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Multi-objective Mathematical model for Suppliers selection in Cement Production Industry with hybrid number of coefficient parameters

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Abstract
In this study, we have introduced a multiobjective mathematical model for supplier selection problems with multi-supplier and multi-item delivery in the cement production industry. The selection of the right suppliers is the most important part of the problem in choosing the percentage of expected defect rate of items and the flexibility of on-time delivery by different suppliers. Sometimes, we consider suppliers of their late-deliver items, as well as the percentage of expected defect rate of different items due to trustable, dutiful, and believable suppliers and, in the next time, maintain a better quality of their items as well as on-time deliveries of their products. In this situation, some percentage of on-time delivery defect items are assumed to be reworked. Hence the coefficient parameters are taken as triangular fuzzy numbers together with stochastic variables that is, hybrid numbers. Then the said problem is transformed into a fuzzy mathematical model and uses a fuzzy decision-making method to solve this problem. Numerical examples have been provided to support the said problem.

Keywords: Multi-objective Model, Hybrid Number, Nonlinear Programming,Supplier Selection, Fuzzy decision-making Method.

1 Introduction

Supplier selection is one of the major parts of economic development and the smooth running of industrial production. In most situations, a single supplier may not meet all buyer criteria. Appropriate suppliers always try to deliver on time, at reasonable price, quality and quantitative measures of items, that is, multi-criteria decision-making situations arise.Bellman & Zadeh (1970) proposed a concept of fuzzy set theory for decision-making problems. In 1974, Tanaka et.al. (1974) applied Bellman and Zadeh's fuzzy set optimization theory. After four years, Zimmermann (1978) developed a procedure for solving linear fuzzy problems.Amid et. al. (2006) introduced a fuzzy multi-objective supplier selection model in supply chain management. Various supplier selection decision making techniques developed by Weber et. al. (1991), Degraeve et. al. (2000), Deboer et. al. (2001). A brief Review of supplier selection methods in Manufacturing Industries described by Tahriri et.al. (2008).Mandal& Roy (2006) formulated a Multi-item imperfect production lot size model with hybrid number cost parameters, Demirtas & Ustun (2008) introduced multi-supplies and multi-sourcing decision-making problems. Wu et. al. (2010) developed a Supplier Selection and Risk model with Fuzzy Multi-Objective Possibility approach. Ho et.al. (2010) introduce and review of many suppliers' evaluation and selection in multi-objective optimization techniques. Amid et. al. (2011) developed a Fuzzy Multi-Objective Supplier Selection technique by using Weighted Max–Min Model. Shirkouhi et.al. (2013) studied Two-Phase Fuzzy Multi-Objective method for Supplier Selection and Order Allocation Problem. Umarusman & Turkmen (2013) using Global

Criterion De Novo technique for Building Optimum Production settings. Inventory lot size, Supplier Selection and Carrier Selection model with Goal Programming technique was developed by Choudhary & Shankar (2014). Jadidi et.al. (2015) developed a model of supplier selection problems with multi-choice goal programming approach. Rajaprasad & Chalapathi (2015) proposed a Criteria Based Supplier Selection Model in a Manufacturing Organization by using Integer Linear Programming method. Govindan et.al. (2017) developed a Fuzzy Multi-Objective approach for the Optimal Selection of Suppliers and Transportation Decisions in an Eco-Efficient Closed Loop Supply Chain Network. El-Hiri et.al. (2019) developed a Suppliers selection process in consideration of risks by using Neural Network. Zakeri et.al. (2022) applying Alternative Ranking Process in a Supplier Selection problem. Choudhary & Agrawal (2022) developed a Supplier selection model for small and medium scale enterprises by using interpretive modelling and analytical hierarchy method. Forghania et.al. (2022) introduced a Supplier Selection problem for complementary and conditional products. Azimli & Cebi (2022) introduced a dynamic Supplier selection and evaluation model in a Company for Decision-Making Process. Zakeri et.al. (2023) applying the Triangular Fuzzy-Grey Numbers in a Supplier selection problem.

The basic structure of our paper is as follows: In Section 2, gives a mathematical formulation of the Suppliers Selection Model with multi-supplier and multi-item. In Section 3, we have introduced the basic definition and properties of Hybrid Number and its application present for Suppliers Selection Model in Section 4. A Fuzzy nonlinear mathematical programming methodology is described in Section 5 and the solution procedure of the said problem present in section 6 and finally, a numerical illustration is provided to support the said mathematical Model in Section 7.

