The Aligarh Journal of Statistics ISSN 0971-0388
Vol. 45 (2025), pp. 17-26

Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model With Multiple Vacations and Orbital Search

J. Indhumathi[1], **A. Muthu Ganapathi Subramanian**[2], **Gopal Sekar**[3]
[1,2,3]Pondicherry University, Pondicherry, India

Corresponding author: J. Indhumathi, *E-mail: indhuspb10@gmail.com*

AMS Subject Classification: 60J22, 60J25
http://www.ams.org/mathscinet/msc/msc2020.html

Received: 06/01/2022 Accepted: 21/08/2024

Abstract

In this research paper, we considered a transient behaviour of finite source retrial model with multiple vacations and orbital search in which the primary arrival rate λ which follows a Poisson distribution and the service time μ follows an exponential distribution. We have assumed that the finite calling population size being M. The vacation time follows an exponential distribution with parameter α. All transitions are assembled by using an infinitesimal generator matrix. Fundamental Matrix method is used to obtain the time dependent and steady state solutions. Numerical studies have been done for time dependent mean number of customers in the orbit, transient probabilities of server free/busy/vacation for various values of $\mathrm{M}, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t.

Keywords: Finite Source Queues, Multiple Vacation, Orbital Search, Retrial Queue, Fundamental Matrix.

1 Introduction

The main aim of this paper is to analyze the transient behaviour of finite source Retrial model with multiple vacations and orbital search by new computational approach. If the arriving customers who find the server busy then go for invisible queue (orbit) and retry for a service after some random time again and again from the orbit till to get the service which is known as retrial queues. Application of retrial queues are found in various areas such as call centres, LAN, Telephone systems, Retrial shopping queue etc. For example, in a call centre, if a customer makes a phone call when all the agents are busy, the customer will try to make a phone call again after some random time.

Retrial queues and its studies have been found in Falin (1990) and Artalejo (2010). Miller (1983) discussed about Matrix-geometric solutions in stochastic Models. Considerable attention on finite source queues studies from various researchers such as Velika I. Dragieva; Tuan Phung-Duc (2020) studied A finite-source M/G/1 retrial queue with outgoing calls. Indhumathi Muthu Ganapathi Subramanian & Gopal Sekar (2021) have analysed Finite Source queues with catastrophe. Shan Gao & Jiting Wang (2020) explained stochastic analysis of a pre-emptive retrial queue with orbital search and multiple vacations. Rajadurai Sundararaman Haripriya& Saranya (2017) studied M/G/1 retrial queueing system with orbit search under single vacation and starting failures. Transient analysis and its studies by Sherif Ammar (2015) explained Transient Analysis of an M/M/1 Queue with Impatient Behaviour and Multiple Vacations.

Muthu Ganapathi Subramanian Gopal Sekar, & Indhumathi (2019) analysed Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model with Impatient Customers. Indhumathi

Muthu Ganapathi Subramanian & Gopal Sekar (2020) studied Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model with Exhaustive Type Single Vacation, in Indhumathi Muthu Ganapathi Subramanian & Gopal Sekar (2018) they analysed finite source queues with catastrophe. Also, in Indhumathi Muthu Ganapathi Subramanian & Gopal Sekar (2021) they worked on Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model with Multiple Vacations and Bernoulli Schedule. Shan Gao & Jiting Wang (2020) worked on stochastic analysis of a preemptive retrial queue with orbital search and multiple vacation.

2 The Mathematical Model and Its Solutions

2.1 Model description

Consider a finite source retrial queuing system with multiple vacation and orbital search in which the primary arrival rate λ follows a Poisson distribution and the service time follows an exponential distribution with parameter μ. Further, we have assumed that the calling population is finite of size M. If the server is idle then the primary arrival will be served immediately and after completion of the service, it leaves the system. An effective algorithm is used to obtain transient probabilities and time dependent system performance measures. All the numerical values are done by using the software **SCILAB**.

2.2 Retrial queues

If the server is not idle then the arriving customer goes to orbit becomes a group of repeated customers. This group of sources of repeated customers may be viewed as a sort of queue. Every such source produces a Poisson process of repeated customers with intensity σ. If incoming repeated customers find the server free, it is served and leaves the system after service, while the source which produced this repeated customer disappears.

