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Discriminating Overlapping Models: An Empirical Study on Skewed and Leptokurtic Stock Prices

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Abstract

One of the important problems in modeling the data is the identification of suitable statistical model that fits the observed data. Many a time, the choice depends on the literature, type of distribution parameters that are needed, some specific distributional properties that the distribution should have etc. We consider four probability distributions to model skewed and leptokurtic data, and use the ratio of maximized likelihood statistic (RML) and the Kolmogorov-Smirnov (KS) distance to discriminate among the various overlapping family of distributions. Out of the probability models considered, we found that both RML and KS distance supports Log-logistic and Cauchy as a suitable fit for the stock prices. We also conduct a Wald test for samples from Log-logistic distribution.

Keywords: Cauchy distribution, Kolmogorov-Smirnov distance, Log-logistic distribution, overlapping models, ratio of maximum likelihood, Wald test

1 Introduction

The identification of suitable statistical model that fits the observed data helps in providing reliable inferences. However, identifying the appropriate model is an important problem in statistics, as the choice depends on the literature or some specific distributional properties that the distribution should attain. The discrimination problem between statistical models was first addressed by Cox (1961, 1962). Cox used the likelihood ratio (LR) statistic to discriminate between two-parameter Log-normal distribution and the Exponential distribution. Using the asymptotic distribution of LR statistic, he also obtained the probability of correct selection (PCS). One of the drawbacks of Cox's approach was, it is difficult to get the asymptotic distribution of the discrimination statistic analytically for most pairs of distributions. Secondly, the PCS were valid only for large samples. Dumonceaux and Antle (1973a) discussed the problem of discriminating between Log-normal and Weibull distributions. Ashkar and Aucoin (2012) deliberated the problem of discriminating between Log-normal and Log-logistic distributions in the context of hydrological events. Gupta and Kundu (2004) used maximized likelihood ratio in discriminating Gamma and Generalized exponential distributions. Gupta and Kundu (2003) considered the problem of discriminating Weibull and generalized exponential distributions. Kundu and Manglick (2004) studied the problem of discriminating Weibull and Log-normal distributions. Kundu et al. (2005) discussed the discrimination problem between Log-normal and generalized exponential distributions. Raqab et. al (2017) discussed the closeness of Lindley distribution to Weibull and Gamma distribution using likelihood ratio (LR), asymptotic LR and minimum Kolmogorov distance. Niu et. al (2023) discussed the discrimination problem between three positively skewed lifetime distributions using LR, minimum Kolmogorov distance and sequential probability ratio test. Ghosh (2023) considered the discrimination problem between two bounded distribution namely generalized beta distribution and the Kumaraswamy distribution using LR and pseudo-distance methods. A general discussion on discriminating two distribution functions can be seen in Atkinson (1969, 1970), Chambers and Cox (1967), Chen (1980), Dumonceaux et al. (1973b), Jackson (1968) and Dyer (1973).

It is well known that the basic properties of the data (or distribution) can be identified by analyzing the four characteristics namely central tendency, dispersion, skewness and kurtosis. The characteristics skewness (positive or negative) and kurtosis deals with the shape of the distribution. According to DeCarlo (1997) the two components of kurtosis are tailedness and peakedness. But it can also show the effect of any one of these components say heavy tails. Dyson (1943) and Finucan (1964) showed that the presence of excess kurtosis in the data leads to overlapping of densities. If the data has these properties, most of the time normal distribution (skewness zero and kurtosis 3) may not be a suitable model to describe the data.

Several statistical models like Gamma, Log-logistic, Log-normal distributions are available for modeling skewed data (Johnson et al. (1995)). Here, we consider four models namely the Log-logistic (LL) distribution, the Log-normal (LN) distribution, the normal distribution and the Cauchy distribution. It is observed that the shape of both the probability density functions (pdf) of LL and LN can be similar in nature for certain values of the parameters. Based on the histogram if it is obvious that the data are positively skewed then either LL or LN can be used for modeling. The normal distribution is a symmetric distribution whereas the Cauchy distribution is a fat-tailed distribution. An excellent survey on these distributions can be found in Johnson et al. (1995).

