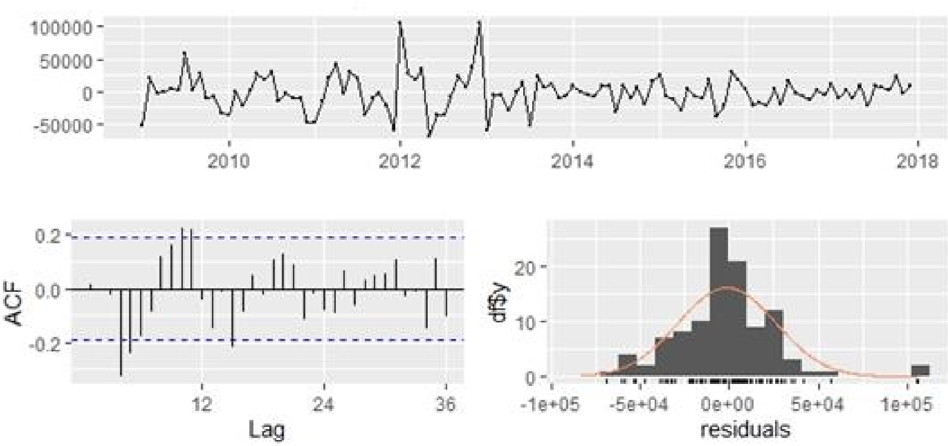

Figure 2: Residuals plot of Holt's Winter model for Air India.

Seasonal ARIMA Model: The decomposition of the time series is given below:

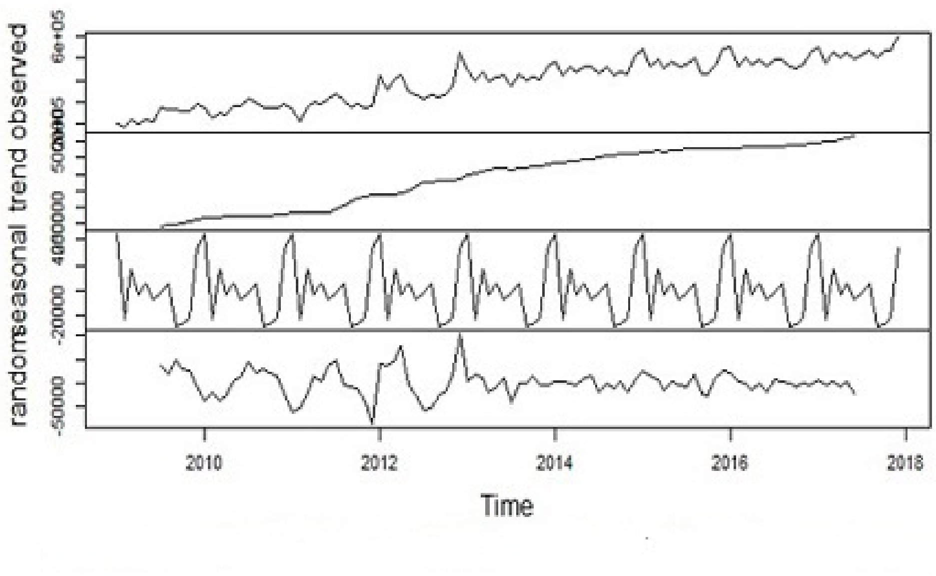

Figure 3: Decomposition of Time Series of Air India.

The autocorrelation and partial autocorrelation functions were evaluated and are presented below in Figure 4:
The plots clearly showed a monthly effect but did not clearly show a pattern, so the ACF and PACF of the 12th differences (seasonal differencing) were examined.

Based on the variations in the 12^{th} differences ACF and PACF, we identified the model as ARIMA (1,0,0)(0,1,1)$_{12}$. The estimated parameters are $\phi_1 = 0.5883$ and $\Theta_1 = -0.7925$ with standard errors

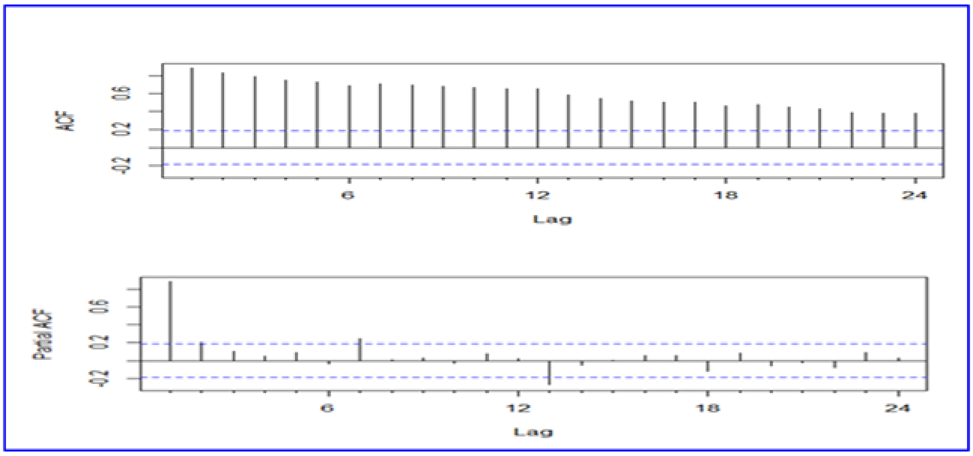

Figure 4: ACF and PACF plots of Air India.