2.3 Multiple Vacation

The concept of multiple vacations is incorporated in this work in such a way that after completion of a service if the server finds no one in the orbit then he goes for a vacation. After completion of the vacation period, if the server finds atleast one customer in the orbit then he returns to the system and waiting for service otherwise he will go for vacation once again. The vacation time follows an exponential distribution with parameter α.

2.4 Orbital Search

After the completion of service, the server may search for the customer in orbit in sequence with some probability p to provide service right away (called orbit search) or with a certain probability 1-p remains idle. When the sever cannot find any customers during search of customers in orbit then he goes for vacation.

2.5 Retrial Policy

The retrial policy states that the probability of an customers from an orbit to try for service during the time interval (t, $t + \Delta t$) given that there were n customers in orbit at time t is $n\sigma \; \Delta t + O(\Delta t)$. This regulation for accessing the server from the retrial group is called a classical retrial policy.

3 Representation of Random Process

Let C(t) be the random variable which represents the number of customers in the orbit at time t and S(t) be the random variable which represents the status of the server at time t.

The random process is described as
$\{<$C(t), S(t)$>$ / C(t) = 1,2, 3. . . , F$_s$; S(t) = 0$\}\bigcup\{<$C(t), S(t)$>$ / C(t) = 0,1,2, 3. . . , F$_s$-1; S(t) = 1$\}$
$\bigcup\{<$C(t), S(t)$>$ / C(t) = 0,1,2, 3. . . , F$_s$; S(t) = 2$\}$, where
S(t) = 0 if the server is free/idle at time t,
S(t) = 1 if the server is busy at time t &
S(t) = 2 if the server is in vacation at time t.
We define,
$P_{n0}(t)$: Probability that the server is idle when there are n customers in the orbit at t.
$P_{n1}(t)$: Probability that the server is busy when there are n customers in the orbit at t.
$P_{n2}(t)$: Probability that the server in vacation when there are n customers in the orbit at t.
Kolmogorov balanced equations of this model are given below
The server is in idle:

$$\left.\begin{array}{l} {P_{10}}'(t) = -[(M-1)\lambda + \sigma]P_{10}(t) + \mu(1-p)P_{11}(t) + \alpha P_{12}(t) \\ {P_{20}}'(t) = -[(M-2)\lambda + 2\sigma]P_{20}(t) + \mu(1-p)P_{21}(t) + \alpha P_{22}(t) \\ ... \\ {P_{M0}}'(t) = -[M\sigma]P_{M0}(t) + \alpha P_{M2}(t) \end{array}\right\} \tag{1}$$

The server is in Busy:

$$\left.\begin{array}{l} {P_{01}}'(t) = -[(M-1)\lambda + \mu]P_{01}(t) + \sigma P_{10}(t) + \mu p P_{11}(t) \\ {P_{11}}'(t) = -[(M-2)\lambda + \mu(1-p) + \mu p]P_{11}(t) + (M-1)\lambda P_{10}(t) + (M-1)\lambda P_{01}(t) + 2\sigma P_{20}(t) + \mu p P_{21}(t) \\ {P_{21}}'(t) = -[(M-3)\lambda + \mu(1-p) + \mu p]P_{21}(t) + (M-2)\lambda P_{20}(t) + (M-2)\lambda P_{11}(t) + 3\sigma P_{30}(t) + \mu p P_{31}(t) \\ ... \\ {P_{M-11}}'(t) = -[(M-M)\lambda + \mu(1-p) + \mu p]P_{M-11}(t) + (M-(M-1))\lambda P_{M-10}(t) \\ \qquad +(M-(M-1))\lambda P_{M-21}(t) + M\sigma P_{M0}(t) \end{array}\right\} \tag{2}$$

The server is in vacation:

$$\left.\begin{array}{l} {P_{02}}'(t) = -[M\lambda]P_{02}(t) + \mu P_{01}(t) \\ {P_{12}}'(t) = -[(M-1)\lambda + \alpha]P_{12}(t) + M\lambda P_{02}(t) \\ {P_{22}}'(t) = -[(M-2)\lambda + \alpha]P_{22}(t) + (M-1)\lambda P_{12}(t) \\ .... \\ {P_{M2}}'(t) = -\alpha P_{M2}(t) + \lambda P_{M-12}(t) \end{array}\right\} \tag{3}$$