The paper is organized as follows. In section 2, we briefly give an overview of the LL, the LN, the normal and the Cauchy distributions. We briefly discuss the ratio of maximum likelihoods and the KS distance in section 3. Analysis of real data is performed in section 4. The conclusion is provided in section 5.

2 Overview of Log-logistic, Log-normal, Normal and Cauchy Distributions

In economics, the LL distribution, also known as Fisk distribution, is applied to model wealth in a society or income. It is also used to describe events whose rate increases initially and then decreases, extreme events such as annual maximum one day rainfall etc. For certain values of the parameters the shapes of both LL and LN distributions are similar in nature and also provides a good approximation to LN distribution. A continuous random variable X is said to follow LL distribution denoted by $LL(\eta, \omega)$ if it has the following pdf

$$f_{LL}(X) = \frac{\left(\frac{\eta}{\omega}\right)\left(\frac{x}{\omega}\right)^{\eta-1}}{\left[1+\left(\frac{x}{\omega}\right)^{\eta}\right]^2}, \quad 0 \le x < \infty,\ \eta > 0,\ \omega > 0$$

where (η, ω) are respectively the shape and scale parameters of the LL distribution. The distribution is positively skewed and depends only on the shape parameter η.

To model income, exchange rates, stock prices, time to repair a maintainable system, extreme observations like monthly maximum rainfall, natural growth process, etc. are modeled using the celebrated LN distribution. It is a continuous probability distribution of a random variable whose natural logarithm is normally distributed. A continuous random variable X is said to follow the LN distribution denoted by $LN(\mu, \sigma)$ if it has the following pdf

$$f_{LN}(x) = \frac{1}{\sqrt{2\pi}\,x\sigma} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), \quad x, \sigma > 0,\ -\infty < \mu < \infty$$

where (μ, σ) are respectively the mean and standard deviation of the variable's natural logarithm. The distribution is positively skewed and depends only the scale parameter σ.

The Gaussian (or Normal) distribution, a celebrated symmetric continuous probability distribution that is often used to describe many real phenomena in the natural and social sciences. Due to its unique properties many classical statistical techniques are developed based on the normality assumption. A continuous random variable X is said to follow a normal distribution denoted by $N(\mu_1, \sigma_1)$ if it has the following pdf

$$f_N(x) = \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right), \quad -\infty < x, \mu_1 < \infty,\ \sigma_1 > 0$$

where (μ_1, σ_1) are respectively the mean and standard deviation of the distribution. It is well known that the distribution has skewness zero and kurtosis 3.

Another continuous probability distribution on the real line is the Cauchy distribution. It is well-known that the mean and variance of this distribution does not exist. In computational finance, this distribution is used to model Value at Risk (VaR) and fat tails. A continuous random variable X is said to follow a Cauchy distribution denoted by $C(\delta, \gamma)$ if it has the following pdf

$$f_C(x) = \frac{1}{\pi}\left[\frac{\gamma}{(x-\delta)^2 + \gamma^2}\right], \quad -\infty < x, \delta < \infty,\ \gamma > 0$$

where (δ, γ) are respectively the location and scale parameters of the Cauchy distribution.

3 The Maximized Likelihood Ratio and Kolmogorov-Smirnov Distance

In this section, we use ratio of maximized likelihoods and minimum KS distance criteria to choose best suitable model among the three distributions. Let $(\hat{\eta}, \hat{\omega})$, $(\hat{\mu}, \hat{\sigma})$, $(\hat{\mu_1}, \hat{\sigma_1})$ and $(\hat{\delta}, \hat{\gamma})$ denotes the ML estimates (MLE) of (η, ω), (μ, σ), (μ_1, σ_1) and (δ, γ) respectively. Since no analytical solution to the likelihood equation is available in the case of LL and Cauchy distributions, the ML estimates of the parameters (η, ω) and (δ, γ) can be obtained numerically.

3.1 The Maximized Likelihood Ratio Procedure

Let $f(x; \alpha, \beta)$ be the pdf of the random variable X and $g(x; \nu, \tau)$ be the pdf of the competing model where α,β,ν and τ are the parameters of the models f and g respectively. The objective is to identify a suitable model for X or discriminate between f and g. Let L_f and L_g denotes the likelihood functions of f and g respectively. Then the ratio of maximized likelihoods (RML) evaluated at the respective MLEs of the parameters of f and g is defined as $\text{ML} = \frac{L_f(\hat{\alpha}, \hat{\beta})}{L_g(\hat{\upsilon}, \hat{\tau})}$ where $(\hat{\alpha}, \hat{\beta})$ and $(\hat{\nu}, \hat{\tau})$ are the MLEs of the parameters of f and g. Also, let $T = \ln(\text{RML})$. Then the decision rule for discriminating f and g distributions is to choose f if $T > 0$ otherwise choose g as preferred model.