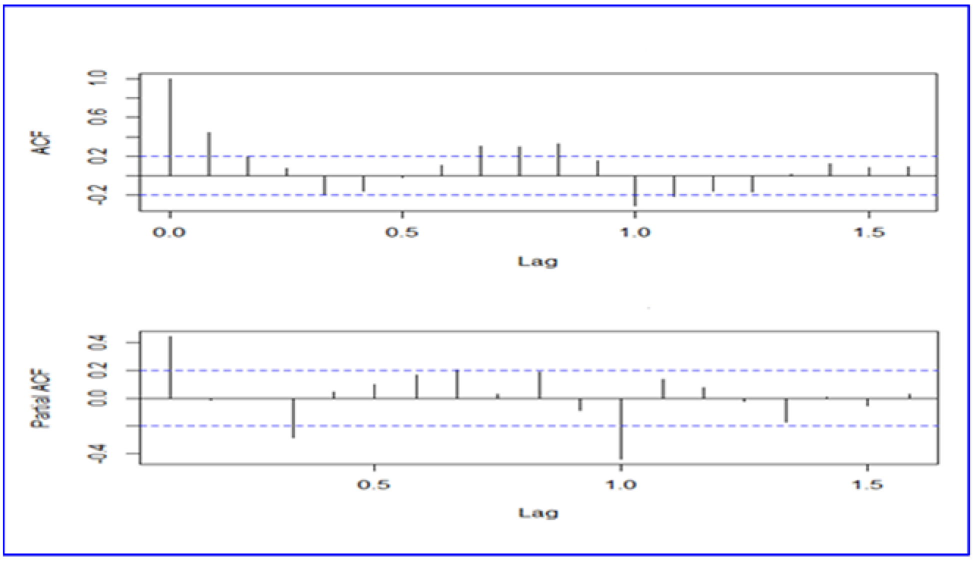

Figure 5: ACF and PACF plots of differenced data for Air India.

0.0842 and 0.1598. The model is:

$$(1 - 0.5883B)(1 - B^S)Y_t = (1 - 0.7925B)\epsilon_t$$

3 Model Building for Spice Jet

The number of domestic air passengers travelled in each month through SpiceJet during the period 2009 to 2019, is considered for model building and forecasting. The size of the dataset is 132 months (11 years). The average number of passengers traveled in a month is 993571.6 with a standard deviation 388715.86 and coefficient of variation in the data is 0.39%. The range of passengers traveled is (378758, 2144156). The data is positively skewed (Sk $= 0.68$) and platy kurtic ($\gamma_2 = 0.21$). The stability in the data was tested using CDVI $= 17.91$.

Data Sequence Plot: The figure below represents the data plot for the data points.

The stationarity in the data was tested through the ADF test and found non-stationarity (DF = -1.4796 with lag = 4, $p = 0.79$).

Holts-Winter Model: The best fitted Holt-Winter model:

$$F_{t+m} = L_t + B_t m + C_{t-c+m}$$

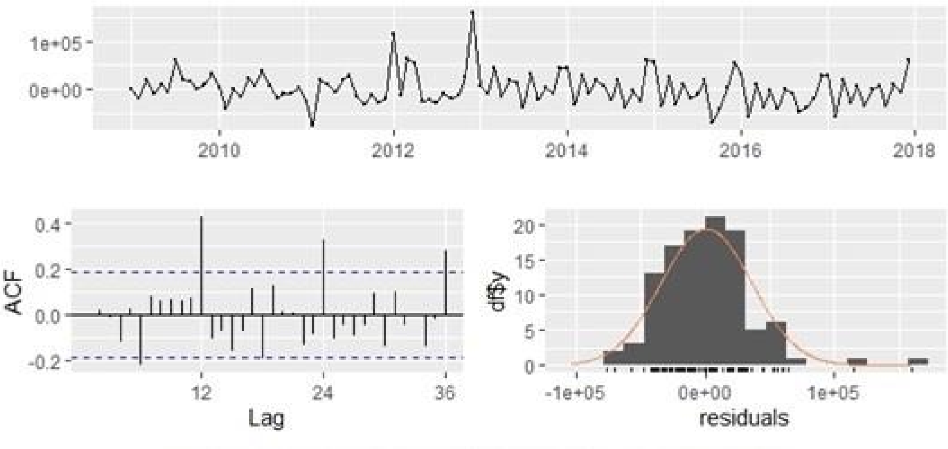

Figure 6: Residual plot of ARIMA (1,0,0)(0,1,1)$_{12}$ for Air India.

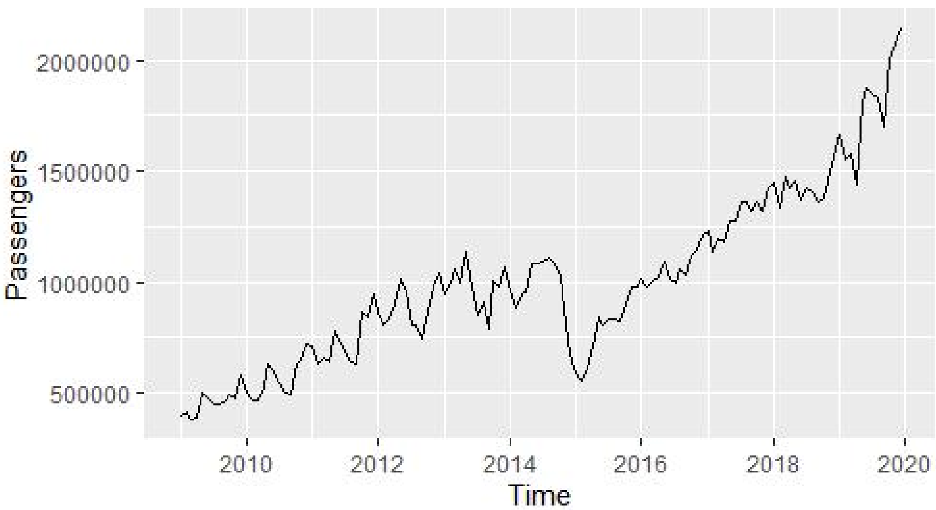

Figure 7: Data Plot of the no. of Passengers Travelled Through SpiceJet.

