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Estimation of the Common Mean of Normal and Logistic Distributions with Known Coefficient of Variation by Ranked Set Sampling.

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Abstract
In this article we have developed a method of estimating the common location parameter μ of several distributions having further a common scale parameter which is proportional to μ using ranked set sampling (RSS). For illustration purpose we estimate the common mean of normal, and logistic distributions with known coefficient of variation using RSS. Efficiency comparison of the proposed estimator using RSS of independent but not identically distributed (inid) random variables with that of using order statistics of inid random variables is also made.

Keywords:Normal distribution; Logistic distribution; Known coefficient of variation; Order statistics of inid random variables; Ranked set sampling of inid random variables.

1 Introduction

The subject of order statistics deals with the properties and applications of ordered random variables and of functions of these variables. If the random variables $X_1, X_2, ..., X_n$ are arranged in ascending order of magnitude as $X_{1:n} \leq X_{2:n} \leq ... \leq X_{n:n}$, then $X_{i:n}$ is called the i^{th} order statistic , $i=1,2,\ldots,n$. More details about order statistics and its applications are discussed in David and Nagaraja(2003) and Balakrishnan and Cohen(1991).

In some of the problems of biological and physical science situations, where the scale parameter is proportional to the location parameter are reported in the literature, see , Glesser and Healy (1976). If the parent distribution is univariate normal then the problem of estimating the mean has been extensively discussed in the available literature See, Searls(1964) , Khan(1968) , Arnholt and Hebert(1995), Kunte(2000) and Guo and Pal(2003).The best linear unbiased estimator of the mean of the normal distribution with known coefficient of variation using order statistics is discussed by Thomas and Sajeevkumar(2003). Estimating the mean of logistic distribution with known coefficient of variation using order statistics is discussed by Sajeevkumar and Thomas(2005). Estimating a parameter of the exponential distribution with known coefficient of variation by order statistics are discussed in Sajeevkumar and Irshad(2013).

The concept of ranked set sampling (RSS) was first introduced by McIntyre(1952) as a process of improving the precision of the sample mean as an estimator of the population mean. The RSS described in McIntyre(1952) is applicable whenever ranking of a set of sampling units can be done easily by judgment method. The ranked set sampling procedure consists of choosing randomly n sets of units each of size n from the population of interest. The units in each set are ranked by judgment method or by using some inexpensive method, without actual measurement of the units. Now the unit ranked as one from the first set is actually measured, the unit ranked as two from second set is measured. The process is continued in this way until the unit ranked as n from the n^{th} set is measured. Then the observations resulting out of actual measurements made on units chosen as described in the above process is known as the ranked set sampling.

RSS offers several advantages over using order statistics (like median , quantiles etc.) from simple random sampling. The following are the advantages of RSS over order statistics.
(i). Higher Efficiency:
RSS provide more precise estimators (lower variance) for population parameters than simple random sampling (SRS) , especially for the mean or median.This provide efficiency comes without increasing sample size.
(ii). Cost Effectiveness:
In RSS , ranking is done without actual measurement (eg. Visual inspection) , and only a subset is measured. This reduces cost and effort when measurement is expensive or destructive.
(iii). Better Utilization of Auxiliary Information:
RSS leverages prior knowledge or easily observable trails to rank units , which improves the representativeness of the sample.
(iv). Improved Estimation of Distributional Properties;
RSS provide better estimators for distribution functions , quantiles and moments than order statistics from RSS.
(v). Applicable to Skewed Distributions:
RSS tends to perform better than order statistics from random samples when dealing with non-normal or skewed distributions.

For a detailed discussion on the theory and applications of ranked set sampling(RSS) and estimating parameters of some distributions using RSS (efficiency better than order statistics) are discussed in Chen et al.(2004) , Lam et.al(1994) , Stokes(1995) and Zheng and Moderres(2006). Estimating the location parameter of the exponential distribution with known coefficient of variation by ranked set sampling are discussed by Irshad and Sajeevkumar(2011).