2 Formulation of Mathematical Model:

The Suppliers Selection Model(SSM) with multi-supplier and multi-item mathematical model is considered under the notations and assumptions as follows:

n**:** Number of suppliers.

m**:** Number of items.

c_{ij}**:** Per-unit net purchase cost of item i from supplier j.

a_{ij}**:** Percentage of expected defect rate of item i delivered by supplier j.

o_{ij}**:** Percentage of on-time delivery of item i by supplier j.

U_{ij}**:** Capacity of item i for supplier j.

D_i**:** Aggregate demand of item i.

z_{ij}**:** Quantity of item i ordered from supplier j (decision variable).

The SSM model minimizes net purchased cost, the expected defect rate of the items and at the same time the maximum percentage of on-time deliveries items under the limited capacity of suppliers, aggregate demand and prohibits negative orders of the delivery items which can be stated as follows:

$$\text{Minimize} \quad P(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} c_{ij}\, z_{ij}, \ldots\ldots\ldots (1)$$

$$\text{Minimize} \quad E(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} a_{ij}\, z_{ij},$$

$$\text{Maximize} \quad O(z) = \sum_{j=1}^{m}\sum_{i=1}^{n} o_{ij}\, z_{ij},$$

subject to:

$$z_{ij} \le U_{ij}, \quad i = 1, \ldots, m,\ j = 1, \ldots, n,$$

$$\sum_{j=1}^{n} z_{ij} = D_i, \quad i = 1, \ldots, m,$$

$$z_{ij} \ge 0, \quad i = 1, \ldots, m,\ j = 1, \ldots, n.$$

Now SSM model (1) can be formulated in crisp and fuzzy environments depending upon the nature of the coefficients of objectives, constraints and goals.

3 Basic Definition and properties of Hybrid Number (HN):

The SSM model is introduced with considerations of net purchased cost, expected defect rate, on-time delivery of items etc. given in specific certain data i.e. crisp. But in real-life scenarios, there is much diversified data caused by uncertainty in decisions, considerations of the expected defect and late deliveries items, market demand etc. Sometimes it becomes impossible to get this applicable specific data. So, this imprecise data is not always well characterized by stochastic variables chosen from a probability distribution. Hence the different types of data of the SSM problem may be considered to be fuzzy (vague) as well as randomness i.e. hybrid environment.

3.1 Fuzzy Number

Let us consider an example in industrial management, say 'production cost' which can be imprecisely defined as 'production cost is about C', i.e., it can have a value within the interval $(C - \delta_-, C + \delta_+)$ (say). Here, the left and right spreads of C are δ_- and δ_+, respectively. This imprecise cost, denoted by $\tilde{C} = (C - \delta_-, C + \delta_+)$, may be expressed by the fuzzy set $(z, \mu_{\tilde{C}}(z))$ with the membership function $\mu_{\tilde{C}}(z)$. The $\tilde{C}$ represents a triangular fuzzy number $\tilde{C} = (C_L, C, C_R)$ where $C_L = C - \delta_-$ and $C_R = C + \delta_+$ and $\delta_-, \delta_+ > 0$.

3.2 Triangular Fuzzy Number(TFN)

A fuzzy number $\tilde{C}$ is called TFN if $\tilde{C} = (C_L, C, C_R)$ where the lower and upper bounds of $\tilde{C}$ are C_L and C_R with mode C is a fuzzy set in Real line $\mathbb{R}$ and its membership function is $\mu_{\tilde{C}}(z)$ where $\mu_{\tilde{C}}(z) : R \to [0, 1]$ and defined as follows:

$$\mu_{\tilde{C}}(z) = \begin{cases} \mu^L_{\tilde{C}}(z) = \dfrac{z - C_L}{C - C_L}, & C_L \le z < C, \\ \mu^R_{\tilde{C}}(z) = \dfrac{C_R - z}{C_R - C}, & C < z \le C_R, \\ 0, & \text{otherwise.} \end{cases}$$

Here, the left membership function $\mu^L_{\tilde{C}}(z) : [C_L, C] \to [0, 1]$ is a monotonic increasing function and the right membership function $\mu^R_{\tilde{C}}(z) : [C, C_R] \to [0, 1]$ is a monotonic decreasing function (Figure 1)