where

$$L_t = (0.9999)(Y_t - C_{t-c}) + (1 - 0.9999)(L_{t-1} + B_{t-1}),$$

$$B_t = (0.0001)(L_t - L_{t-1}) + (1 - 0.0001)B_{t-1},$$

and

$$C_t = (0.0001)(Y_t - L_t) + (1 - 0.0001)C_{t-c}.$$

Seasonal Arima Model: The decomposition of the time series is given below.

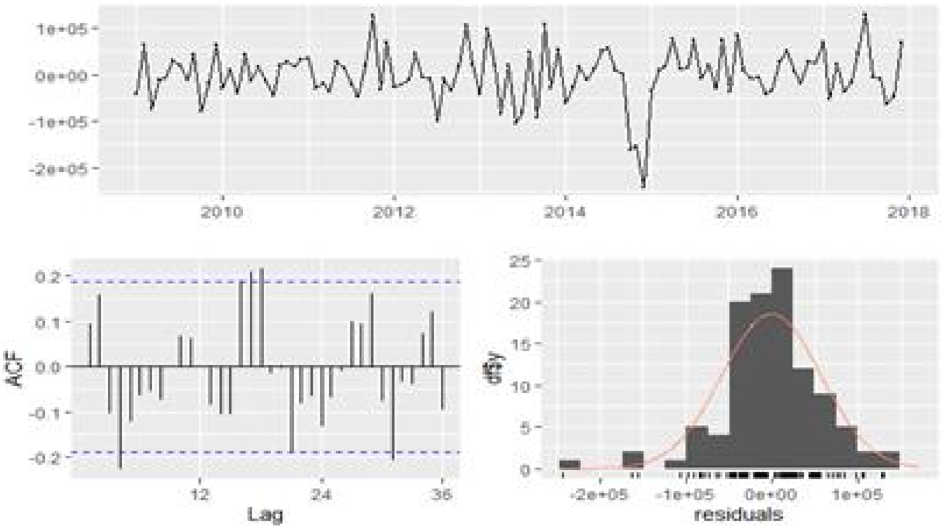

Figure 8: Residuals Plot of Holts Winter Model for SpiceJet.

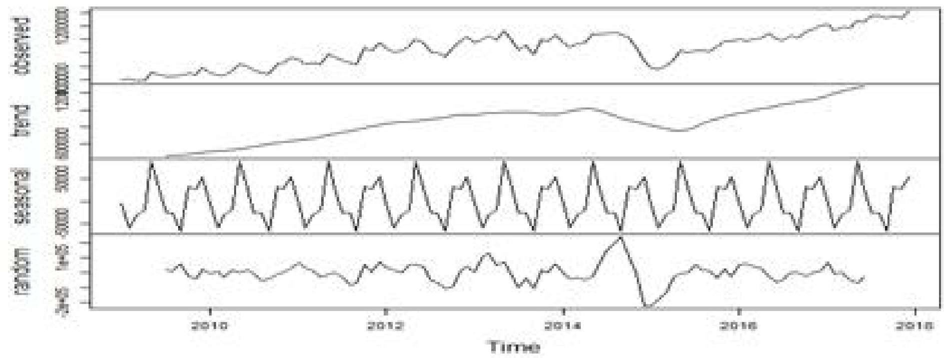

Figure 9: Decomposition of Time Series of SpiceJet.

The autocorrelation and partial autocorrelation functions were evaluated and are presented below.

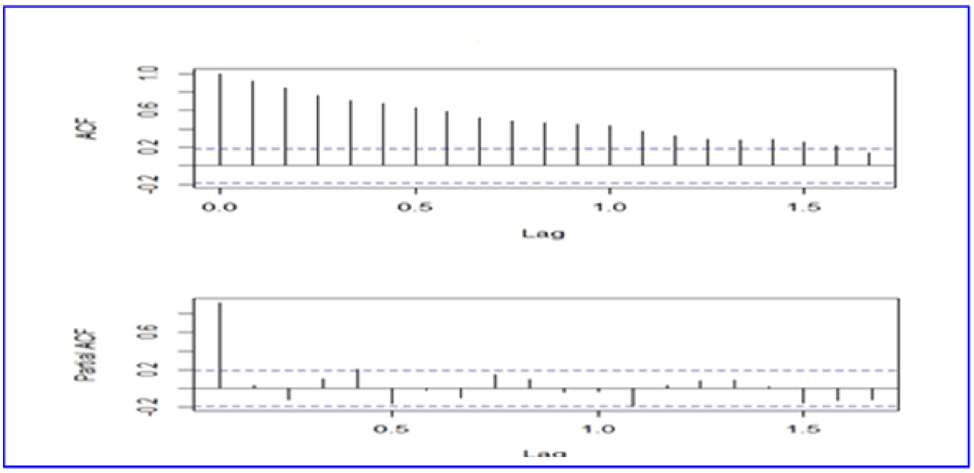

Figure 10: ACF and PACF plot of SpiceJet.

The plots clearly showed the monthly effect but did not clearly show a pattern, so the ACF and PACF of the 12th differences (seasonal differencing) were examined.