The problem of estimating the common location and common scale parameter of several distributions using order statistics of independent but not identically distributed (inid) random variables are discussed by Sajeevkumar and Thomas(2005). The ranked set sampling(RSS) of independent non-identically distributed (inid) random variables was suggested by Priya and Thomas(2016) .
RSS offers several advantages in the multiple population context – such as comparing two or more groups, populations , or treatments.Hence are the main benefits.
(a). Increased Efficiency Across Population:
RSS improve estimation precision (eg. Means , variances , quintiles) for each population individually , when comparing multiple populations , it lead to more powerful statistical tests (eg. ANOVA , t-tests).
(b). Better Resource Allocation:
In multiple population studies , RSS allows you to minimize measurement costs while still obtaining reliable comparisons .Especially useful when measurements are expensive but ranking is cheap.
(c). Enhanced Detection of Differences:
RSS boosts the ability to detect small differences between population parameters (eg . mean differences , variance differences). This results in higher statistical power for hypothesis tests across population.
(d). Stratified –Like Sampling Efficiency:
RSS acts somewhat like a stratified sampling method, maintaining good representation from each population even when variability within population differs.
Estimation of common location parameter μ of several distributions when their common scale parameter is proportional to μ using order statistics of inid random variables are discussed in Sajeevkumar and Thomas(2006).In this article our aim is to estimate the common location parameter μ of several distribu-

tions when their common scale parameter is proportional to μ using ranked set sampling of inid random variables.

2 Estimation of the Common Location Parameter μ of Several Distributions When the Scale Parameter of Each of the Distributions is $c\mu$ for Known c Using Order Statistics

The estimation of a common mean across multiple population is statistically significant in many applied fields because it allows for a pooled understanding of a shared parameter. Here's why it matters.
(i). Efficient Data Utilization:
When populations are believed to have similar or identical means , estimating a common mean allows for combining data from all groups , increasing statistical power and precision.
(ii). Simplified Modeling:
A common mean reduces model complexity. Instead of modeling each group separately , one parameter represents the central tendency of all , making interpretation simpler.
(iii). Cost Reduction:
In fields like agriculture , medicine or industry , assuming a common mean means fewer measurements may be needed from each population , reducing costs and resources.
(iv). Basis for Testing Homogeneity:
Estimating a common mean provides a base line for testing whether populations are actually homogeneous or differ significantly.For example
(i). In clinical trails : Combining treatments effects from different hospitals. (ii). In environmental studies : Estimating average pollutant levels across multiple location.

The distribution theory of order statistics of independent but not identically distributed (inid) random variables are discussed in Vaughan and Venables (1972). Using this distribution theory Sajeevkumar and Thomas(2005) derived the best linear unbiased estimator of common location and common scale parameters of several distributions using order statistics of inid random variables.

Let $X_{i1}, X_{i2}, ..., X_{in_i}$ be random sample of size n_i drawn from a distribution with *pdf* of the form

$$f_i\left(x_i;\mu,c\mu\right)=\frac{1}{c\mu}f_{i0}\left(\frac{x_i-\mu}{c\mu}\right)\ ,\ \mu>0\ ,\ c>0, \tag{1}$$

where the form of $f_{i0}\left(.\right)$ is known and $i=1,2,\ldots,k$. For convenience we assume that all k distributions defined by the *pdf* defined in (1) have same support . Let $n = n_1+n_2+...+n_k$. Let $\underline{X}^{(p)} = \left(X_{1:n(p)}, X_{2:n(p)}, ..., X_{n:n(p)}\right)'$ be the vector of order statistics of the pooled sample of all n observations. Then the above vector of order statistics can be considered as vector of order statistics of n independent non-identically distributed (inid) random variables. The distribution theory of order statistics of inid random variables are discussed by Vaughan and Venables(1972). Let $Y_{i1}, Y_{i2}, ..., Y_{in_i}$ are the observations of a random sample of size n_i drawn from a population with pdf $f_{i0}\left(y_i\right)$, $i=1,2,...,k$ and let $Y_{1:n(p)}, Y_{2:n(p)}, ..., Y_{n:n(p)}$ are the order statistics of the pooled sample of the random samples , then $\left(Y_{1:n(p)}, Y_{2:n(p)}, ..., Y_{n:n(p)}\right)' \stackrel{d}{=} \left(\frac{X_{1:n(p)}-\mu}{c\mu}, \frac{X_{2:n(p)}-\mu}{c\mu}, ..., \frac{X_{n:n(p)}-\mu}{c\mu}\right)'$. Let $E(Y_{r:n(p)}) = \alpha_{r:n(p)}$, for$r = 1,2,\ldots,n$, $V(Y_{r:n(p)}) = \beta_{r,r:n}$, $for\, r = 1,2,...,n$ and $Cov(Y_{r:n(p)}, Y_{s:n(p)}) = \beta_{r,s,n(p)}$ $1 \leq r < s \leq n$ .Then the BLUE of the common location parameter is given in the following theorem.