. . . .
In general,

$$\left.\begin{array}{l} {P_{n0}}'(t) = -[(M-n)\lambda + n\sigma]P_{n0}(t) + \mu(1-p)P_{n1}(t) + \alpha P_{n2}(t); n = 1,2,...,M-1 \\ {P_{n1}}'(t) = -[(M-(n+1))\lambda + \mu(1-p) + \mu p]P_{n1}(t) + (M-n)\lambda P_{n0}(t) + (M-n)\lambda P_{n-11}(t) \\ \qquad +(n+1)\sigma P_{n+10}(t) + \mu p P_{n+11}(t); for\, n = 1,2,...,M-2 \\ {P_{n2}}'(t) = -[(M-n)\lambda + \alpha]P_{n2}(t) + (M-(n-1))\lambda P_{n-12}(t); n = 1,2,...,M-1 \end{array}\right\} \tag{4}$$

The infinitesimal generator matrix L for this model is given below

$$L = \begin{pmatrix} L_{00} & L_{01} & L_{02} & L_{03} & L_{04} & ...... & L_{0M} \\ L_{10} & L_{11} & L_{12} & L_{13} & L_{14} & ...... & L_{1M} \\ L_{20} & L_{21} & L_{22} & L_{23} & L_{24} & ...... & L_{2M} \\ L_{30} & L_{31} & L_{32} & L_{33} & L_{34} & ...... & L_{3M} \\ . & . & . & . & . & ...... & ...... \\ L_{M-10} & L_{M-11} & L_{M-12} & L_{M-13} & L_{M-14} & .... & L_{M-1M} \\ L_{M0} & L_{M1} & L_{M2} & L_{M2} & L_{M3} & .... & L_{MM} \end{pmatrix}$$

The matrices $L_{00,}L_{01}, L_{10}, L_{11}, L_{21}, L_{22}, L_{32}, L_{33}, ..., L_{M-1M-1}, L_{M0}, L_{M2}$ are described in the L.

The infinitesimal transition rates of process ***X*** as follows

$$L_{00} = \begin{bmatrix} -((M-1)\lambda+\mu) & 0 \\ \mu & -M\lambda \end{bmatrix}$$

$$L_{i\,i} = \begin{bmatrix} -((M-i)\lambda+i\sigma) & \mu(1-p) & \alpha \\ (M-i)\lambda & -((M-(i+1))\lambda+\mu(1-p)+\mu p & 0 \\ 0 & 0 & -((M-i)\lambda+\alpha) \end{bmatrix} for\, i=1,2,...,M-1$$

$$L_{i-1\,i} = \begin{bmatrix} (M-i)\lambda & 0 \\ 0 & (M-(i-1))\lambda \end{bmatrix}; for\ i=1,2,...,M-1$$

$$L_{i\ i+1} = \begin{bmatrix} i\sigma & \mu p \\ 0 & 0 \end{bmatrix} for\ i=1,2,...,M-2$$

$$L_{M-1M} = \begin{bmatrix} M\sigma & 0 \\ 0 & 0 \end{bmatrix}$$

$$L_{M0} = \begin{bmatrix} -M\sigma & \alpha \end{bmatrix}; L_{M2} = \begin{bmatrix} \lambda & 0 & -\alpha \end{bmatrix}$$

Remaining all other entries are zero.
The equations (1), (2), (3) & (4) can be combined and expressed as

$$X'(t) = LX(t),\ \ where\ L = R^T$$
$$\&\ [X(t)]^T = [P_{01}(t), P_{02}(t),\ P_{10}(t), P_{11}(t), P_{12}(t), ..., P_{M-1\,0}(t), P_{M-1\,1}(t), P_{M-1\,2}(t), P_{M0}(t), P_{M2}(t)]$$

Solving the equations, we get, $X(t) = e^{tL} X_0$
When t = 0, $X_0 = X(0) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 \end{bmatrix}^T$

4 Description of Computational Method

The following effective computational procedure is used to find the Time dependent probabilities of number of customers in the orbit at time *t*. The Time dependent Probabilities is denoted by

$$X(t) = [P_{01}(t), P_{02}(t)\,, P_{10}(t), P_{11}(t), P_{12}(t), ..., P_{M-1\,0}(t)P_{M-1\,1}(t)P_{M-1\,2}(t), P_{M0}(t), P_{M2}(t)]^T$$