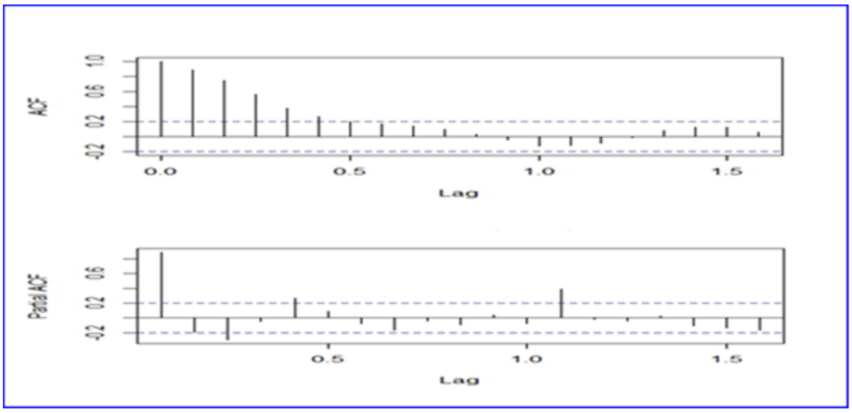

Figure 11: ACF and PACF plot of differenced data of SpiceJet.

Based on the variations in the 12th differences ACF and PACF, we identified the model as ARIMA $(3,0,0)(2,1,0)_{12}$ with the estimated parameters:

$$\phi_1 = 1.022, \quad \phi_2 = 0.0997, \quad \phi_3 = -0.2446$$

with standard errors:

$$SE(\phi_1) = 0.1016, \quad SE(\phi_2) = 0.15, \quad SE(\phi_3) = 0.15$$

The seasonal parameters are:

$$\Phi_1 = -0.4881, \quad \Phi_2 = 0.2454$$

with standard errors:

$$SE(\Phi_1) = 0.1056, \quad SE(\Phi_2) = 0.0994.$$

The model is given by:

$$(1 - 1.022B - 0.0997B^2 + 0.2446B^3)(1 + 0.4881B - 0.2454B^2)(1 - B^S)Y_t = 0.$$

4 Bayesian Approach for Parameters Estimation

Let Y denote the observed time series, $Y_t \sim N(\mu, \sigma^2)$, where the location parameter μ follows a Normal distribution and the scale parameter σ follows a Student-t distribution. The AR (ϕ) and MA (θ) parameters follow a Normal distribution. Similarly, seasonal parameters AR (Φ) and MA (Θ) follow a Normal distribution.

Let $f(Y, \theta)$ be the distribution of Y, where θ is unknown and $f(\theta)$ is the prior distribution of parameter θ. The joint density function of the observed sample Y given θ is called the likelihood function, denoted by $f(Y|\theta)$. The posterior distribution $f(\theta|Y)$ can be evaluated based on the prior and likelihood using Bayes' rule. Initially, the value of the parameter θ is estimated based on the observed sample and the likelihood function $f(Y|\theta)$ is evaluated. Then the normalized constant

$$f(Y) = \int f(\theta) \cdot f(Y|\theta)\, d\theta$$

is calculated by generating all possible samples through simulation using the Metropolis-Hastings algorithm. This algorithm generates sample values from a probability distribution, getting closer to the intended distribution as the number of simulated sample values increases. A candidate for the next sample value is chosen by the method depending on the current sample value at each iteration, generating these sample values repeatedly. Using a comparison of the likelihoods of the present sample values with the intended distribution, each stage's chance of acceptance for the candidate is determined.

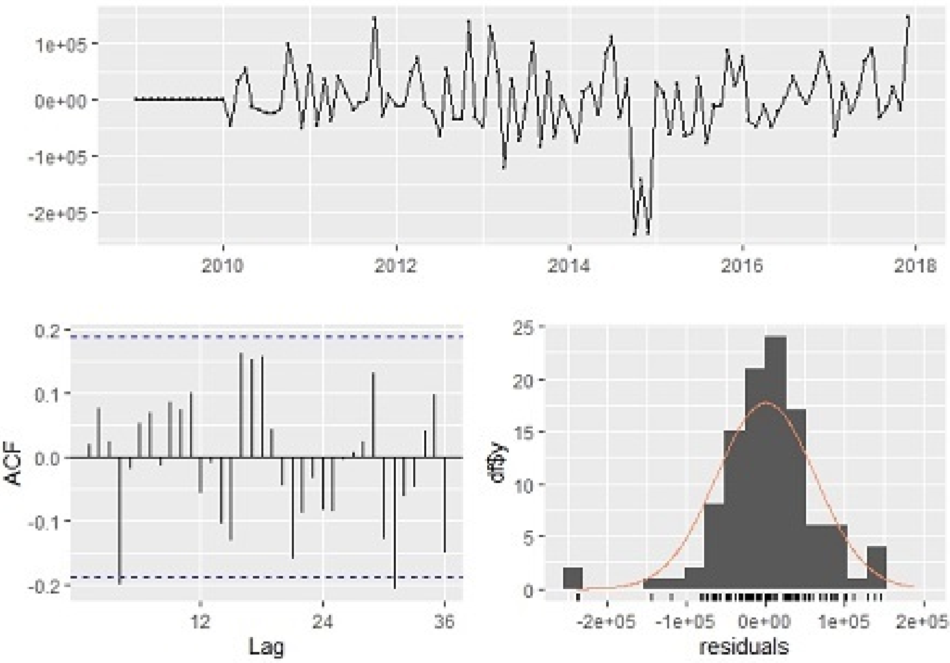

Figure 12: Residuals plot of ARIMA(3,0,0)(2,1,0)$_{12}$ for SpiceJet.