Here, we try to discriminate a suitable statistical model for the stock prices that exhibits 1) positive skewness with high kurtosis and 2) negative skewness with moderate/high kurtosis.

Case 1: Positive Skewness with High Kurtosis

Let $\{x_1, x_2, \ldots, x_n\}$ is independently and identically distributed (iid) random sample of size n assumed to be generated from a $LL(\eta, \omega)$ distribution or a $LN(\mu, \sigma)$ distribution. Consider the hypothesis H_0: data from LL distribution against H_1:data from LN distribution. In other words, we are interested to test the following hypothesis: $H_0 : X \sim LL(\eta, \omega)$ against $H_1 : X \sim LN(\mu, \sigma)$. For a random sample of size n, the likelihood functions of LL and LN distributions are respectively given by:

$$L_{LL} = \left(\frac{\eta}{\omega}\right)^n \prod_{i=1}^{n} \frac{\left(\frac{x_i}{\omega}\right)^{\eta-1}}{\left[1+\left(\frac{x_i}{\omega}\right)^{\eta}\right]^2}$$

and

$$L_{LN} = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n \prod_{i=1}^{n} \frac{\exp\left(-\frac{(\ln x_i-\mu)^2}{2\sigma^2}\right)}{x_i}.$$

Then we have, $T = \ln(L_{\mathrm{LL}}(\hat{\eta},\hat{\omega})) - \ln(L_{\mathrm{LN}}(\hat{\mu},\hat{\sigma}))$. Simplifying we get,

$$T = n\ln\left(\frac{\hat{\eta}\sqrt{2\pi}\,\hat{\sigma}}{\hat{\omega}^{\hat{\eta}}}\right) + \hat{\eta}\sum_{i=1}^{n}\ln x_i - 2\sum_{i=1}^{n}\ln\left(1+\left(\frac{x_i}{\hat{\omega}}\right)^{\hat{\eta}}\right) + \sum_{i=1}^{n}\frac{(\ln x_i-\hat{\mu})^2}{2\hat{\sigma}^2}. \quad (3.1)$$

The decision rule for discriminating $LL(\eta,\omega)$ and $LN(\mu,\sigma)$ distributions is to choose LL distribution if $T > 0$ otherwise choose LN distribution as preferred model.

Case 2: Negative Skewness with Moderate/High Kurtosis

Let $\{x_1, x_2, \ldots, x_n\}$ is independently and identically distributed (iid) random sample of size n assumed to be generated from a $N(\mu_1,\sigma_1)$ distribution or a $C(\delta,\gamma)$ distribution. Consider the hypothesis H_0: data from normal distribution against H_1:data from Cauchy distribution. In other words, we are interested to test the following hypothesis: $H_0 : X \sim N(\mu_1,\sigma_1)$ against $H_1 : X \sim C(\delta,\gamma)$. For a random sample of size n, the likelihood functions of normal and Cauchy distributions are respectively given by:

$$L_N = \left(\frac{1}{\sqrt{2\pi}\,\sigma_1}\right)^n \prod_{i=1}^{n} \exp\left(-\frac{(x_i-\mu_1)^2}{2\sigma_1^2}\right)$$

and

$$L_C = \left(\frac{\gamma}{\pi}\right)^n \prod_{i=1}^{n} \frac{1}{(x_i-\delta)^2+\gamma^2}.$$

Then we have, $T = \ln\left(L_N(\hat{\mu}_1,\hat{\sigma}_1)\right) - \ln\left(L_C(\hat{\delta},\hat{\gamma})\right)$. Simplifying we get,

$$T = -\,n\ln\left(\sqrt{\frac{2}{\pi}}\,\hat{\sigma}_1\hat{\gamma}\right) - \sum_{i=1}^{n}\left(\frac{x_i-\hat{\mu}_1}{2\hat{\sigma}_1}\right)^2 + \sum_{i=1}^{n}\ln\left[(x_i-\hat{\delta})^2+\hat{\gamma}^2\right]. \quad (3.2)$$

The decision rule for discriminating $N(\mu_1,\sigma_1)$ and $C(\delta,\gamma)$ distributions is to choose normal distribution if $T > 0$ otherwise choose Cauchy distribution as preferred model.

