
Benefits of Preservation Technology and Inflation on an EPQ Model for Deteriorating Items with Price and Stock Dependent Demand for Covid19

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Abstract

The COVID-19 pandemic has profoundly disrupted daily life, causing significant difficulties for people worldwide. Amid the crisis, the rates of deterioration and inflation for essential goods, such as medicines, have surged. In response, this study investigates production inventory models that incorporate inflation effects and leverage preservation technology to manage deteriorating goods, where demand is influenced by price and stock levels. The model's effectiveness is demonstrated through numerical analysis conducted using MATHEMATICA 12.0. Sensitivity analysis are performed by varying one parameter at a time while keeping others constant, providing insights into the model's robustness. The findings indicate that the model is robust for real-world applications, particularly in managing inventory during crisis situations like the COVID-19 pandemic. The inclusion of inflation and preservation technology considerations allows for more accurate and cost-effective inventory management. The paper concludes with a discussion on potential future research directions, highlighting areas for further exploration and model enhancement.

Keywords: Inventory, Deterioration, Preservation Technology, Inflation, COVID-19.

1 Introduction

The Economic Production Quantity (EPQ) model is widely used in inventory management to determine optimal production levels that minimize total costs, including setup and holding costs. However, during the COVID-19 pandemic, significant changes in economic conditions and supply chain dynamics have highlighted the need to refine these models to address additional complexities. The crisis led to fluctuations in demand patterns, inflationary pressures, and heightened awareness of supply chain vulnerabilities, especially for items with a deteriorating nature, such as medical supplies, perishables, and other essential commodities. In such a volatile environment, inventory management must account for the effects of inflation and preservation technology on production and stock management. Inflation increases the costs of raw materials, labour, and transportation, directly impacting production and inventory holding costs. It necessitates frequent updates to pricing strategies and inventory policies to maintain profitability. At the same time, preservation technology can help mitigate losses by extending the shelf life of deteriorating items, thereby reducing waste and enhancing supply chain resilience. This technology can significantly impact the EPQ model for deteriorating items by optimizing the production cycle and reducing the need for excessive stock replenishment.

Furthermore, during the pandemic, demand for many items became dependent not only on price but also on stock availability, as shortages led to panic buying and strategic stockpiling. This situation highlights the necessity of incorporating price- and stock-dependent demand factors into the EPQ model. By doing

so, the model can more accurately reflect the real-world dynamics of consumer behaviour during crises and provide strategies for optimizing inventory levels.

This paper examines the benefits of incorporating preservation technology and inflation considerations into an EPQ model for deteriorating items with price- and stock-dependent demand during COVID-19. It aims to develop a more resilient and adaptive inventory management approach, helping businesses maintain efficient operations and meet fluctuating demand under challenging economic conditions.

2 Literature Review

Research on the Economic Production Quantity (EPQ) model has evolved to address various real-world complexities, including preservation technology, inflation, and demand variations. Several studies have extended the classic EPQ model to incorporate the impact of deteriorating items, emphasizing the importance of preservation techniques in reducing spoilage and inventory costs. The influence of inflation on inventory management has also been extensively examined, with findings indicating that inflationary effects necessitate dynamic pricing and cost adjustments to maintain optimal inventory policies. Additionally, models incorporating price- and stock-dependent demand have gained attention, particularly in response to disruptions such as the COVID-19 pandemic, which significantly altered consumer behaviour and supply chain stability. However, integrating preservation technology and inflation into EPQ models for deteriorating items with price- and stock-dependent demand during crises remains an area requiring further exploration.

2.1 Inventory models based on preservation technology

Preservation technology has emerged as a crucial factor in inventory management, particularly for deteriorating items such as perishable goods, pharmaceuticals, and other time-sensitive products. The application of preservation techniques extends the shelf life of products, thereby reducing the rate of deterioration and inventory losses. Studies have shown that incorporating preservation technology into inventory models can significantly enhance cost efficiency by lowering the frequency of stock replenishment and minimizing waste. For instance, refrigeration, controlled-atmosphere storage, and chemical treatments have been widely explored in the literature as means to delay spoilage and maintain product quality over extended periods. Several inventory models have integrated preservation technology investments to address various complexities associated with deteriorating items. Shastri *et al.* (2014) explored supply chain management strategies for two-level trade credit financing with demand dependent on the selling price, considering the impact of preservation technology. Kumar *et al.* (2015) presented a production-inventory model for two-level trade credit financing, incorporating the effects of preservation technology and learning in the supply chain. Singh & Rathore (2016) developed an inventory model that explicitly considered investment in preservation technology to reduce deterioration and optimize inventory costs. Yadav *et al.* (2016) developed a multi-item, multi-constraint supply chain inventory model with demand influenced by multiple variables, incorporating the effects of preservation technology. Mishra *et al.* (2017) extended this approach by creating a model that integrates price- and stock-dependent demand, controls the deterioration rate, addresses shortages, and incorporates preservation technology investment, offering a more comprehensive framework for inventory management. Iqbal *et al.* (2018) proposed a two-echelon supply chain model for rapidly deteriorating products, focusing on the use of preservation technology across different levels of the supply chain to improve efficiency and minimize losses. Similarly, Li *et al.* (2019) studied a joint pricing, replenishment, and preservation technology investment problem for non-instantaneously deteriorating goods, demonstrating that coordinated decision-making can significantly enhance inventory performance. Harit *et al.* (2020) examined the role of preservation technology in optimizing two-warehouse inventory models for deteriorating items, highlighting its impact on reducing the overall deterioration rate and associated costs. Yadav *et al.* (2021) proposed a sustainable supply chain model that reduces waste and carbon emissions by selecting items based on the cross-price elasticity of demand, incorporating preservation technology. Mashud *et al.* (2021) explored the potential applications