4.1 Parameters Estimation for Air India

Initially, the ARIMA model parameters are obtained as ARIMA (1,0,0)(0,1,1)$_{12}$ using the maximum likelihood method. For the simulation, as per the assumptions, the distributions of AR parameter ϕ_1 and the seasonal MA parameter Θ_1 follow Normal distributions with parameters:

$$\phi_1 \sim N(0.5883, 0.8), \quad \Theta_1 \sim N(-0.7925, 2.76).$$

A large-scale sample is generated from these specified populations and normalized constants are evaluated. Then the posterior probabilities of the parameters are calculated. The posterior estimates of the parameters are:

$$\phi_1 = 0.8201, \quad \Theta_1 = -0.519.$$

Simulation trace plots of parameters and density estimates in the ARIMA (1,0,0)(0,1,1)$_{12}$ from Bayesian inference are plotted.

The estimates of the SARIMA parameters from Bayesian and maximum likelihood approach for Air India are presented below: A comparison of the residual behavior under the Bayesian and Maximum

Parameter	Bayesian Approach			Maximum Likelihood Method		
	Mean	**S.E.**	**C.I. at 95%**	**Mean**	**S.E.**	**C.I. at 95%**
μ	0.0039	0.0216	-4.8466, 4.9377	-	-	-
σ	35337.41	18.069	31345.89, 39765.08	-	-	-
AR1	0.8201	0.0004	0.7318, 0.8994	0.5883	0.0842	0.4497, 0.7268
SMA1	-0.519	0.0006	-0.647, -0.3811	-0.7925	0.1598	-1.0554, -0.5296

Table 1: Comparison of Bayesian and Maximum Likelihood Approaches (Air India)

Likelihood approaches is provided in Figures 13 and 14, highlighting differences in model adequacy and fit. In addition, Figure 16 depicts the actual versus predicted traffic volumes, contrasting the performance of the classical and Bayesian approaches.

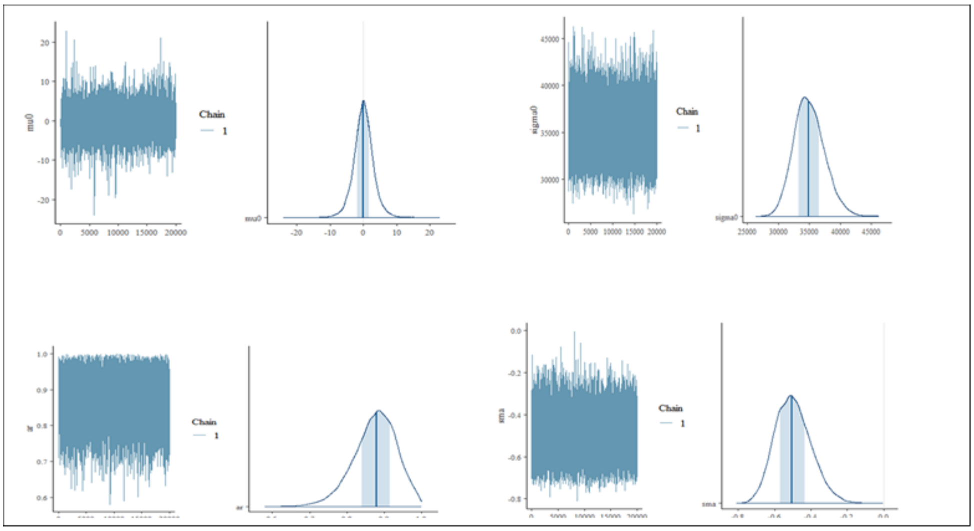

Figure 13: Air India.

Method	RMSE	MASE	MAPE	MAE
Bayesian Approach	24290.33	0.5537	0.0341	19212.51
Maximum Likelihood Method	62468.10	1.5167	0.0878	52625.93

Table 2: Comparison of Errors in Prediction (Air India)

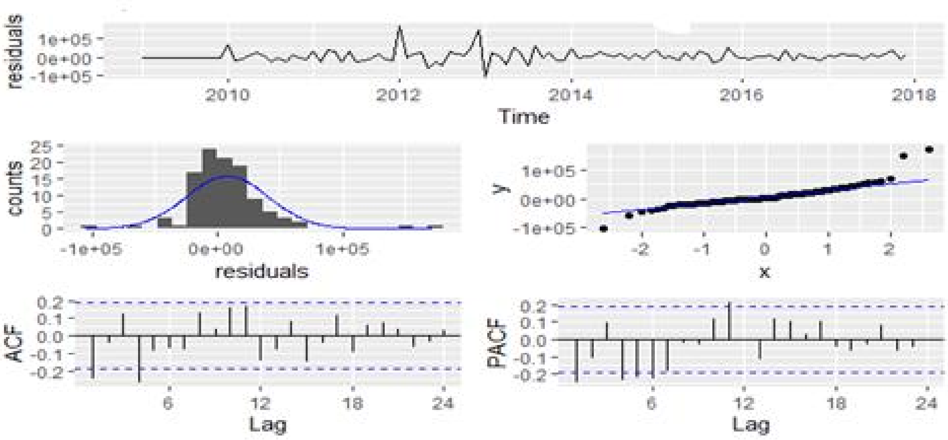

Figure 14: Bayesian Approach (Air India).