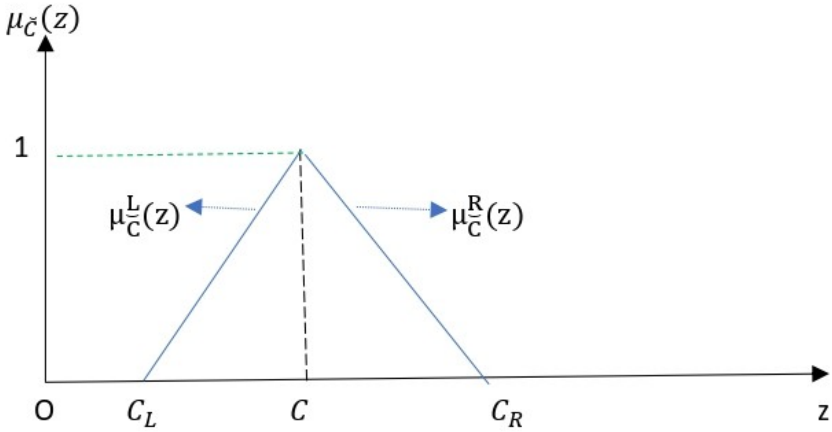

Figure 1: TFN $\tilde{C} = (C_L, C, C_R)$

3.3 Addition and Scalar multiplication of TFNs

Suppose $\tilde{C} = (C_L, C, C_R)$ and $\tilde{D} = (D_L, D, D_R)$ are two TFNs, then the max-min convolution of $\tilde{C}$ and $\tilde{D}$ is $\mu_{\tilde{C}(+)\tilde{D}}(z) = \cup_{z=u+v}(\mu_{\tilde{C}}(u) \cap \mu_{\tilde{D}}(v))$ for all $z, u, v \in R$. Therefore, $\tilde{C}(+)\tilde{D} = (C_L, C, C_R)(+)(D_L, D, D_R) = (C_L + D_L, C + D, C_R + D_R)$ and $\mu_{k\tilde{C}}(z) = \cup_{z=ku}(\mu_{\tilde{C}}(u) \cap \mu_k(u))$ where

$$k\tilde{C} = \begin{cases} (kC_L, kC, kC_R), & \text{for} \quad k \geq 0 \\ (kC_R, kC, kC_L), & \text{for} \quad k \leq 0 \end{cases}$$

According to Kaufman & Gupta (1988), the approximation of TFN $C = (C_L, C, C_R)$ is
$\tilde{C} = \dfrac{(C_L + 2C + C_R)}{4}$

3.4 Addition and Scalar multiplication of Random Variables

Suppose $R_C = (m_C, v_C)$ and $R_D = (m_D, v_D)$ are a Random Variable(RV) where mean $= (m_C, m_D)$, variance $= (v_C, v_D)$ and corresponding probability density functions(pdf) $f_{R_C}(r_c)$ and $f_{R_D}(r_d)$ respectively, if R_C and R_D are two independent RV then addition is $R_C <+> R_D = (m_C, v_C) <+> (m_D, v_D) = (m_C + m_D, v_C + v_D)$.
Now sum-product convolution $R(= R_C <+> R_D)$ be a RV with mean $m = m_C + m_D$ and variance $v = v_C + v_D$ and same type of pdf $f_R(r) = \int f_{R_C}(r - r_d) f_{R_D}(r_d) dr_d$ where integral over the region R. Similarly scalar multiplication $kR_C = (km_C, k^2 v_C)$ and RV of R_C and kR_C are both same type of pdf.

3.5 Hybrid Number (HN)

A fuzzy number $\tilde{C}$ and a RV R_C with pdf $f_{R_C}(r_c)$ together represent a hybrid number denoted by $\bar{\tilde{C}}$ and defined by $\bar{\tilde{C}} = (\tilde{C}, R_C)$ where the addition to a fuzzy number with an RV without loss of available information and does not alter the characteristic.

Now consider two HN, say $\bar{\tilde{C}} = (\tilde{C}, R_C)$ and $\bar{\tilde{D}} = (\tilde{D}, R_D)$ in R, where $f_{R_C}(r_c)$ and $f_{R_D}(r_d)$ are their pdfs. Then the hybrid convolution for addition is

$$\bar{\tilde{C}} \uplus \bar{\tilde{D}} = (\tilde{C}, R_C) \uplus (\tilde{D}, R_D) = \big(\tilde{C}(+)\tilde{D},\ R_C <+> R_D\big),$$

where $(+)$ and $<+>$ denote the max–min convolution for additive fuzzy subsets and the sum–product convolution for RV addition.