Theorem 1. *Let* $\underline{X}^{(p)} = \left(X_{1:n(p)}, X_{2:n(p)}, ..., X_{n:n(p)}\right)'$ *be the vector of order statistics of the pooled sample of k samples , where the* i^{th} *sample contain* n_i *observations drawn from the distribution with pdf defined in equation (1) , for i=1,2,...,k and* $n = n_1+n_2+...+n_k$ *.Let* $\underline{\alpha}^{(p)} = \left(\alpha_{1:n(p)}, \alpha_{2:n(p)}, ..., \alpha_{n:n(p)}\right)'$

and $V^{(p)} = \left(\left(\beta_{r,s:n(p)}\right)\right)$ *be the vector of means and dispersion matrix of vector of order statistics of the pooled sample arising from* $f_{i0}\left(y_i\right)$ *,* $i = 1, 2, ..., k$ *and* $n = n_1 + n_2 + ... + n_k$*. Then the BLUE of* μ *namely* $\hat{\mu}$ *and its variance are given by*

$$\hat{\mu} = \frac{(c\,\underline{\alpha}^{(p)} + \underline{1})'(V^{(p)})^{-1}}{(c\,\underline{\alpha}^{(p)} + \underline{1})'(V^{(p)})^{-1}(c\,\underline{\alpha}^{(p)} + \underline{1})}\underline{X}^{(p)} \tag{2}$$

and

$$V(\hat{\mu}) = \frac{c^2\mu^2}{(c\,\underline{\alpha}^{(p)} + \underline{1})'(V^{(p)})^{-1}(c\,\underline{\alpha}^{(p)} + \underline{1})}. \tag{3}$$

3 Estimation of Common Location Parameter μ of Several Distributions When the Scale Parameter of Each of the Distributions is Equals to $c\mu$ for known c using RSS.

The ranked set sampling of independent non-identically distributed (inid) random variables was suggested by Priya and Thomas(2016) and it is described as below:
Suppose there are k populations and the i^{th} population has a distribution function $F_i\left(x\right)$ with pdf $f_i\left(x\right)$ and suppose we fix up the ultimate number of observations to be drawn from the i^{th} population of size n_i , i=1,2,..,k , with $\sum\limits_{i=1}^{k} n_i = n$ .Then we define RSS from k populations in the following manner.
Choose randomly n sets of units where each set contain n units comprising of further k subsets of units such that the i^{th}subset consists of n_i units drawn from the i^{th} population for $i = 1, 2, \ldots, k$. Now by an inexpensive method of rank the units of each set .From the r^{th} set choose the unit with rank r and measure the characteristic of interest, which is denoted by $X_{(r:n(p))r}$ for $r = 1, 2, \ldots, n$. Then the observations $X_{(1:n(p))1}, X_{(2:n(p))2}, ..., X_{(n:n(p))n}$ taken together is said to form the ranked set sampling drawn from k populations with n_i observations from the i^{th} population, $i = 1, 2, \ldots, k$.Using this Priya and Thomas(2016) estimated the common location and common scale parameter of several distributions using RSS of inid random variables. In this article we have used the procedure suggested by Priya and Thomas(2016) to estimate the common location Parameter μ of several distributions , when the Scale Parameter of each of the distributions is equals to $c\mu$ for known c using RSS of inid random variables.

Let $X_{i1}, X_{i2}, ..., X_{in_i}$ be a random sample of size n_i drawn from a distribution with *pdf* of the form

$$f_i\left(x_i, \mu, c\mu\right) = \frac{1}{c\mu} f_{i0}\left(\frac{x_i - \mu}{c\mu}\right) \,,\, \mu > 0\,,\, c > 0, \tag{4}$$

where the form of $f_{i0}\left(.\right)$ is known and i=1,2,...,k . Let $n = n_1+n_2+...+n_k$.Let $X_{(1:n(p))1}, X_{(2:n(p))2}, ...,$ $X_{(n:n(p))n}$ be the RSS of inid random variables from equation(4).Define $Y_{(r:n(p))r} = \left(\frac{X_{(r:n(p))r}-\mu}{c\mu}\right)$, $r = 1, 2, ..., n$. Then clearly $Y_{(1:n(p))1}, Y_{(2:n(p))2}, ..., Y_{(n:n(p))n}$ be the RSS of inid random variables from $f_{i0}\left(y_i\right)$,$i = 1, 2, ..., k$. Now clearly $\left(Y_{(1:n(p))1}, Y_{(2:n(p))2}, ..., Y_{(n:n(p))n}\right)' \stackrel{d}{=} (\frac{X_{(1:n(p))1}-\mu}{c\mu}, \frac{X_{(2:n(p))2}-\mu}{c\mu}, ..., \frac{X_{(n:n(p))n}-\mu}{c\mu})'$.Let $E(Y_{(r:n(p))r}) = \alpha_{r:n(p)}$ and $V(Y_{(r:n(p))r}) = \beta_{r,r:n(p)}$, $r = 1, 2, ..., n$. Also $Cov(Y_{(r:n(p))r}, Y_{(s:n(p))s}) = 0\, for\, 1 < r < s < n$.Let $\underline{\alpha}^{(p)} = (\alpha_{1:n(p)}, \alpha_{2:n(p)}, ..., \alpha_{n:n(p)})'$ and $B^{(p)} = \left(\left(\beta_{r,s:n(p)}\right)\right)$. Then the BLUE of μ using RSS is given in the following theorem.