4.2 Parameters Estimation for Spice Jet

Initially, the ARIMA model parameters are obtained as ARIMA(3,0,0)(2,1,0)$_{12}$ using the maximum likelihood method. For the simulation, as per the assumptions, the distributions of AR parameter ϕ_1 and SAR parameter Φ_1 follow Normal distributions with the following parameters:

$$\phi_1 \sim N(1.0220, 1.1), \quad \phi_2 \sim N(0.0997, 1.5), \quad \phi_3 \sim N(-0.2446, 1.3),$$

$$\Phi_1 \sim N(-0.4881, 1.20), \quad \Phi_2 \sim N(-0.2454, 1.63).$$

A large-scale sample is generated from these specified populations and normalized constants are evaluated. Then, the posterior probabilities of the parameters are calculated. The posterior estimates of the parameters are:

$$\phi_1 = 0.9305, \quad \phi_2 = 0.2194, \quad \phi_3 = -0.2730, \quad \Phi_1 = -0.0251, \quad \Phi_2 = 0.0012.$$

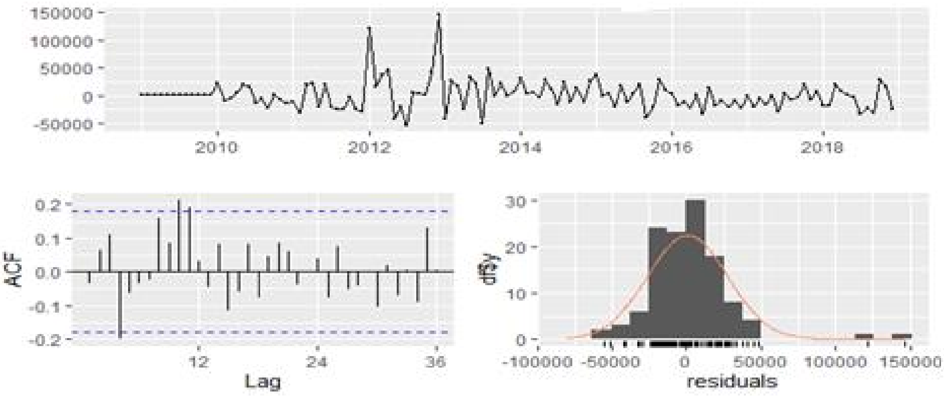

Figure 15: Maximum Likelihood Approach (Air India).

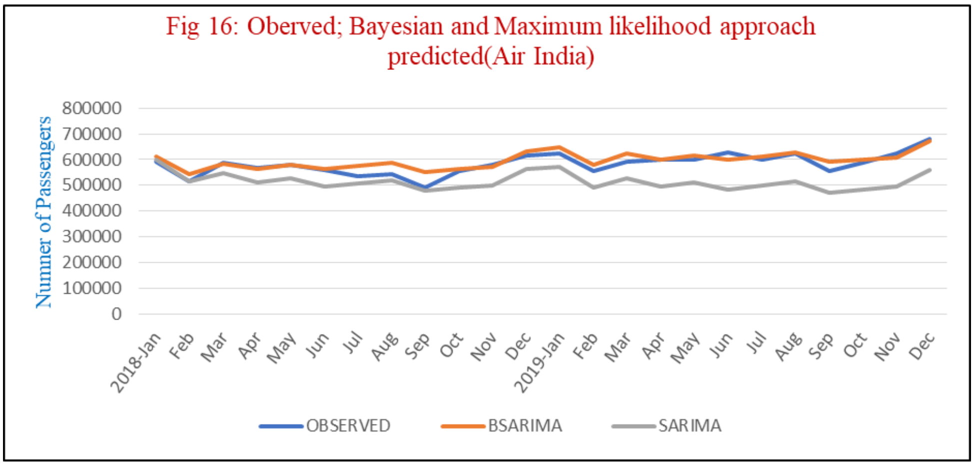

Figure 16: Observed, Bayesian and Maximum likelihood approach predicted(Air India).

Simulation trace plots of parameters and density estimates in the ARIMA(3,0,0)(2,1,0)$_{12}$ from Bayesian inference are plotted.
The estimates of the SARIMA parameters from Bayesian and maximum likelihood approach for SpiceJet are presented below:

Parameter	**Bayesian Approach**			**Maximum Likelihood Method**		
	Mean	**S.E.**	**C.I. at 95%**	**Mean**	**S.E.**	**C.I. at 95%**
μ	0.0098	0.0214	-4.8838, 4.7983	-	-	-
σ	76513.39	38.87	67979.99, 86120.57	-	-	-
AR1	0.9305	0.0004	0.8240, 0.9946	1.0220	0.1016	0.8549, 1.1891
AR2	0.2194	0.0008	0.0375, 0.4112	0.0997	0.1500	-0.1471, 0.3464
AR3	-0.2730	0.0007	-0.4371, -0.1093	-0.2446	0.1500	-0.4113, -0.0779
SAR1	-0.0251	0.0003	-0.0978, 0.0484	-0.4881	0.1056	-0.6617, -0.3144
SAR2	0.0012	0.0004	-0.0837, 0.0872	0.2454	0.0994	-0.4089, -0.0819

Table 3: Comparison of Bayesian and Maximum Likelihood Approaches (Spice Jet)