Step 1: Find the Eigen values this finite order matrix tL.
Step 2: Find the corresponding Eigen vectors x_i (t) = e^{λ_i} x_i
Step 3: The matrix ϕ(t) = (x_1 (t), x_2 (t) ... x_M (t)) is called fundamental matrix and it is non-singular.
Step 4: Find x (t) =ϕ (t)$\overline{c}$, where $\overline{c}$=$(c_1, c_2, \ldots, c_M)'$ is a constant column vector.
Step 5: using x (0) = x_0, we obtain $\overline{c}$=$\phi^{-1}(0)$ x_0,
Step 6: put$\overline{c}$ in step 4, we get x (t) =ϕ (t)$\phi^{-1}(0)$ x_0.
Step 7: we know that X (t) = e^{At} x_0
Step 8: substitute X (t) in step 4 and comparing x_0, we get e^{At} =ϕ(t)[ϕ (0)$]^{-1}$
Step 9: Extract the first column of this Exponential matrix tL & store in X (t).
Step 10: This probability vector **X(t)** provides time dependent probabilities of number of customers in the queue at time t.

5 System Performance Measures

The following system measures are used to bring out the transient behaviour of the finite source retrial queueing model with multiple vacation and orbital search under Computational study for various values of M, $\lambda, \mu, \sigma, \alpha$, p and t.

1. Probability that the server is idle at time t = $P_{idle}(t) = \sum_{n=1}^{M} P_{n0}(t)$

2. Probability that the server is busy at time t = $P_{busy}(t) = \sum_{n=0}^{M-1} P_{n1}(t)$

3. Probability that the server is in vacation at time t = $P_{vacation}(t) = \sum_{n=0}^{M} P_{n2}(t)$

4. Mean number of customers in the orbit at time t =$L_q(t)$=$\sum_{n=1}^{M} n\left(P_{n0}(t) + P_{n2}(t)\right)$+$\sum_{n=1}^{M-1} nP_{n1}(t)$

6 Numerical Computations

Transient probabilities and system performance measures of this model have been done and expressed in the form of Tables for various values $\mathrm{M}, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t are given below:

Table 1: Transient probability distribution of number of customers in the orbit when the server is idle in the system for λ=*3*, μ=*4*, α=*2*, σ=*2*, *p=0.3, M=5* and various values of t.

t	P10(t)	P20(t)	P30(t)	P40(t)	P50(t)
0.1	0.0492	0.0167	0.0029	0.0002	0.0000
0.2	0.0622	0.0469	0.0184	0.0032	0.0001
0.3	0.0491	0.0602	0.0386	0.0112	0.0007
0.4	0.0348	0.0599	0.0541	0.0223	0.0019
0.5	0.0248	0.0547	0.0633	0.0335	0.0035
0.6	0.0184	0.0489	0.0681	0.0433	0.0049
0.7	0.0145	0.0439	0.0706	0.0513	0.0058
0.8	0.0119	0.0401	0.0720	0.0577	0.0062
0.9	0.0102	0.0373	0.0728	0.0627	0.0060
1	0.0090	0.0352	0.0734	0.0666	0.0056

Table 2: Transient probability distribution of number of customers in the orbit when the server is idle in the system for λ=*5*, μ=*6*, α=*4*, σ=*3*, *p=0.5, M =7* and various values of t.

T	P10(t)	P20(t)	P30(t)	P40(t)	P50(t)	P60(t)	P70(t)
0.1	0.0255	0.0254	0.0156	0.0059	0.0013	0.0001	0.0000
0.2	0.0086	0.0194	0.0270	0.0237	0.0123	0.0031	0.0002
0.3	0.0028	0.0095	0.0210	0.0299	0.0257	0.0109	0.0010
0.4	0.0011	0.0049	0.0145	0.0282	0.0336	0.0197	0.0021
0.5	0.0006	0.0029	0.0105	0.0252	0.0373	0.0269	0.0028
0.6	0.0003	0.0020	0.0081	0.0227	0.0390	0.0321	0.0028
0.7	0.0002	0.0015	0.0068	0.0210	0.0399	0.0358	0.0024
0.8	0.0002	0.0012	0.0060	0.0199	0.0404	0.0382	0.0019
0.9	0.0002	0.0011	0.0056	0.0192	0.0407	0.0398	0.0014
1	0.0001	0.0010	0.0053	0.0189	0.0409	0.0407	0.0010