of credit financing combined with preservation technology to lower the rate of deterioration while providing flexible financing options for retailers, thereby enhancing financial and inventory management. Bhatnagar *et al.* (2022) presented a single-stage cleaner production system incorporating waste management, reworking, preservation technology, and partial backlogging under the influence of inflation. Singh *et al.* (2022a) considered an inflationary environment and investigated the joint effects of order cost reduction and preservation technology investment under various carbon emission regulation policies, demonstrating the relevance of preservation technology in sustainable inventory management. Singh *et al.* (2022b) developed an inventory model addressing price-sensitive demand and preservation investment, considering partial backlogging and low carbon emissions. Kaushik (2023) developed a model for deteriorating items with trapezoidal-type demand, where preservative measures played a key role in regulating the deterioration rate. Sharma *et al.* (2024) introduced an inventory model with stock-varying demand under an advanced payment plan, incorporating preservation technology investments to further improve inventory control and reduce losses.

2.2 Inventory models based on inflation

The impact of inflation on inventory management has been extensively studied, as it directly affects various cost components, including production, holding, and ordering costs. Inflation leads to fluctuating prices for raw materials, labour, and logistics, making it essential for inventory models to incorporate these changing economic conditions to maintain optimal decision-making. Several studies have extended traditional inventory models to include the effects of inflation, demonstrating its significant influence on inventory policies and cost optimization. Moreover, various inventory models have been developed to incorporate the effects of inflation on deteriorating items, highlighting its crucial role in shaping inventory costs and policies. Tripathi (2018) examined an inventory model for deteriorating items where demand was inventory-driven, selling price was exponentially time-sensitive, and holding costs were time-induced, showcasing the complex interactions between inflation and inventory parameters. Kumar (2020) developed an Economic Production Quantity (EPQ) model for imperfect items considering inflation, where demand, deterioration rate, and production rate were constant. Their results demonstrated the importance of accounting for inflation in production planning to maintain cost efficiency. Handa *et al.* (2021) further expanded this field by exploring a production–inventory model under inflation, with fluctuating demand and shortages, emphasizing how economic conditions influence production strategies. Sharma & Sharma (2022) analysed the influence of inflation in an inventory model with partially backlogged shortages, incorporating nonlinear holding costs and nonlinear demand. Their findings underscored the need for adaptive inventory policies that consider complex cost behaviours under inflation. Padiyar *et al.* (2022) explored a three-echelon supply chain management model addressing the challenges of deteriorating products in the context of inflation, highlighting its implications for inventory control and decision-making processes in supply chains. Bhawaria & Rathore (2023) presented an EPQ model for deteriorating items that incorporated both inflation and trade credit policy, along with preservation technology, offering insights into optimizing inventory costs in financially constrained environments. More recently, Pal *et al.* (2024) investigated a two-warehouse inventory problem for non-instantaneously deteriorating items, considering allowable delay in payments under inflation. Their study addressed the unique challenges posed by inflation in managing multi-warehouse systems and emphasized strategies for payment flexibility and inventory optimization.

2.3 Inventory models based on price and stock dependent demand

Price- and stock-dependent demand models have gained considerable attention in inventory management, as they more accurately reflect real-world consumer behaviour where demand is influenced by both the selling price and available inventory levels. These models acknowledge that higher prices typically suppress demand, while lower prices can stimulate it. At the same time, stock availability can impact demand, with higher inventory levels often signalling abundance and increasing customer confidence, while lower stock levels may induce urgency or stockpiling behaviour. Mashud *et al.* (2018) developed