Therefore,

$$\mu_{\tilde{C}(+)\tilde{D}}(z) = \bigcup_{z=u+v} \left(\mu_{\tilde{C}}(u) \cap \mu_{\tilde{D}}(v)\right), \quad \forall\, z, u, v \in R,$$

and

$$f_R(r) = \int f_{R_C}(r - r_d)\, f_{R_D}(r_d)\, dr_d,$$

where the integral is taken over the region R.
Remark1: $0 = (0, 0)$ is the neutral for addition of hybrid numbers.
Remark2: If trivial (zero) is the RV with probabilities

$$P(r_c) = \begin{cases} 1, & r_c = 0 \\ 0, & r_c \neq 0 \end{cases}$$

then HN $\bar{\bar{C}} = (\tilde{C}, 0)$ is called a Fuzzy number.
Remark3: If trivial (zero) fuzzy number with membership function

$$\mu_0(z) = \begin{cases} 1, & z = 0 \\ 0, & z \neq 0 \end{cases}$$

then HN $\bar{\bar{C}} = (0, R_c)$ is called a RV.

4 SSM with Hybrid Coefficients

The SSM problem (1) with HN as the coefficients of net purchased cost, the expected defect rate of the items, and percentage of on-time deliveries items parameters of objective functions due to different thought of human and also measurable in the sense of statistical data which can be represented as follows:

$$\text{Minimize } P(z) = \sum_{j=1}^{n} \sum_{i=1}^{m} \tilde{c}_{ij}\, z_{ij}, \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\text{Minimize } E(z) = \sum_{j=1}^{n} \sum_{i=1}^{m} \bar{\bar{a}}_{ij} z_{ij},$$

$$\text{Minimize } O(z) = \sum_{j=1}^{n} \sum_{i=1}^{m} \bar{\bar{o}}_{ij} z_{ij},$$

subject to:

$$z_{ij} \leq U_{ij}, \quad i = 1, \ldots, m,\ j = 1, \ldots, n,$$

$$\sum_{j=1}^{n} z_{ij} = D_i, \quad i = 1, \ldots, m,$$

$$z_{ij} \geq 0, \quad i = 1, \ldots, m,\ j = 1, \ldots, n.$$

Here, $P(z)$, $E(z)$, and $O(z)$ are hybrid-valued objective functions where $\tilde{c}_{ij} = (c_{ijL}, c_{ij}, c_{ijR})(+)'(0, 0)$ are triangular fuzzy numbers (special types of HNs), whereas $\bar{\bar{a}}_{ij} = (a_{ijL}, a_{ij}, a_{ijR})(+)'(m_{a_{ij}}, v_{a_{ij}})$ and $\bar{\bar{o}}_{ij} = (o_{ijL}, o_{ij}, o_{ijR})(+)'(m_{o_{ij}}, v_{o_{ij}})$ are hybrid numbers.

The objective functions of (2) are formulated as:

Minimize $P(z) = \hat{P}_0(z)$ where $\hat{P}_0(z) = \sum_{j=1}^{n}\sum_{i=1}^{m} \hat{c}_{ij}\, z_{ij}$ with $\hat{c}_{ij} = \dfrac{c_{ijL} + 2c_{ij} + c_{ijR}}{4}, \ldots\ldots (3)$

Minimize $E(z) = \hat{E}_0(z)(+)^{'}(0, Ve(z))$ where $\hat{E}_0(z) = \sum_{j=1}^{n}\sum_{i=1}^{m}(\hat{a}_{ij} + m_{a_{ij}})\, z_{ij}$

with $\hat{a}_{ij} = \dfrac{a_{ijL} + 2a_{ij} + a_{ijR}}{4}$, and $Ve(z) = \sum_{j=1}^{n}\sum_{i=1}^{m} v_{a_{ij}} z_{ij}^2$