Table 3: Transient probability distribution of number of customers in the orbit when the server is idle in the system for λ=*6*, μ=*7*, α=*5*, σ=*4*, *p=0.8, M =8* and various values of t.

t	P10(t)	P20(t)	P30(t)	P40(t)	P50(t)	P60(t)	P70(t)	P80(t)
0.1	0.0076	0.0105	0.0096	0.0059	0.0024	0.0006	0.0001	0.0000
0.2	0.0013	0.0037	0.0074	0.0104	0.0099	0.0060	0.0019	0.0002
0.3	0.0003	0.0012	0.0034	0.0073	0.0109	0.0106	0.0056	0.0010
0.4	0.0001	0.0005	0.0018	0.0050	0.0097	0.0124	0.0085	0.0016
0.5	0.0001	0.0003	0.0012	0.0038	0.0087	0.0131	0.0102	0.0017
0.6	0.0000	0.0002	0.0009	0.0031	0.0080	0.0134	0.0114	0.0014
0.7	0.0000	0.0002	0.0007	0.0028	0.0076	0.0136	0.0121	0.0010
0.8	0.0000	0.0001	0.0007	0.0026	0.0074	0.0138	0.0126	0.0007
0.9	0.0000	0.0001	0.0006	0.0025	0.0073	0.0138	0.0128	0.0004
1	0.0000	0.0001	0.0006	0.0025	0.0072	0.0139	0.0130	0.0003

From Tables 1, 2, and 3 show transient probabilities of number of customers in the orbit when the server is idle in the system for various values of $\mathrm{M}, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t, we infer that the value of t increases then the Transient state goes to steady state (i.e.) $P_{idle}(t) \to P_{idle}$.

Table 4: Transient probability distribution of number of customers in the orbit when the server is busy in the system λ=*3*, μ=*4*, α=*2*, σ=*2*, *p=0.3, M=5* and various values of t.

t	P01(t)	P11(t)	P21(t)	P31(t)	P41(t)
0.1	0.2220	0.3175	0.1617	0.0368	0.0032
0.2	0.0664	0.2238	0.2403	0.1176	0.0222
0.3	0.0295	0.1543	0.2465	0.1854	0.0554
0.4	0.0175	0.1130	0.2329	0.2327	0.0948
0.5	0.0121	0.0868	0.2150	0.2641	0.1348
0.6	0.0090	0.0695	0.1978	0.2843	0.1716
0.7	0.0071	0.0577	0.1829	0.2968	0.2034
0.8	0.0058	0.0497	0.1710	0.3047	0.2294
0.9	0.0050	0.0440	0.1619	0.3098	0.2499
1	0.0044	0.0401	0.1552	0.3133	0.2654

Table 5: Transient probability distribution of number of customers in the orbit when the server is busy in the system for λ=*5*, μ=*6*, α=*4*, σ=*3*, *p=0.5, M=7* and various values of t.

t	P01(t)	P11(t)	P21(t)	P31(t)	P41(t)	P51(t)	P61(t)
0.1	0.0485	0.1748	0.2496	0.1964	0.0891	0.0219	0.0023
0.2	0.0081	0.0517	0.1402	0.2250	0.2203	0.1221	0.0294
0.3	0.0026	0.0195	0.0721	0.1688	0.2522	0.2212	0.0872
0.4	0.0011	0.0090	0.0404	0.1208	0.2379	0.2818	0.1533
0.5	0.0005	0.0049	0.0255	0.0905	0.2152	0.3122	0.2100
0.6	0.0003	0.0032	0.0182	0.0725	0.1967	0.3266	0.2517
0.7	0.0002	0.0023	0.0143	0.0621	0.1841	0.3338	0.2796
0.8	0.0002	0.0019	0.0121	0.0560	0.1762	0.3377	0.2972
0.9	0.0002	0.0016	0.0109	0.0525	0.1714	0.3401	0.3079
1	0.0001	0.0015	0.0103	0.0504	0.1687	0.3417	0.3143