Note that since the exact distribution of the test statistic T is difficult to obtain, Dey and Kundu (2009) showed that the distribution of T can be approximated asymptotically to a normal distribution with asymptotic mean $AM_{LL}(T) = \bar{T}$ and asymptotic variance $AV_{LL}(T) = V(T)$.

3.2 The Kolmogorov-Smirnov (KS) Procedure

The KS distance is a preferred way of measuring how similar the two probability distributions are. Let $F_n(x)$ denote the empirical cumulative distribution function (CDF) computed based on the data and $F(x)$ denotes the theoretical CDF. Then KS distance is defined as

$$D_n = \sup_x \left|F_n(x) - F(x)\right|.$$

Let $F(\hat{\alpha}, \hat{\beta})$ and $G(\hat{\nu}, \hat{\tau})$ denotes the CDFs computed at the MLEs of the parameters of pdfs of f and g respectively. The KS distance associated with f and g are defined as:

$$KS_f = \sup_x \left| F(\hat{\alpha}, \hat{\beta}) - F_n(x) \right|$$

and

$$KS_g = \sup_x \left| G(\hat{\nu}, \hat{\tau}) - G_n(x) \right|.$$

The decision rule for discriminating between f and g distributions is to choose the preferred model as f if $\min(KS_f, KS_g) = KS_f$ otherwise the preferred model as g.

4 Discrimination of Statistical Models using Real Data Set

Here we consider adjusted closing prices of 5 companies from different industries like pharma, entertainment etc. from 1st April, 2019 to 29th March, 2023 and 29th November, 2022 to 29th November, 2023 (`https://finance.yahoo.com/quote/%5ENSEI/history/`). The gross returns were calculated using $R = p_{t-1}^{-1} p_t$. A statistical summary of the returns is provided in table 1.

Industry/Company	n	Min.	Median	Mean	Max.	SD	Skewness	Kurtosis
Entertainment	990	0.848	0.999	1.004	1.20	0.035	1.345	9.064
Medical	990	0.850	1.000	1.002	1.154	0.024	0.600	9.994
Pharma	247	0.964	0.999	1.001	1.049	0.016	0.474	5.331
Airline	247	0.954	1.001	1.002	1.047	0.015	-0.053	3.412
Life Insurance	247	0.891	1.000	1.001	1.067	0.016	-0.433	12.235

Table 1: Summary statistics of stock price returns

It is observed from the above table that the average returns and the SDs are more or less same for companies selected from the above-mentioned industries. We also check the presence of any outliers or extreme observations and we found that (see figure 1) there are several extreme observations in the data. It can also be observed that some returns are positively skewed and some are negatively skewed but exhibits high kurtosis. One reason for this phenomenon may be the presence of extreme values in the data. In fact, for airline stock prices, the skewness is almost near to zero and the kurtosis is almost near to 3 in comparison with life insurance.

Data Set 1: This data set consists of adjusted closing prices of the company chosen from the entertainment industry for the period 1st April, 2019 to 29th March, 2023. For comparison, we plot (see figure 2) the histogram and the empirical cumulative distribution functions (CDF) of the data. It is obvious from the plot that the fitted densities of LL and LN are very close to each other and a discrimination of these models is next to impossible. Based on the observed data, the ML estimates of the parameters, log-likelihoods and the KS distance values of the LL and LN distributions are provided in the following table 2.