Theorem 2. *Let* $\underline{X}^{(p)} = (X_{(1:n(p))1}, X_{(2:n(p))2}, ..., X_{(n:n(p))n})'$ *be the vector of RSS arising from equation (4). Also let* $\underline{Y}^{(p)} = (Y_{(1:n(p))1}, Y_{(2:n(p))2}, ..., Y_{(n:n(p))n})'$ *be the RSS arising from* $f_{i0}\left(y_i\right)$ *,* $i = 1, 2, ..., k.$ *.Let* $\underline{\alpha}^{(p)} = (\alpha_{1:n(p)}, \alpha_{2:n(p)}, ..., \alpha_{n:n(p)})'$ *and* $B^{(p)} = \left(\left(\beta_{r,s:n(p)}\right)\right)$ *be the vector of means and dispersion matrix of* $\underline{Y}^p$ *respectively .Then the BLUE of* μ *and its variance is given by*

$$\tilde{\mu} = \frac{(c\underline{\alpha}^{(p)} + \underline{1})'(B^{(p)})^{-1}}{(c\underline{\alpha}^{(p)} + \underline{1})'(B^{(p)})^{-1}\left(c\underline{\alpha}^{(p)} + \underline{1}\right)}\underline{X}^{(p)} \tag{5}$$

and

$$V(\tilde{\mu}) = \frac{c^2\mu^2}{(c\underline{\alpha}^{(p)} + \underline{1})'(B^{(p)})^{-1}\left(c\underline{\alpha}^{(p)} + \underline{1}\right)}. \tag{6}$$

Proof : Let $\underline{X}^{(p)} = (X_{(1:n(p))1}, X_{(2:n(p))2}, ..., X_{(n:n(p))n})'$ be the vector of RSS arising from equation (4). Define $Y_{(r:n(p))r} = \left(\frac{X_{r:n(p)}-\mu}{c\mu}\right)$, $r = 1, 2, .., n$.. Then $\underline{Y}^p = (Y_{(1:n(p))1}, Y_{(2:n(p))2}, ..., Y_{(n:n(p))n})'$ be the RSS arising from $f_{i0}(y_i)$, $i = 1, 2, ..., k$.. Now clearly $(Y_{(1:n(p))1}, Y_{(2:n(p))2}, ..., Y_{(n:n(p))n})' \overset{d}{=} \left(\frac{X_{(1:n(p))1}-\mu}{c\mu}, \frac{X_{(2:n(p))2}-\mu}{c\mu}, ..., \frac{X_{(n:n(p)n}-\mu}{c\mu}\right)'$. Let $E(Y_{(r:n(p))r}) = \alpha_{r:n(p)}$ and $V(Y_{r:n(p)r}) = \beta_{r,r:n(p)}, r = 1, 2, ..., n$.. Also clearly $Cov(Y_{(r:n(p))r}, Y_{(s:n(p))s}) = 0$ for $1 < r < s < n$. Now we have the following

$$E(X_{(r:n(p))r}) = (c\alpha_{r:n(p)} + 1)\mu\,,\; r = 1, 2, ..., n \tag{7}$$

$$V(X_{(r:n(p))r}) = c^2\mu^2\beta_{r,r:n}, r = 1, 2, ..., n \tag{8}$$

Also clearly

$$Cov(X_{(r:n(p))r}, X_{(s:n(p))s}) = 0\,,\; 0 < r < s < n. \tag{9}$$

Using (7) to (9) we can write

$$E(\underline{X}^{(p)}) = (c\underline{\alpha}^{(p)} + \underline{1})\mu \tag{10}$$

and

$$D(\underline{X}^{(p)}) = B^{(p)}c^2\mu^2, \tag{11}$$

where $\underline{1}$ is a column vector of n ones, $\underline{\alpha}^{(p)} = (\alpha_{1:n(p)}, \alpha_{2:n(p)}, ..., \alpha_{n:n(p)})'$ and $B^{(p)} = \left(\left(\beta_{r,s:n(p)}\right)\right)$ .Then by Generalized Gauss Markov theorem , we got the required result.
Remark: $\underline{\alpha}^{(p)}$ defined in equations (2) and (3) is same as that defined in equations (5) and (6) , while $V^{(p)}$ defined in equations (2) and (3) is not same as that of $B^{(p)}$ defined in equations (5) and (6) . In $B^{(p)}$ all the elements except the diagonal elements are zeros.