Maximize $O(z) = \hat{O}_0(z)(+)^{'}(0, Vo(z))$ where $\hat{O}_0(z) = \sum_{j=1}^{n}\sum_{i=1}^{m}(\hat{o}_{ij} + m_{o_{ij}})\, z_{ij}$

with $\hat{o}_{ij} = \dfrac{o_{ijL} + 2o_{ij} + o_{ijR}}{4}$, and $Vo(z) = \sum_{j=1}^{n}\sum_{i=1}^{m} v_{o_{ij}} z_{ij}^2$

subject to the same constraints and restrictions as in (2). Hence (3) reduced to

$$\text{Minimize } \hat{P}_0(z) = \sum_{j=1}^{n}\sum_{i=1}^{m} \hat{c}_{ij}\, z_{ij} \ldots\ldots (4)$$

$$\text{Minimize } \hat{E}_0(z) = \sum_{j=1}^{n}\sum_{i=1}^{m}(\hat{a}_{ij} + m_{a_{ij}})\, z_{ij}$$

$$\text{Minimize } Ve(z) = \sum_{j=1}^{n}\sum_{i=1}^{m} v_{a_{ij}} z_{ij}^2$$

$$\text{Maximize } \hat{O}_0(z) = \sum_{j=1}^{n}\sum_{i=1}^{m}(\hat{o}_{ij} + m_{o_{ij}})\, z_{ij}$$

$$\text{Maximize } Vo(z) = \sum_{j=1}^{n}\sum_{i=1}^{m} v_{o_{ij}} z_{ij}^2$$

subject to same constraints and restrictions of (2).

5 Solution Methodology: Fuzzy Nonlinear Programming method

Let us consider m linear / non-linear objective functions and all objectives transform into the Vector Minimization Form as below

$$\begin{aligned} &\text{Minimize } g(z) = [g_1(z), g_2(z), \ldots, g_m(z)] \ldots\ldots\ldots\ldots (5) \\ &\text{subject to} \quad z \in Z = \{z : h_k(z) \leq d_k, k = 1, 2, \ldots\ldots, n\} \end{aligned}$$

Using Zimmermann (1978) max–min fuzzy decision approach, the solution of (5) is

$$\begin{aligned} &\text{Find } z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6) \\ &\text{so as to satisfy } g_r(z) \tilde{\leq} L_r, r = 1, 2, \ldots\ldots, m, \\ &z \in Z \end{aligned}$$

Here L_r are the goals of $g_r(z)$ that are to be minimized, and we consider these objective functions of (5) as fuzzy constraints. Let $\mu_r(g_r(z))$ denote the membership functions corresponding to $g_r(z)$ and this $\mu_r(g_r(z))$ be strictly monotonic decreasing functions and hence formulate a payoff table using the Ideal

Solution of objective functions and constraints. Therefore, a set of feasible solutions is represented by its membership function as follows.

$$\mu_{\tilde{D}}(z) = \text{ Minimize } \mu_{\tilde{D}}\{\mu_1(g_1(z)), \mu_2(g_2(z)), \ldots, \mu_m(g_m(z))\}.$$

So, a decision maker uses a multi-objective fuzzy optimization technique and hence takes the highest level of $\mu_{\tilde{D}}(z)$ of the feasible set of solutions, which is

$$\begin{aligned} &\text{Maximize } \mu_{\tilde{D}}(z) \ldots\ldots\ldots\ldots\ldots\ldots (7)\\ &\text{subject to } z \in Z \end{aligned}$$

Now let α be the satisfactory level of all compromises where $0 \leq \alpha \leq 1$, then problem (7) is reformulated as

$$\begin{aligned} &\text{Maximize } \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)\\ &\text{such that } \mu_r(g_r(z)) \geq \alpha, \text{ for } r = 1, 2, \ldots\ldots, m.\\ &z \in Z. \end{aligned}$$

The nonlinear problem (5) does not always have a complete optimal solution because many times conflicts with each other of the objective functions of the problem. So, we used another concept of another solution called Pareto Optimal Solution as follows:

5.1 Pareto Optimal Solution (POS)

A set of solutions z^* is called a POS for the problem (5) iff there does not exist another $z \in Z$ such that for all $r = 1, 2, \ldots\ldots, m, g_r(z^*) \leq g_r(z)$ and for at least one $j, j \in < 1, 2, \ldots\ldots, m >, g_j(z) \neq g_j(z^*)$. Hence, to solve (8), we get Pareto Optimal Solution.

6 Solution Procedure of SSM (Using section 4).

The vector minimization form of the problem (4) is

Minimize $[\tilde{P}_0(z), \hat{E}_0(z), Ve(z), -\hat{O}_0(z), -Vo(z)]$ .... (9)

subject to same constraints and restrictions of (2).