Model	ML estimates	Log-likelihoods	KS distance
Log-logistic	$(\hat{\eta} = 56.336,\ \hat{\omega} = 1.001)$	1990.145	0.048
Log-normal	$(\hat{\mu} = 0.0032,\ \hat{\sigma} = 0.0345)$	1923.923	0.083

Table 2: ML estimates of the parameters, log-likelihoods and KS distance

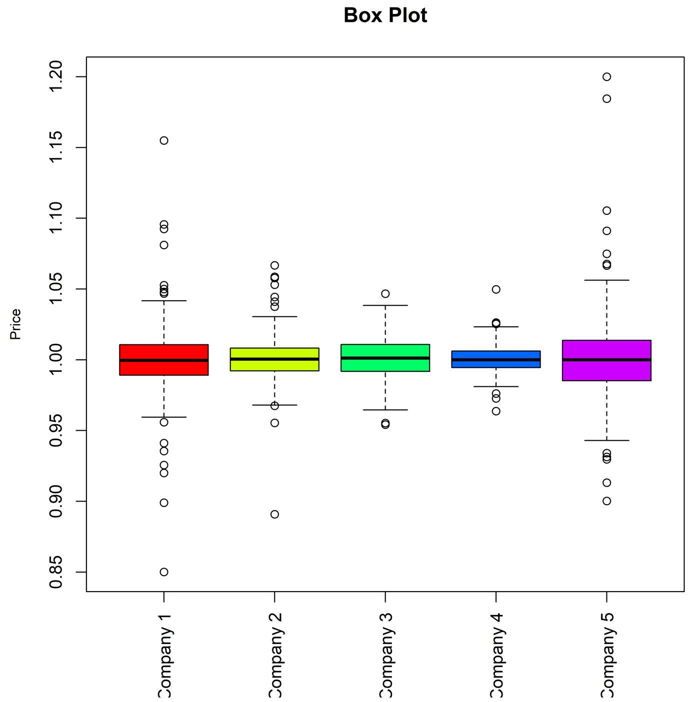

Figure 1: Boxplot of all the stocks

Since $T = 66.222 > 0$ indicates that LL is the preferred model. It can also be observed from the above table that $\min(KS_{LL}, KS_{LN}) = 0.048$, indicating that Log-logistic distribution is the preferred model.

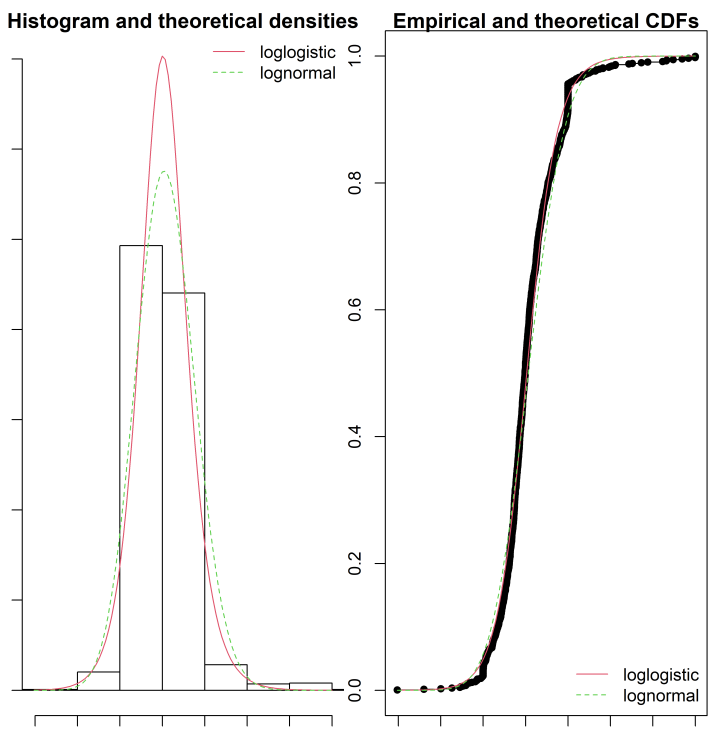

Figure 2: Histogram super imposed with the respective densities (left) and the empirical versus theoretical CDFs (right)

Data Set 2: This data set consists of adjusted closing prices of the hospital chosen from the medical industry for the period 1st April, 2019 to 29th March, 2023. It is obvious from the plot (see figure 3) that the fitted densities of LL and LN are very close to each other and a discrimination of these models is next to impossible. Based on the observed data, the ML estimates of the parameters, log-likelihoods and the KS distance values of the LL and LN distributions are provided in the following table 3.