Table 6: Transient probability distribution of number of customers in the orbit when the server is busy in the system for λ=*6*, μ=*7*, α=*5*, σ=*4*, *p=0.8, M =8* and various values of t.

t	P01(t)	P11(t)	P21(t)	P31(t)	P41(t)	P51(t)	P61(t)	P71(t)
0.1	0.0262	0.1062	0.2039	0.2357	0.1718	0.0775	0.0199	0.0022
0.2	0.0038	0.0219	0.0707	0.1543	0.2311	0.2282	0.1341	0.0356
0.3	0.0010	0.0067	0.0273	0.0813	0.1761	0.2642	0.2462	0.1074
0.4	0.0004	0.0028	0.0132	0.0474	0.1284	0.2486	0.3070	0.1822
0.5	0.0002	0.0015	0.0079	0.0321	0.1006	0.2279	0.3339	0.2383
0.6	0.0001	0.0010	0.0056	0.0249	0.0855	0.2133	0.3458	0.2739
0.7	0.0001	0.0008	0.0046	0.0214	0.0774	0.2047	0.3514	0.2945
0.8	0.0001	0.0007	0.0041	0.0195	0.0732	0.2000	0.3544	0.3059
0.9	0.0001	0.0006	0.0038	0.0186	0.0710	0.1975	0.3561	0.3120
1	0.0001	0.0006	0.0037	0.0181	0.0698	0.1963	0.3571	0.3152

From Table 4, 5, and 6 show transient probabilities of number of customers in the orbit when the server is in busy for various values of $\mathrm{M}, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t, We infer that the value of t increases then the Transient state goes to steady state (i.e.)$P_{busy}(t) \rightarrow P_{busy}$.

Table 7: Transient probability distribution of number of customers in the orbit when the server is vacation with orbital search in the system for *λ=3, μ=4, α=2, σ=2, p=0.3, M =5* and various values of t.

t	P02(t)	P12(t)	P22(t)	P32(t)	P42(t)	P52(t)
0.1	0.0874	0.0657	0.0284	0.0071	0.0010	0.0001
0.2	0.0418	0.0598	0.0552	0.0310	0.0096	0.0013
0.3	0.0176	0.0328	0.0469	0.0436	0.0231	0.0053
0.4	0.0083	0.0160	0.0294	0.0395	0.0318	0.0112
0.5	0.0047	0.0081	0.0164	0.0288	0.0327	0.0168
0.6	0.0031	0.0046	0.0090	0.0187	0.0283	0.0204
0.7	0.0023	0.0030	0.0052	0.0115	0.0221	0.0216
0.8	0.0018	0.0022	0.0033	0.0071	0.0161	0.0209
0.9	0.0015	0.0017	0.0023	0.0045	0.0113	0.0190
1	0.0013	0.0015	0.0018	0.0030	0.0079	0.0165

Table 8: Transient probability distribution of number of customers in the orbit when the server is vacation with orbital search in the system for *λ=5, μ=6, α=4, σ=3, p=0.5, M=7* and various values of t.

t	P02(t)	P12(t)	P22(t)	P32(t)	P42(t)	P52(t)	P62(t)	P72(t)
0.1	0.0209	0.0347	0.0390	0.0292	0.0144	0.0045	0.0008	0.0001
0.2	0.0028	0.0063	0.0136	0.0232	0.0281	0.0223	0.0104	0.0021
0.3	0.0007	0.0012	0.0027	0.0068	0.0145	0.0218	0.0198	0.0081
0.4	0.0002	0.0004	0.0006	0.0016	0.0048	0.0117	0.0186	0.0136
0.5	0.0001	0.0002	0.0002	0.0004	0.0014	0.0048	0.0124	0.0155
0.6	0.0001	0.0001	0.0001	0.0002	0.0004	0.0017	0.0069	0.0142
0.7	0.0000	0.0001	0.0001	0.0001	0.0002	0.0006	0.0035	0.0116
0.8	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0016	0.0087
0.9	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0008	0.0063
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0044

Table 9: Transient probability distribution of number of customers in the orbit when the server is vacation with orbital search in the system *λ=6, μ=7, α=5, σ=4, p=0.8, M =8*and various values of t.