Here, the membership functions

$$\mu_{\tilde{P}_0}(\tilde{P}_0(z)),\ \mu_{\hat{E}_0}(\hat{E}_0(z)),\ \mu_{Ve}(Ve(z)),\ \mu_{-\hat{O}_0}(-\hat{O}_0(z)),\ \mu_{-Vo}(-Vo(z))$$

for the objective functions

$$\tilde{P}_0(z),\ \hat{E}_0(z),\ Ve(z),\ -\hat{O}_0(z),\ -Vo(z)$$

respectively, are defined as follows:

$$\mu_{\tilde{P}_0}(\tilde{P}_0(z)) = \begin{cases} 1, & \tilde{P}_0(z) \leq L_{\tilde{P}_0}, \\ \dfrac{U_{\tilde{P}_0} - \tilde{P}_0(z)}{U_{\tilde{P}_0} - L_{\tilde{P}_0}}, & L_{\tilde{P}_0} < \tilde{P}_0(z) < U_{\tilde{P}_0}, \\ 0, & \tilde{P}_0(z) \geq U_{\tilde{P}_0}, \end{cases}$$

$$\mu_{\hat{E}_0}(\hat{E}_0(z)) = \begin{cases} 1, & \hat{E}_0(z) \leq L_{\hat{E}_0}, \\ \dfrac{U_{\hat{E}_0} - \hat{E}_0(z)}{U_{\hat{E}_0} - L_{\hat{E}_0}}, & L_{\hat{E}_0} < \hat{E}_0(z) < U_{\hat{E}_0}, \\ 0, & \hat{E}_0(z) \geq U_{\hat{E}_0}, \end{cases}$$

$$\mu_{Ve}(Ve(z)) = \begin{cases} 1, & Ve(z) \leq L_{Ve}, \\ \dfrac{U_{Ve} - Ve(z)}{U_{Ve} - L_{Ve}}, & L_{Ve} < Ve(z) < U_{Ve}, \\ 0, & Ve(z) \geq U_{Ve}, \end{cases}$$

$$\mu_{-\hat{O}_0}(-\hat{O}_0(z)) = \begin{cases} 1, & -\hat{O}_0(z) \leq -U_{-\hat{O}_0}, \\ \dfrac{-L_{-\hat{O}_0} - (-\hat{O}_0(z))}{-L_{-\hat{O}_0} - (-U_{-\hat{O}_0})}, & -U_{-\hat{O}_0} \leq -\hat{O}_0(z) \leq -L_{-\hat{O}_0}, \\ 0, & -\hat{O}_0(z) \geq -L_{-\hat{O}_0}, \end{cases}$$

$$\mu_{-Vo}(-Vo(z)) = \begin{cases} 1, & -Vo(z) \leq -U_{-Vo}, \\ \dfrac{-L_{-Vo} - (-Vo(z))}{-L_{-Vo} - (-U_{-Vo})}, & -U_{-Vo} \leq -Vo(z) \leq -L_{-Vo}, \\ 0, & -Vo(z) \geq -L_{-Vo}, \end{cases}$$

where $U_{\tilde{P}_0},\ U_{\hat{E}_0},\ U_{Ve},\ U_{-\hat{O}_0},\ U_{-Vo}$ and $L_{\tilde{P}_0},\ L_{\hat{E}_0},\ L_{Ve},\ L_{-\hat{O}_0},\ L_{-Vo}$ denote the upper and lower bounds of the objective functions $\tilde{P}_0(z),\ \hat{E}_0(z),\ Ve(z),\ -\hat{O}_0(z),\ -Vo(z)$, which are calculated using the estimated payoff table.

If β be the all-compromise satisfactory level then problem (9) reduced to:

$$\begin{aligned} &\text{Maximize } \beta, \\ &\text{subject to } \ \tilde{P}_0(z) \leq U_{\tilde{P}_0} - \beta(U_{\tilde{P}_0} - L_{\tilde{P}_0}), \\ &\quad \hat{E}_0(z) \leq U_{\hat{E}_0} - \beta(U_{\hat{E}_0} - L_{\hat{E}_0}), \\ &\quad Ve(z) \leq U_{Ve} - \beta(U_{Ve} - L_{Ve}), \\ &\quad \hat{O}_0(z) \geq L_{-\hat{O}_0} + \beta(U_{-\hat{O}_0} - L_{-\hat{O}_0}), \\ &\quad Vo(z) \geq L_{-Vo} + \beta(U_{-Vo} - L_{-Vo}), \\ &\quad z \in Z, \quad \beta \in [0,1]. \end{aligned}$$

subject to same constraints and restrictions of (2). Solving this optimization yields the best compromise (Pareto optimal) solution at maximum β.