Model	ML estimates	Log-likelihoods	KS distance
Log-logistic	$(\hat{\eta} = 86.106,\ \hat{\omega} = 1.000)$	2401.265	0.040
Log-normal	$(\hat{\mu} = 0.0013,\ \hat{\sigma} = 0.0233)$	2313.845	0.088

Table 3: ML estimates of the parameters, log-likelihoods and KS distance

Since $T = 87.42 > 0$ indicates that LL is the preferred model. It can also be observed from the above table that $\min(KS_{LL}, KS_{LN}) = 0.040$, indicating that Log-logistic distribution is the preferred model.

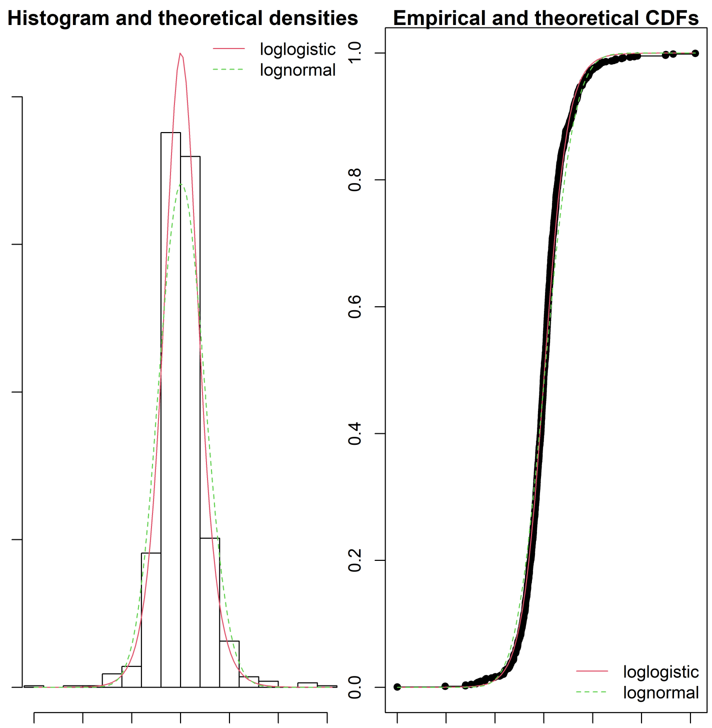

Figure 3: Histogram super imposed with the respective densities (left) and the empirical versus theoretical CDFs (right)

Data Set 3: This data set consists of adjusted closing prices of a pharma company for the period 29th November, 2022 to 29th November, 2023. It is obvious from the plot (see figure 4) that the fitted densities of LL and LN are very close to each other and a discrimination of these models is next to impossible. Based on the observed data, the ML estimates of the parameters, log-likelihoods and the KS distance values of the LL and LN distributions are provided in the following table 4.

Model	ML estimates	Log-likelihoods	KS distance
Log-logistic	$(\hat{\eta} = 183.365,\ \hat{\omega} = 1.000)$	792.222	0.035
Log-normal	$(\hat{\mu} = 0.0006,\ \hat{\sigma} = 0.0100)$	785.793	0.053

Table 4: ML estimates of the parameters, log-likelihoods and KS distance

Since $T = 6.43 > 0$ indicates that LL is the preferred model. It can also be observed from the above table that $\min(KS_{LL}, KS_{LN}) = 0.035$, indicating that Log-logistic distribution is the preferred model.