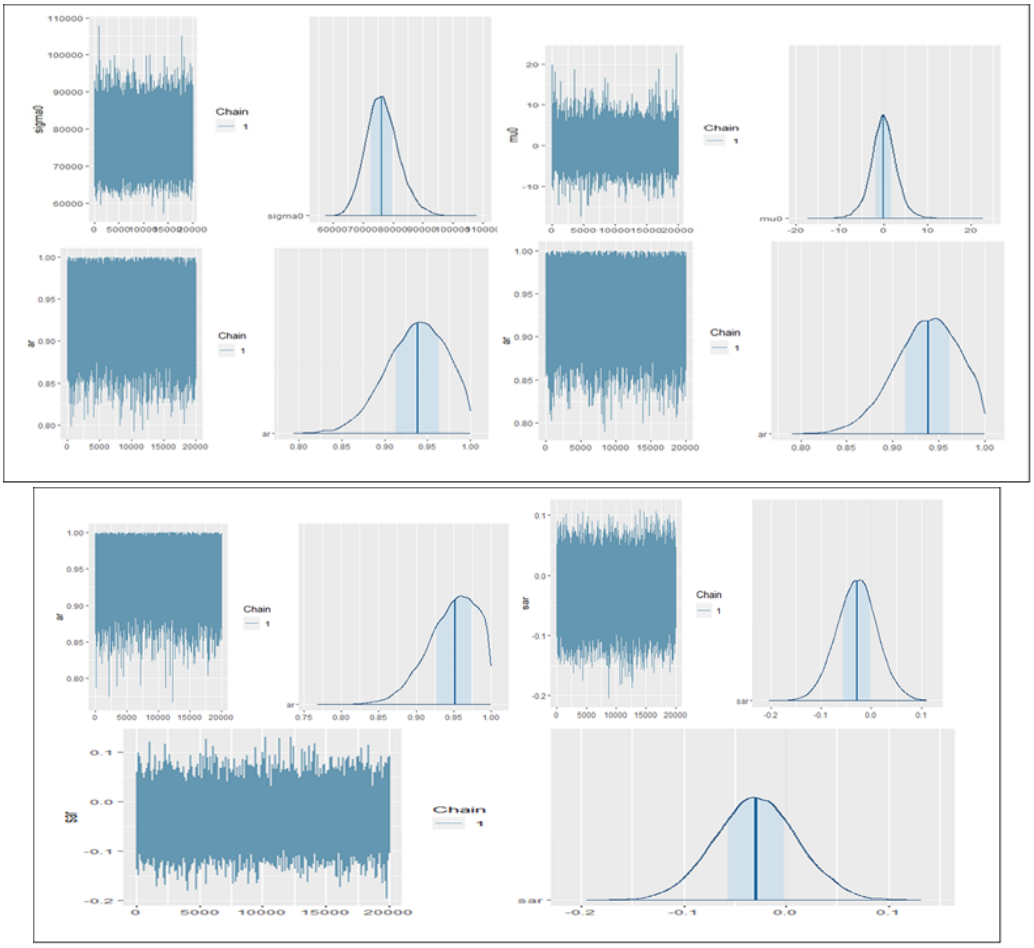

Figure 17: : Spice Jet

Method	RMSE	MASE	MAPE	MAE
Bayesian Approach	56142.32	0.2136	3.6521	26351.02
Maximum Likelihood Method	62314.64	0.2444	4.9836	43531.82

Table 4: Comparison of Errors in Prediction (Spice Jet)

Figures 18 and 19 present the residual plots obtained from the Bayesian and Maximum Likelihood approaches, respectively. Furthermore, Figure 20 illustrates the actual versus predicted traffic volumes under both the classical and Bayesian approaches.

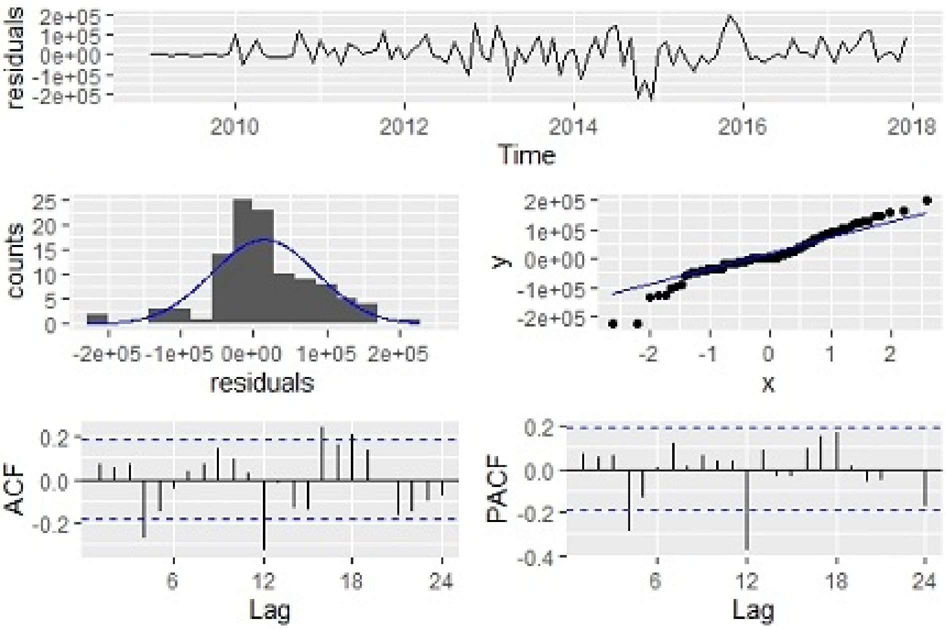

Figure 18: Bayesian Approach (Spice Jet).