4 Estimation of Common Mean of Normal and Logistic Distributions with Known Coefficient of Variation by RSS.

A continuous random variable X_1 is said to have a Normal distribution with location parameter μ and scale parameter $c\mu$, if its *pdf* is given by

$$f(x_1; \mu, c\mu) = \frac{\exp\{-\frac{(x_1-\mu)^2}{2c^2\mu^2}\}}{c\mu\sqrt{2\pi}}\;\;,\; x_1 \in R\,,\; \mu > 0\,,\; c > 0. \tag{12}$$

Also a continuous random variable X_2 is said to have Logistic distribution with location parameter μ and scale parameter $c\mu$, if its *pdf* is given by

$$g(x_2; \mu, c\mu) = \frac{\pi\exp\{-\frac{\pi}{\sqrt{3}}\left(\frac{x_2-\mu}{c\mu}\right)\}}{c\mu\sqrt{3}\,(1 + \exp\{-\frac{\pi}{\sqrt{3}}\left(\frac{x_2-\mu}{c\mu}\right)\})^2}\,,\; x_2 \in R\,,\; \mu > 0,\; c > 0. \tag{13}$$

Clearly both the populations defined in (12) and (13) have common mean and common standard deviation $c\mu$.

5 Simulation Study.

We take sample of size n_1 from (12) and take sample of size n_2 taken from (13), such that $n = n_1 + n_2$ and using the means , variances and covariance's of order statistics of inid random variables of standard normal and standard logistic distributions available in Sajeevkumar(2005), we have calculated the BLUE $\tilde{\mu}$ of μ using RSS of inid random variables and calculate the relative efficiency of our estimator $\tilde{\mu}$ using RSS of inid random variables with that of the estimator $\hat{\mu}$ using order statistics of inid random variables(defined in (2) and Sajeevkumar and Thomas(2006)) were calculated (for $1 \le n_1 < 5$, $1 \le n_2 < 5$ such that $n = n_1 + n_2 \le 5$ and for c=0.1(0.05)0.3) , "Using R software" and are presented in Table 4.1 and Table 4.2 respectively. For example consider the case $n_1 = 1$, $n_2 = 2$, such that $n = 3$ and $c = 0.3$,the the R programme for evaluating the coefficient of the BLUE $\tilde{\mu}$,$Var(\tilde{\mu})$ using RSS , and using order statistics are given below.

$c < -0.3$
$> \alpha < -matrix(c(-0.83385, 0, 0.83385), nrow = 3, byrow = TRUE)$
$> \alpha$
$[, 1]$
$[1,] - 0.83385$
$[2,]0.00000$
$[3,]0.83385$

$> B < -matrix(c(0.59970, 0, 0, 0, 0.41000, 0, 0, 0, 0.59970), nrow = 3, byrow = TRUE)$
$> B$
$[, 1][, 2][, 3]$
$[1,]0.5997, 0.00, 0.0000$
$[2,]0.0000, 0.41, 0.0000$
$[3,]0.0000, 0.00 , 0.5997$
l = matrix(c(1,1,1),nrow=3,byrow=TRUE)
l
[,1]
[1,] 1
[2,] 1
[3,] 1
m1=t(c*$\alpha + l)\% * \%solve(B)$
$> m1$
$[, 1][, 2][, 3]$
$[1,]1.250367, 2.439024, 2.084634$
$>$
$> m2 < -t(c * \alpha + l)\% * \%solve(B)\% * \%(c * \alpha 1)$
$> m2$
$[, 1]$
$[1,]5.982721$
$>$
$> Coefficient < -solve(m2)\% * \%m1$
$> Coefficient$
$[, 1]\ [, 2]\ [, 3]$
$[1,]0.2089963, 0.4076781, 0.3484424$
$>$
$> v1 < -c^2 * solve(m2)$
$> v1$
$[, 1]$
$[1,]0.01499032$
$> V < -matrix(c(0.59970, 0.26528, 0.16475, 0.26528, 0.41000, 0.26528, 0.16475, 0.26528, 0.59970), nrow =$