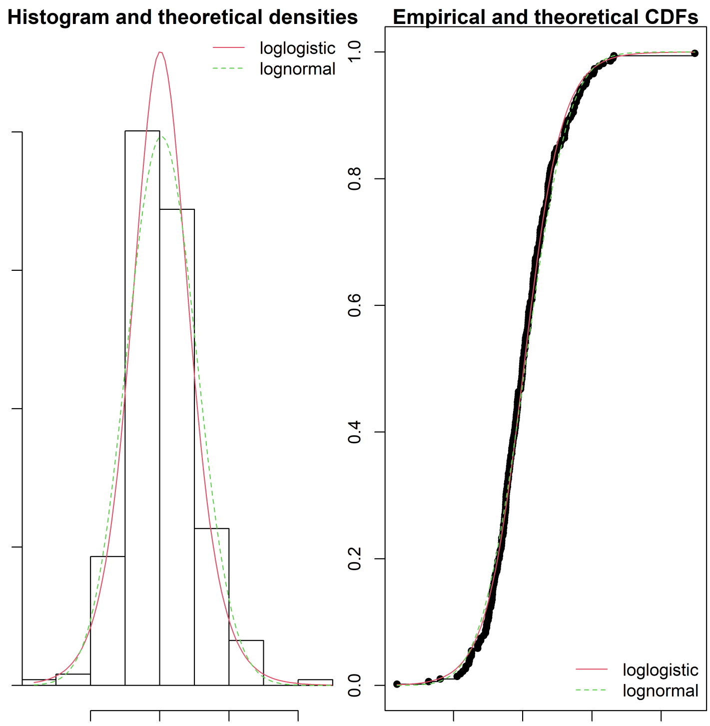

Figure 4: Histogram super imposed with the respective densities (left) and the empirical versus theoretical CDFs (right)

Data Set 4: This data set consists of adjusted closing prices of an airline company from the aviation industry for the period 29th November, 2022 to 29th November, 2023. It is obvious from the plot (see figure 5) that the fitted densities of normal and Cauchy are somewhere close to each other. Based on the observed data, the ML estimates of the parameters, log-likelihoods and the KS distance values of the normal and Cauchy distributions are provided in the following table 5.

Model	ML estimates	Log-likelihoods	KS distance
Normal	$(\hat{\mu}_1 = 1.002,\ \hat{\sigma}_1 = 0.015)$	679.408	0.037
Cauchy	$(\hat{\delta} = 1.002,\ \hat{\gamma} = 0.009)$	643.170	0.069

Table 5: ML estimates of the parameters, log-likelihoods and KS distance

Since $T = 36.24 > 0$ indicates that Normal is the preferred model. It can also be observed from the above table that $\min(KS_N, KS_C) = 0.037$, indicating that normal distribution is the preferred model.

Data Set 5: This data set consists of adjusted closing prices of the life insurance company from the insurance sector for the period 29th November, 2022 to 29th November, 2023. It is obvious from the plot (see figure 6) that the fitted densities of normal and Cauchy are somewhat close to each other. Based on the observed data, the ML estimates of the parameters, log-likelihoods and the KS distance values of the normal and Cauchy distributions are provided in the following table 6.

Model	ML estimates	Log-likelihoods	KS distance
Cauchy	$(\hat{\delta} = 1.000,\ \hat{\gamma} = 0.007)$	675.861	0.071
Normal	$(\hat{\mu}_1 = 1.001,\ \hat{\sigma}_1 = 0.016)$	664.253	0.102

Table 6: ML estimates of the parameters, log-likelihoods and KS distance

Since $T = -11.61 < 0$ indicates that Cauchy is the preferred model. It can also be observed from the above table that $\min(KS_N, KS_C) = 0.071$, indicating that Cauchy distribution is the preferred model.

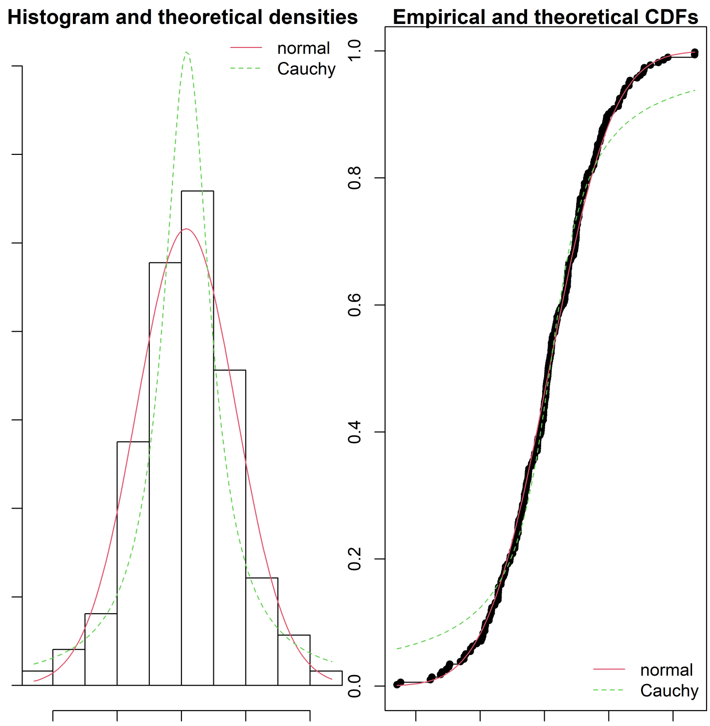

Figure 5: Histogram super imposed with the respective densities (left) and the empirical versus theoretical CDFs (right)