7 Numerical illustration

Suppose a cement manufacturing company needs three raw materials (Limestone, Clay and Chemical powder.) to select appropriate suppliers based on the three purchasing criteria which are net purchasing cost, percentage of expected defect rate and on-time delivery of the raw materials. In a certain period, the company demand for Limestone, Clay and Chemical powder is 28000, 32000 and 12000 metric tons. The related information about the suppliers is as follows(Table 1):

Suppliers	Items(Ton)	purchasing cost (lakh)	% expected defect rate	% on-time delivery	Capacity of items(mton)
I	Limestones(A)	0.25	0.07	0.65	12000
I	Clay(B)	0.18	0.08	0.75	19000
I	Chemical powder(C)	—	—	—	—
II	Limestones(A)	0.22	0.06	0.68	25000
II	Clay(B)	0.16	0.09	0.77	22000
II	Chemical powder(C)	0.97	0.04	0.85	11000
III	Limestones(A)	0.26	0.04	0.84	24000
III	Clay(B)	—	—	—	—
III	Chemical powder(C)	0.91	0.07	0.69	8000

Table 1: Suppliers related information

The Company Manager selects the percentage of order materials of ith ($i = 1, 2, 3$) item from jth ($j = 1, 2, 3$) supplier for a particular time period, therefore the mathematical formulation of hybrid data

of different coefficients, so the model is as follows:
Minimize $P(z) = \tilde{(}0.25)z_{11} + \tilde{(}0.18)z_{21} + \tilde{(}0.22)z_{12} + \tilde{(}0.16)z_{22} + \tilde{(}0.97)z_{32} + \tilde{(}0.26)z_{13} + \tilde{(}0.91)z_{33}$,
Minimize $E(z) = \bar{(}0.07)z_{11} + \bar{(}0.08)z_{21} + \bar{(}0.06)z_{12} + \bar{(}0.09)z_{22} + \bar{(}0.04)z_{32} + \bar{(}0.04)z_{13} + \bar{(}0.07)z_{33}$,
Maximize $O(z) = \bar{(}0.65)z_{11} + \bar{(}0.75)z_{21} + \bar{(}0.68)z_{12} + \bar{(}0.77)z_{22} + \bar{(}0.85)z_{32} + \bar{(}0.84)z_{13} + \bar{(}0.69)z_{33}$
,

$$\begin{aligned} \text{subject to } & z_{11} + z_{12} + z_{13} = 28000, \\ & z_{21} + z_{22} = 32000, \\ & z_{32} + z_{33} = 12000, \\ & z_{11} \le 12000, \quad z_{21} \le 19000, \quad z_{12} \le 25000, \\ & z_{22} \le 22000, \quad z_{32} \le 11000, \quad z_{13} \le 24000, \\ & z_{33} \le 800, \quad z_{ij} \ge 0,\ i = 1,2,3,\ j = 1,2,3. \end{aligned}$$