$3, byrow = TRUE)$
$> V$
$[, 1]\ [, 2]\ [, 3]$
$[1,]0.59970, 0.26528, 0.16475$
$[2,]0.26528, 0.41000, 0.26528$
$[3,]0.16475, 0.26528, 0.59970$
$>$
$> m3 < -t(c * \alpha + l)\% * \%solve(V)$
$> m3$
$[, 1][, 2][, 3]$
$[1,]0.2629589, 1.35449, 1.413229$
$>$
$> m4 < -t(c * \alpha + l)\% * \%solve(V)\% * \%(c * \alpha + 1)$
$> m4$
$[, 1]$
$[1,]3.318424$
$> v2 < -c^2 * solve(m4)$
$> v2$
$[, 1]$
$[1,]0.0269731$

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for $c = 0.1$.

n_1	n_2	n	Coefficients				
			a1	a2	a3	a4	a5
1	1	2	0.47061	0.52628			
1	2	3	0.26365	0.42072	0.31162		
2	1	3	0.27218	0.40150	0.32212		
1	3	4	0.16429	0.30614	0.32385	0.20145	
2	2	4	0.17212	0.29730	0.31483	0.21125	
3	1	4	0.18028	0.28811	0.30542	0.22147	
1	4	5	0.11072	0.22510	0.27316	0.24713	0.13954
2	3	5	0.11661	0.22212	0.26544	0.24420	0.14707
3	2	5	0.12288	0.21881	0.25755	0.24091	0.15506
4	1	5	0.12955	0.21515	0.24949	0.23722	0.16356

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for $c = 0.15$.

n_1	n_2	n	Coefficients				
			a1	a2	a3	a4	a5
1	1	2	0.45492	0.53811			
1	2	3	0.25041	0.41863	0.32201		
2	1	3	0.25834	0.39941	0.33286		
1	3	4	0.15418	0.30012	0.32653	0.20962	
2	2	4	0.16143	0.29128	0.31743	0.21980	
3	1	4	0.16899	0.28211	0.30793	0.23040	
1	4	5	0.10295	0.21841	0.27168	0.25127	0.14596
2	3	5	0.10838	0.21537	0.26393	0.24831	0.15380
3	2	5	0.11415	0.21201	0.25602	0.24497	0.16214
4	1	5	0.12029	0.20833	0.24793	0.24123	0.17099

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for $c = 0.2$

n_1	n_2	n	Coefficients				
			a1	a2	a3	a4	a5
1	1	2	0.43868	0.54900			
1	2	3	0.23683	0.41573	0.33163		
2	1	3	0.24414	0.39651	0.34277		
1	3	4	0.14387	0.29353	0.32849	0.21724	
2	2	4	0.15053	0.28470	0.31929	0.22774	
3	1	4	0.15746	0.27554	0.30968	0.23868	
1	4	5	0.09506	0.21133	0.26964	0.25482	0.15198
2	3	5	0.10002	0.20822	0.26185	0.25180	0.16010
3	2	5	0.10528	0.20482	0.25390	0.24839	0.16873
4	1	5	0.11087	0.20109	0.24578	0.24457	0.17789

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for $c = 0.25$

n_1	n_2	n	Coefficients				
			a1	a2	a3	a4	a5
1	1	2	0.42196	0.55891			
1	2	3	0.22299	0.41207	0.34045		
2	1	3	0.22967	0.39285	0.35182		
1	3	4	0.13344	0.28644	0.32973	0.22428	
2	2	4	0.13948	0.27761	0.32041	0.23505	
3	1	4	0.15746	0.27554	0.30968	0.23868	
1	4	5	0.08711	0.20392	0.26706	0.25777	0.15757
2	3	5	0.09158	0.20074	0.25923	0.25466	0.16593
3	2	5	0.09632	0.19727	0.25123	0.25117	0.17480
4	1	5	0.10136	0.19350	0.24308	0.24726	0.18422

Table 4.1: Coefficients of the BLUE $\tilde{\mu}$ for $c = 0.30$

n_1	n_2	n	Coefficients				
			a1	a2	a3	a4	a5
1	1	2	0.40486	0.56783			
1	2	3	0.20900	0.40768	0.34844		
2	1	3	0.21503	0.38846	0.35997		
1	3	4	0.12294	0.27891	0.33026	0.23073	
2	2	4	0.12838	0.27006	0.32082	0.24169	
3	1	4	0.13407	0.26093	0.31097	0.25308	
1	4	5	0.07914	0.19624	0.26398	0.26011	0.16271
2	3	5	0.08312	0.19298	0.25609	0.25691	0.17127
3	2	5	0.08735	0.18945	0.24805	0.25331	0.18034
4	1	5	0.09184	0.18562	0.23985	0.24928	0.18994