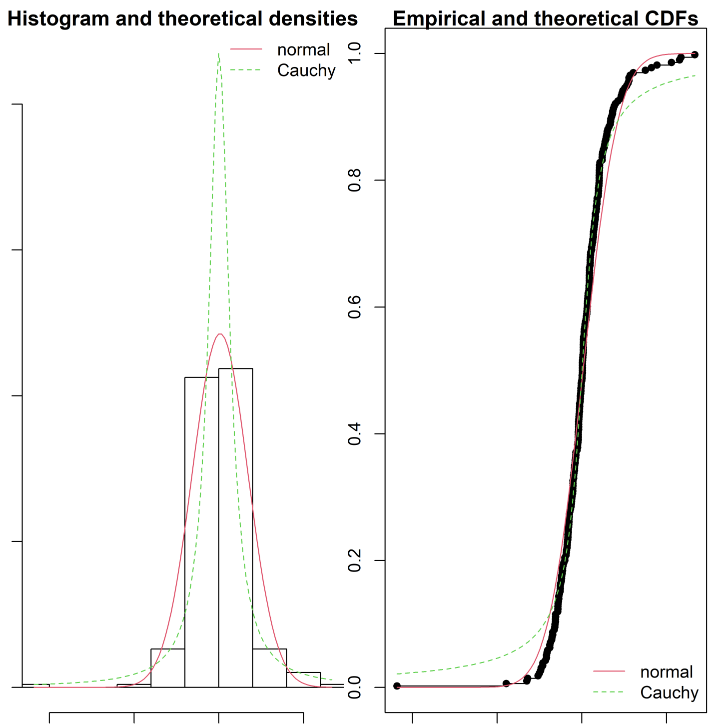

Figure 6: Histogram super imposed with the respective densities (left) and the empirical versus theoretical CDFs (right)

4.1 A Test of Hypothesis for Log-logistic samples

Here we assume that the sample returns are coming from LL population. Let θ denote the population mean return. Then, the hypothesis to be tested is: $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$ i.e., whether the average return is different from zero. We take the case of the company from the entertainment industry where the returns are LL distributed. We conduct a Wald's test to test the above hypothesis. It is well-known that the test statistic

$$W = \frac{(\hat{\theta} - \theta_0)^2}{\mathrm{Var}(\hat{\theta})} \sim \chi_1^2 \quad \text{or} \quad \sqrt{W} = \frac{\hat{\theta} - \theta_0}{\mathrm{SE}(\hat{\theta})} \sim \mathcal{N}(0, 1)$$

asymptotically. Replacing $\hat{\theta} = \bar{R}$ and $\mathrm{Var}(\hat{\theta}) = \mathrm{Var}(R)$ where R is the returns, we have $W = \frac{\bar{R}^2}{\mathrm{Var}(R)} \sim \chi_1^2$ asymptotically. Since the p-value calculated based on the χ_1^2 distribution is zero, we reject H_0 at 5% level of significance and conclude that the average stock price is different from zero.

5 Conclusion

In this paper, we try to discriminate statistical models using the RML criteria in the presence of extreme observations. We have derived the conditions for discriminating two statistical models. We have taken stock prices of five companies from different industries that have either positively or negatively distributed and exhibit high kurtosis. It has been found that LL distribution is preferred for positively skewed with high kurtosis stock prices against the competing LN distribution. But in the case of negatively skewed with high kurtosis the suitable model is Cauchy distribution in comparison with the normal distribution. Since the stock prices corresponding to airline company are slightly negatively skewed (≈ 0) and mesokurtic a normal distribution fit can be expected. We also carried out Wald test and concluded that the stock prices are different from zero. The methodology can aid analysts and researchers in making informed decisions about the most appropriate distribution for modelling stock prices.

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7 Conflicts of Interest

"The authors declare no conflict of interest."

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