t	P02(t)	P12(t)	P22(t)	P32(t)	P42(t)	P52(t)	P62(t)	P72(t)	P82(t)
0.1	0.0089	0.0175	0.0265	0.0288	0.0220	0.0115	0.0039	0.0008	0.0001
0.2	0.0009	0.0017	0.0037	0.0081	0.0148	0.0199	0.0181	0.0099	0.0024
0.3	0.0002	0.0003	0.0005	0.0011	0.0030	0.0074	0.0135	0.0153	0.0080
0.4	0.0001	0.0001	0.0001	0.0002	0.0005	0.0017	0.0053	0.0111	0.0112
0.5	0.0000	0.0000	0.0001	0.0001	0.0001	0.0004	0.0016	0.0057	0.0106
0.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0025	0.0082
0.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0010	0.0057
0.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004	0.0038
0.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0024
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0015

From Table 7, 8, and 9 show transient probabilities of number of customers in the orbit when the server is vacation in the system for various values of $\mathrm{M}, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t, we infer that the value of t increases then the transient state goes to steady state (i.e.)$P_{vacation}(t) \rightarrow P_{vacation}$.

Table 10: Time dependent system measures for λ = *3*, μ = *4*, α = *2*, σ = *2*, *p = 0.3*, *M = 5* & various values of t.

t	$\mathbf{P}_{idle}$(t)	$\mathbf{P}_{busy}$(t)	$\mathbf{P}_{vacation}$ (t)	$\mathbf{L}_q$(t)
0.1	0.0691	0.7412	0.1897	1.0045
0.2	0.1309	0.6704	0.1987	1.6788
0.3	0.1598	0.6710	0.1692	2.1345
0.4	0.1730	0.6909	0.1362	2.4481
0.5	0.1797	0.7129	0.1074	2.6657
0.6	0.1837	0.7322	0.0841	2.8167
0.7	0.1862	0.7480	0.0658	2.9209
0.8	0.1879	0.7606	0.0515	2.9924
0.9	0.1890	0.7706	0.0404	3.0410
1	0.1898	0.7783	0.0319	3.0735

Table 11: Time dependent system measures for λ=*5*, μ=*6*, α=*4*, σ=*3*, *p=0.5*, *M=7* &various values of t.

t	$\mathbf{P}_{idle}$(t)	$\mathbf{P}_{busy}$(t)	$\mathbf{P}_{vacation}$ (t)	$\mathbf{L}_q$(t)
0.1	0.0738	0.7825	0.1437	2.1830
0.2	0.0944	0.7969	0.1087	3.3846
0.3	0.1009	0.8236	0.0755	4.0836
0.4	0.1042	0.8442	0.0516	4.4920
0.5	0.1061	0.8589	0.0350	4.7294
0.6	0.1071	0.8692	0.0237	4.8662
0.7	0.1076	0.8764	0.0160	4.9443
0.8	0.1079	0.8813	0.0109	4.9883
0.9	0.1080	0.8847	0.0074	5.0127
1	0.1080	0.8870	0.0050	5.0259

Table 12: Time dependent system measures for λ=*6*, μ=*7*, α=*5*, σ=*4*, *p=0.8*, *M =8* and various values of t.

t	$\mathbf{P}_{idle}$(t)	$\mathbf{P}_{busy}$(t)	$\mathbf{P}_{vacation}$ (t)	$\mathbf{L}_q$(t)
0.1	0.0367	0.8433	0.1200	2.8592
0.2	0.0409	0.8797	0.0794	4.3077
0.3	0.0405	0.9101	0.0494	5.0739
0.4	0.0397	0.9300	0.0303	5.4784
0.5	0.0390	0.9425	0.0186	5.6907
0.6	0.0384	0.9502	0.0114	5.8011
0.7	0.0381	0.9550	0.0070	5.8581
0.8	0.0378	0.9579	0.0043	5.8871
0.9	0.0377	0.9597	0.0026	5.9016
1	0.0376	0.9608	0.0016	5.9088

From Table 10, 11, and 12 showing that the time dependent system measures for various values of $F_s, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t, We infer that