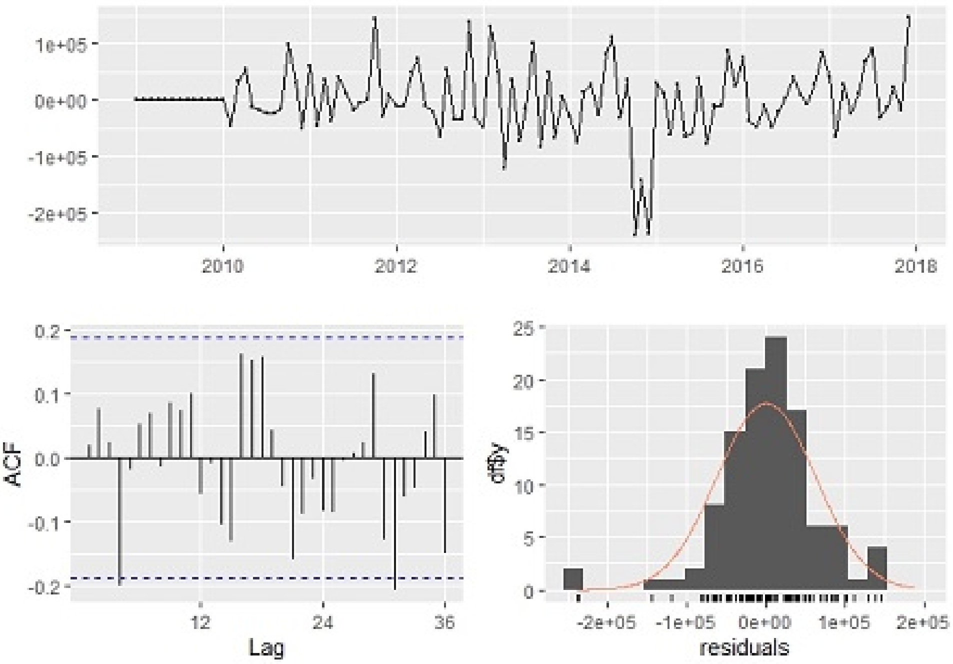

Figure 19: Maximum Likelihood Approach (Spice Jet).

5 Conclusion

1. The significance of autocorrelation in the model SARIMA (1,0,0) $(0,1,1)_{12}$ for Air India was tested through the Ljung-Box test and concluded that there is no satistically significant evidence of residual autocorrelation at α—0.05 with $p-value = 0.06967$ (Test statistic $Q^{*} = 31.245$ with 21 d.f., 3 model parameters and 24 total lags used).

2. The significance of the estimated values of parameters AR1, SMA1 and drift were tested through

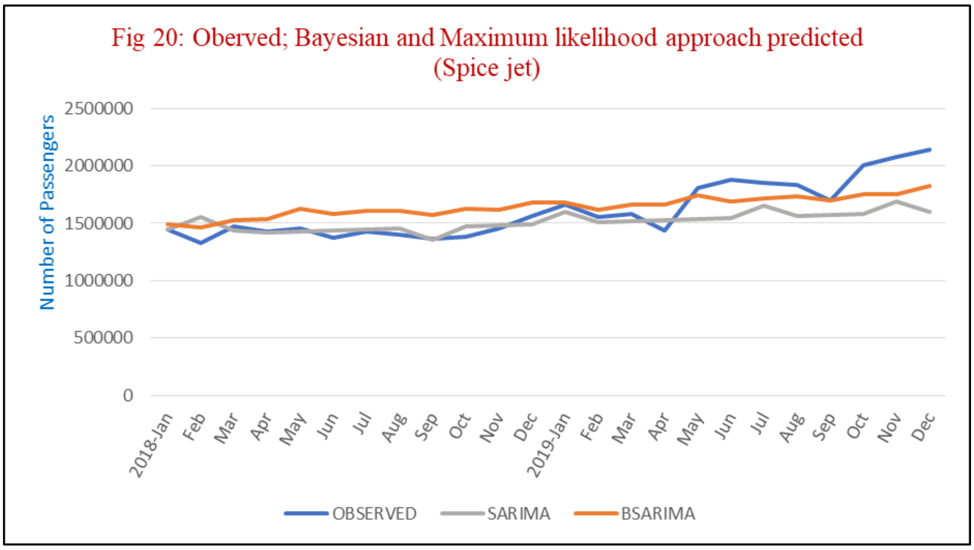

Figure 20: Observed, Bayesian and Maximum likelihood approach predicted (Spice jet).

the LM test. The parameters AR1, SMA1 and Drift with Z-values 6.9838, -4.9583 and 13.7794 are significant with $p - value = 2.872e - 12$, $7.113e - 07$ and $2.2e - 16$ at α—0.05.

3. The autocorrelation in residuals of the SARIMA (3,0,0) $(2,1,0)_{12}$ for Spice Jet was tested using the Ljung-Box test ($Q^* = 28.044$ with 17 d.f., 5 model parameters and total 22 lags) with $p - value = 0.04443 < \alpha = 0.05$, i.e., the null hypothesis is rejected. There is statistically significant correlation in the residuals.

4. The significance in the estimated parameters of AR1, AR2, AR3, SAR1, SAR2 and Drift was examined using the Lagrange Multiplier test with Z-values as 10.0630, 0.6643, -2.4138, -4.6226, -2.4684 and 3.1154 with $p - value = 2.2e - 16$, 0.506505, 0.015785, $3.79e - 06$, 0.013571 and 0.001837 at α—0.05.

5. The RMSE and MASE for the Bayesian approach prediction are the least when compared with the Maximum Likelihood method. The standard error in estimating parameters is also least in the Bayesian approach.

6. The predicted values in both the methods are falling within the 95% confidence interval.

7. When the traffic volume forecasts using classical inference and Bayesian are plotted alongside the original observations, it can be seen that they are close to Bayesian forecasts.

6 Acknowledgement

The authors are thankful to the referee for their valuable suggestions for the improvement of this manuscript.

7 Conflicts of Interest

The authors declare that they have no conflicts of interest to disclose regarding the publication of this paper.

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