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$,$V_2 = \frac{Var(\hat{\mu})}{\mu^2}$,, and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for $c = 0.10$

n_1	n_2	n	V_1	V_2	e_1
1	1	2	0.00343	0.00496	1.44606
1	2	3	0.00172	0.00327	1.90116
2	1	3	0.00172	0.00326	1.89539
1	3	4	0.00103	0.00242	2.34951
2	2	4	0.00103	0.00245	2.37864
3	1	4	0.00103	0.00246	2.38835
1	4	5	0.00068	0.00197	2.89706
2	3	5	0.00068	0.00198	2.91176
3	2	5	0.00069	0.00199	2.88406
4	1	5	0.00069	0.00199	2.88406

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$,$V_2 = \frac{Var(\hat{\mu})}{\mu^2}$,, and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for $c = 0.15$.

n_1	n_2	n	V_1	V_2	e_1
1	1	2	0.00769	0.01104	1.43563
1	2	3	0.00386	0.00725	1.87824
2	1	3	0.00385	0.00724	1.88052
1	3	4	0.00230	0.00537	2.33478
2	2	4	0.00231	0.00542	2.34632
3	1	4	0.00103	0.00246	2.38835
1	4	5	0.00152	0.00438	2.88158
2	3	5	0.00153	0.00440	2.87582
3	2	5	0.00154	0.00440	2.85714
4	1	5	0.00154	0.00440	2.85714

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$,V$_2 = \frac{Var(\hat{\mu})}{\mu^2}$,, and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for $c = 0.20$.

n_1	n_2	n	V_1	V_2	e_1
1	1	2	0.0136	0.01936	1.42353
1	2	3	0.00682	0.01266	1.8563
2	1	3	0.0068	0.01263	1.85735
1	3	4	0.00406	0.00936	2.30542
2	2	4	0.00408	0.00944	2.31373
3	1	4	0.00408	0.00948	2.32353
1	4	5	0.00268	0.00763	2.84701
2	3	5	0.0027	0.00765	2.83333
3	2	5	0.00271	0.00766	2.82657
4	1	5	0.00272	0.00764	2.80882

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$,V$_2 = \frac{Var(\hat{\mu})}{\mu^2}$,, and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for $c = 0.25$.

n_1	n_2	n	V_1	V_2	e_1
1	1	2	0.02110	0.02971	1.40806
1	2	3	0.01056	0.01935	1.83238
2	1	3	0.01053	0.01927	1.83001
1	3	4	0.00629	0.01427	2.26868
2	2	4	0.00631	0.01437	2.27734
3	1	4	0.00408	0.01441	3.53186
1	4	5	0.00415	0.01163	2.80241
2	3	5	0.00418	0.01165	2.78708
3	2	5	0.00420	0.01163	2.76905
4	1	5	0.00421	0.01159	2.75297

Table 4.2: $V_1 = \frac{Var(\tilde{\mu})}{\mu^2}$,V$_2 = \frac{Var(\hat{\mu})}{\mu^2}$,, and the relative efficiency e_1 of $\tilde{\mu}$ relative to $\hat{\mu}$ for $c = 0.30$.

n_1	n_2	n	V_1	V_2	e_1
1	1	2	0.03013	0.04188	1.38997
1	2	3	0.01504	0.02712	1.80319
2	1	3	0.01499	0.02697	1.79919
1	3	4	0.00895	0.01996	2.23017
2	2	4	0.00897	0.02007	2.23746
3	1	4	0.00898	0.02008	2.23608
1	4	5	0.00591	0.01625	2.74958
2	3	5	0.00594	0.01625	2.73569
3	2	5	0.00597	0.01620	2.71356
4	1	5	0.00598	0.01611	2.69398

To clarify the above simulation procedure , we take five sets of random samples of size 2 each from normal distribution as given in equation (12) with mean 5 and standard deviation $5c$ and take five sets of random samples of size three each from logistic distribution given in equation(13) with mean 5 and standard deviation $5c$. Also assume the common coefficient of variation ,$c = 0.3$. The R command for five set of samples from normal distribution given in equation (12) with mean 5 and standard deviation $5c$, where c=0.3 is given below