1. $P_{busy}(t)$ increases as arrival rate λ increases for all values of t.
2. $P_{vacation}(t)$ decreases as arrival rate λ increases for all values of t.
3. As the value of t increases and for various values of $\mathrm{M}, \lambda, \mu, \sigma, \alpha, \mathrm{p}$ and t, then the transient state goes to steady state as follows:
   (i.e.)$P_{idle}(t) \rightarrow P_{idle,}P_{busy}(t) \rightarrow P_{busy,}\ P_{vacation}(t) \rightarrow P_{vacation,}L_q(t) \rightarrow L_q$

7 Special Case

As $\sigma \rightarrow$ 0 and p$\rightarrow$ 0, this model coincides with transient behaviour of single server finite source queueing model with multiple vacation.

8 Conclusion

A new computational approach was used to evaluate the transient solution of finite source retrial queueing model with multiple vacation and orbital search using Fundamental matrix method and infinitesimal generator matrix. Numerical studies have been done in an elaborate manner to determine the transient behaviour of probability distributions and time dependent System performance measures. This work was used by many fields such as manufacturing, computer science, reliability theory, management science and call center etc. In future we can extend this work with multiple servers, including many parameters such as batch arrival, bulk service, priority services etc.

9 Acknowledgement

I wish to express my sincere gratitude and heartfelt thanks to my philosopher Dr.A.Muthu Ganapathi Subramanian. I would like to express my sincere thanks to Dr.Gopal Sekar, HOD. Also, Words are inadequate, to express my heart rendering gratitude to my family members.

10 Conflicts of Interest

"The authors declare no conflict of interest."

References

Artalejo (2010). Accessible bibliography on retrial queues: Progress in 2000-2009, *Mathematical and Computer Modelling*, *51*(9-10), 1071-1081. https://doi.org/10.1016/j.mcm.2009.12.011

Falin, G. (1990). A survey of retrial queues, *Queueing System*, *7*, 127-167. https://doi.org/10.1007/BF01158472

Indhumathi, J., Muthu Ganapathi Subramanian, A. & Gopal Sekar (2018). Analysis of finite source queueing system with catastrophe, *International Journal of Computer Sciences and Engineering*, *7*(5), 5-9. https://doi.org/10.26438/ijcse/v7si5.59

Indhumathi, J., Muthu Ganapathi Subramanian, A. & Gopal Sekar (2020). Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model with Exhaustive Type Single Vacation, *Journal of Scientific Computing*, *9*(4), 65-74. https://jscglobal.org/volume-9-issue-4-2020/

Indhumathi, J., Muthu Ganapathi Subramanian, A. & Gopal Sekar (2021). Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model with Multiple Vacations and Bernoulli Schedule, *GIS Science Journal*, *8*(11), 610-620. https://gisscience.net/volume-8-issue-11-2021/

Miller, D. R. (1983). Matrix-geometric solutions in stochastic models: An algorithmic approach, *The Johns Hopkins University Press, Baltimore*, 352. https://doi.org/10.1002/net.3230130219

Muthu Ganapathi Subramanian, A., Gopal Sekar& Indhumathi,J. (2019). Computational Approach for Transient Behaviour of Finite Source Retrial Queueing Model with Impatient Customers, *Science, Technology and Development*, *8*(X), 7-13. https://journalstd.com/special-issue-1/

Rajadurai , P. Sundararaman , M. Haripriya, T.V. & Saranya, S.(2017). A study on M/G/1 retrial queueing system with orbit search under single vacation and starting failures, *International Journal of Pure and Applied Mathematics*, *133*(7), 102-110. https://doi.org/10.1051/ro/2018117

Shan Gao & Jiting Wang (2020). Stochastic analysis of a preemptive retrial queue with orbital search and multiple vacation, , *KAIRO-Operation Research*, *54*(1), 231-241. https://doi.org/10.1051/ro/2018117

Sherif Ammar (2015). Transient Analysis of an M/M/1 Queue with Impatient Behaviour and Multiple Vacations, *Applied Mathematics and Computations*, *260*, 97-105. https://doi.org/10.1016/j.amc.2015.03.066

Velika I. Dragieva; Tuan Phung-Duc (2020). A finite-source M/G/1 retrial queue with outgoing calls, *Annals of operation Research*, *293*, 101-121. https://link.springer.com/article/10.1007/s10479-019-03359-z