Suppliers	**Items (ton)**	**Fuzzy purcha- -sing cost(lakh)**	**Fuzzy Number**	Random Number		**Hybrid Number**
				Mean	Variance	
I	A	$\tilde{(}0.25)$	(0.20,0.25,0.28)	–	–	$(0.20, 0.25, 0.28)(+)'(0, 0)$
I	B	$\tilde{(}0.18)$	(0.13,0.18,0.21)	–	–	$(0.13, 0.18, 0.21)(+)'(0, 0)$
I	C	—	—	-	–	—
II	A	$\tilde{(}0.22)$	(0.18,0.22,0.26)	–	–	$(0.18, 0.22, 0.26)(+)'(0, 0)$
II	B	$\tilde{(}0.16)$	(0.12,0.16,0.19)	–	–	$(0.12, 0.16, 0.19)(+)'(0, 0)$
II	C	$\tilde{(}0.97)$	(0.94,0.97,0.99)	–	–	$(0.94, 0.97, 0.99)(+)'(0, 0)$
III	A	$\tilde{(}0.26)$	(0.22,0.26,0.29)	—	–	$(0.22, 0.26, 0.29)(+)'(0, 0)$
III	B	—	—	-	–	—
III	C	$\tilde{(}0.91)$	(0.87,0.91,0.96)	–	–	$(0.87, 0.91, 0.96)(+)'(0, 0)$
Suppliers	**Items (ton)**	**% of expected defect rate**	**Fuzzy Number**	Mean	Variance	**Hybrid Number**
I	A	$\bar{(}0.07)$	(0.05,0.07,0.09)	0.08	0.00006	$(0.13, 0.15, 0.17)(+)'(0, 0.00006)$
I	B	$\bar{(}0.08)$	(0.06,0.08,0.11)	0.09	0.00008	$(0.15, 0.17, 0.20)(+)'(0, 0.00008)$
I	C	—	—	-	–	—
II	A	$\bar{(}0.06)$	(0.04,0.06,0.09)	0.05	0.00007	$(0.09, 0.11, 0.14)(+)'(0, 0.00007)$
II	B	$\bar{(}0.09)$	(0.06,0.09,0.12)	0.10	0.00009	$(0.16, 0.19, 0.22)(+)'(0, 0.00009)$
II	C	$\bar{(}0.04)$	(0.02,0.04,0.07)	0.05	0.00006	$(0.07, 0.09, 0.12)(+)'(0, 0.00006)$
III	A	$\bar{(}0.04)$	(0.03,0.04,0.06)	0.04	0.00004	$(0.07, 0.08, 0.10)(+)'(0, 0.00004)$
III	B	—	—	—	-	–
III	C	$\bar{(}0.07)$	(0.05,0.07,0.09)	0.06	0.00003	$(0.11, 0.13, 0.15)(+)'(0, 0.00003)$
Suppliers	**Items (ton)**	**% of On-time delevary**	**Fuzzy Number**	Mean	Variance	**Hybrid Number**
I	A	$\bar{(}0.65)$	(0.62,0.65,0.68)	0.67	0.00008	$(1.29, 1.32, 1.35)(+)'(0, 0.00008)$
I	B	$\bar{(}0.75)$	(0.72,0.75,0.77)	0.74	0.00009	$(1.46, 1.49, 1.51)(+)'(0, 0.00009)$
I	C	—	—	-	–	—
II	A	$\bar{(}0.68)$	(0.65,0.68,0.70)	0.66	0.00007	$(1.31, 1.34, 1.36)(+)'(0, 0.00007)$
II	B	$\bar{(}0.77)$	(0.74,0.77,0.79)	0.78	0.00009	$(1.52, 1.55, 1.57)(+)'(0, 0.00009)$
II	C	$\bar{(}0.85)$	(0.83,0.85,0.87)	0.84	0.0001	$(1.67, 1.69, 1.71)(+)'(0, 0.0001)$
III	A	$\bar{(}0.84)$	(0.82,0.84,0.87)	0.86	0.0002	$(1.68, 1.70, 1.74)(+)'(0, 0.0002)$
III	B	—	—	—	-	–
III	C	$\bar{(}0.69)$	$(0.67, 0.69, 0.71)$	0.70	0.00008	$(1.37, 1.39, 1.41)(+)'(0, 0.00008)$

Table 2: Hybrid Number of Coefficient Parameters

Aspiration Level(β^*)	**Optimal Decision Variables** (z^*_{ij})	$P_0(z^*)$	$E_0(z^*)$	$O_0(z^*)$	$Ve_(z^*)$	$Vo(z^*)$
0.5243650	$z^*_{11} = 0, z^*_{21} = 10000, z^*_{31} = 0,$ $z^*_{12} = 8655.17, z^*_{22} = 22000,$ $z^*_{23} = 0, z^*_{32} = 4000,$ $z^*_{13} = 19344.82, z^*_{33} = 8000$	23270.43	9884.655	111359.2	74652.74	139368.3

Table 3: Pareto Optimal Solution

8 Conclusion

In a competitive business market, the selection of the right Suppliers is one of the most important works for improving the industrial sector. In this article, we consider a multi-objective supplier selection model where coefficient parameters are triangular fuzzy numbers together with stochastic variables of the normal probability density function. Here, the industrial manager wants to optimize the level of necessary objective value and at that moment he needs to optimize the same objective dispersion value. Hence these two types of measures of the same objective help the industrial manager to achieve his targeted value i.e., he will get a concrete idea for finding the optimum value of the several objective functions. Therefore, these types of decision-making procedures will be more realistic situations for the selection of the right suppliers in various industrial production. Further research should add more restrictions and objectives that affect supply chain management and various industrial planning sectors.

9 Conflicts of Interest

The authors declare no conflict of interest.

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