$c < -0.3$
$> mean < -5$
$> set1 < -rnorm(2, 5, c * 5)$
$> set1$
5.998503 , 4.904853
$> set2 < -rnorm(2, 5, c * 5)$
$> set2$
2.824136 , 4.658982
$> set3 < -rnorm(2, 5, c * 5)$
$> set3$
7.388833 , 5.087917
$> set4 < -rnorm(2, 5, c * 5)$
$> set4$
2.722494 , 6.606334

$> set5 < -rnorm(2, 5, c*5)$
$> set5$
4.575996 , 5.215312
The R command for five set of sample from logistic distribution given in equation (13) with mean 5 and standard deviation $5c$, where $c = 0.3$ is given below $> c < -0.3$
$> mu < -5$
$> var < -c^2 * mu^2$
$> scale < -sqrt(3 * var)/pi$
$> scale$
[1]0.8269933
$> set1 < -rlogis(3, location = mu, scale = scale)$
$> set1$
$[1]4.605401, 4.503001, 4.321630$
$> set2 < -rlogis(3, location = mu, scale = scale)$
$> set2$
$[1]4.616442, 4.397085, 6.094110$

$> set3 < -rlogis(3, location = mu, scale = scale)$
$> set3$
$[1]4.368014, 4.919718, 5.105035$
$> set4 < -rlogis(3, location = mu, scale = scale)$
$> set4$
$[1]2.658721, 1.854772, 5.277017$

$> set5 < -rlogis(3, location = mu, scale = scale)$
$> set5$ $[1]5.657509, 4.899696, 6.324829$
Now the corresponding five sets of combined pooled ordered sample set (two from Normal distribution and three from logistic distribution) is given below.
Set 1 =(4.321630, 4.503001,4.605401,4.904853,0.5998503)
Set 2=(2.824136, 4.397085, 4.616442,4.658982,6.094110)
Set 3=(4.368014, 4.918718, 5.105035, 5.087917, 7.388833)
Set 4=(1.854772, 2.658721, 2.722494, 5.277017, 6.06334)
Set 5=(4.578996, 4.899696, 5.215312 , 5.657509, 6.324829)
Now the RSS of inid random variable from the above five sets is given below
(4.321630, 4.397085, 5.105035, 5.277017, 6.324829)

Now the estimated value of the common mean μ is given by (using Table 4.1 , $n_1 = 2$, $n_2 = 3$, $n = 5$, c=0.3), $\tilde{\mu}$=4.321630*0.09158+4.397085*0.20074+5.105035*0.25923+5.277017*0.25466+6.32482*0.16593 = 4.99518≈ 5 (Exactly same as that of the common mean of Normal and Logistic distributions). And estimated value of $Var(\tilde{\mu})$ is given by(using table 4.2 ,$n_1 = 2$, $n_2 = 3$, $n = 5$, c=0.3) Var($\tilde{\mu}$) = $\frac{0.00594}{\tilde{\mu}^2} = \frac{0.00594}{4.995148^2} = \frac{0.00594}{24.95150} = 0.04008$

6 Conclusion

In all the cases, efficiency of the estimator $\tilde{\mu}$ defined in equation (5) , calculated using RSS of inid random variables is much better than that of the estimator defined in equation (2) calculated using order statistics of inid random variables given in Sajeevkumar and Thomas (2006). Also it is very difficult to calculate the covariance between any two order statistics in the inid case, but in the case of RSS covariance between any two RSS in the inid case is zero. For example suppose we have two observations of which one observation from the pdf f(x) with distribution function F(x) and the other observation is from the pdf g(x) with distribution function G(x) . Now Poole the two observations and then order them. Now the *pdf* of the first order statistic is $f_{1:2}\,(x) = f(x)(1-G(x)) + g(x)(1-F(x))$ and the *pdf* of the second order statistic is $f_{2:2}(x) = F(x)g(x) + G(x)f(x)$ .Also the joint *pdf* of first and second order statistic is given by $f_{1,2}(x, y) = 2f(x)g(x)$. To evaluate the BLUE of our estimator using RSS of inid random variables , there is no need to evaluate bi-variate integrals , because the co-variance's are zeros.Hence it is very easy to calculate BLUE of the estimator $\tilde{\mu}$ using RSS of inid random variables than the estimator $\hat{\mu}$ using order statistics of inid random variables. All the computational work in this article are done using "R " software.For future study we wish to estimate the common mean of more than two distributions with known coefficient of variation using RSS of inid random variables.

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8 Conflicts of Interest

"The authors declare no conflict of interest."

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