an inventory model that integrates price- and stock-dependent demand, partially backlogged shortages, and deterioration rates. In their model, demand is influenced by both price and stock levels, except during periods of shortages, when it becomes solely dependent on the product's price. This approach provides a comprehensive framework for managing inventory under varying demand conditions. Das *et al.* (2020) investigated a production–inventory model that incorporates a product replacement policy, where demand depends on price, stock levels, and the replacement period. Their study emphasized optimizing inventory strategies by considering multiple demand drivers simultaneously. Macías-López *et al.* (2021) introduced an inventory model for perishable goods that accounts for shelf life and nonlinear holding costs, along with demand driven by price, stock, and time. This approach offers valuable insights for managing inventory when product perishability and dynamic demand are critical factors. Palanivel *et al.* (2022) explored an inventory model in which market demand rates fluctuate based on stock levels and selling prices. Their study addressed the optimization of order quantity and partial backlogging while aiming to maximize total profit. Additionally, they considered holding costs that vary with storage time. Narang *et al.* (2023) proposed a production–inventory model for instantaneously deteriorating items, where demand is influenced by price, stock levels, and advertising efforts. Their model highlights the interplay between marketing strategies and inventory management. Singh & Singh (2023) investigated a sustainable inventory model incorporating preorder discounts and online payment options, emphasizing the role of controllable carbon emissions in enhancing operational efficiency and environmental sustainability in industrial settings. Pando *et al.* (2024) developed an inventory model aimed at maximizing profitability, incorporating price- and stock-dependent demand alongside time- and stock-quantity-dependent holding costs. Their work provides a framework for aligning inventory policies with profitability goals while addressing complex demand behaviours.

3 Assumptions and Notations

3.1 Assumptions

The following assumptions are used in this model .

- i. The product's demand rate depends on both its price and available stock. Mathematically demand is given by $D = (a - bp + \beta I(t))$ where $a/b > p$, $a, b > 0$, $\beta > 0$.
- ii. The manufactured product / unit is always ready to meet market demand.
- iii. Perishable goods cannot be replaced or repaired once they deteriorate.
- iv. The holding cost of inventory units is assumed to be time-dependent.
- v. $P = k.D$, where $k \geq 1$, D denotes the demand and P is the production rate.
- vi. The COVID-19 pandemic has prompted consideration of preservation technology and the impact of inflation.
- vii. To reduce the deterioration rate, investment in preservation technology is taken into consideration. Mathematically, $\omega\{\xi\} = e^{-v/xi}$, $v > 0$

3.2 Notations and explanations of parameters

Notations	Explanations
A	Setup cost
C_h	Holding cost per unit per time
θ	deterioration rate : $0 \leq \theta \leq 1$
C_p	production cost per unit
C_d	Deterioration cost per unit
P	Production rate (units per unit time)
r_1	Per unit price discount
r	Rate of inflation
$I_1(t)$	Inventory level at time $t \in [0, t_1]$
$I_2(t)$	Inventory level at time $t \in [t_1, T]$
k	Production rate factor
v	Preservation technology factor
ξ	Investment in preservation technology
$\omega(\xi)$	Preservation technology function
a	Demand constant parameter
b	Price elasticity factor
β	Stock dependent parameter
Q	Maximum level of inventory
Decision variables	
t_1	The time at which manufacturing was halted
T	Cycle length

Table 1: Notations and explanations of parameters

4 Model Formulation

Differential equation (1) represents the stock level during the production period $[0, t_1]$, with inventory rising due to production and reaching its maximum at t_1 . During the interval $[t_1, T]$, the inventory level decreases as a result of both demand and deterioration, reaching its minimum at time T . This model is illustrated in figure 1.

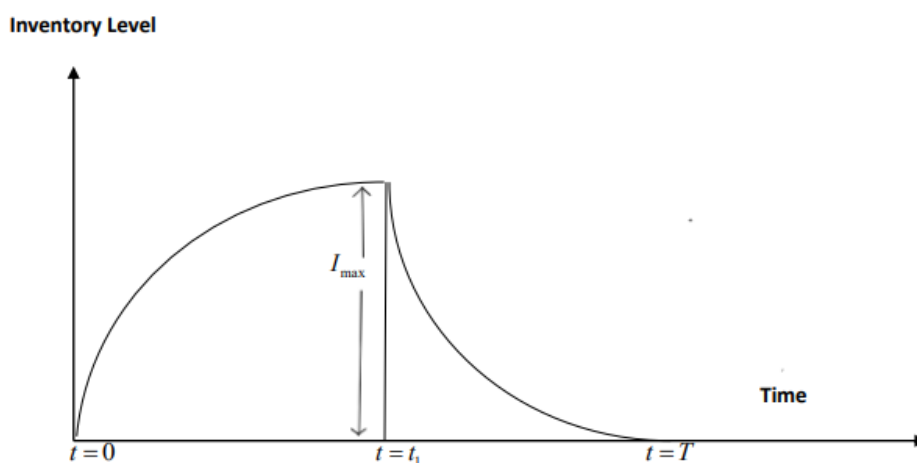


Figure 1: The inventory's graphical representation with respect to time.

$$\frac{dI_1(t)}{dt} + \theta\omega(\xi)I_1(t) = (k - 1)(a - bp + \beta I_1(t)), \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + (\theta\omega(\xi) + \beta)I_2(t) = -(a - bp), \quad t_1 \leq t \leq T \tag{2}$$

Boundary conditions:

$$I_1(0) = 0, \quad I_2(T) = 0$$

$$I_1(t) = \left(\frac{(k - 1)(a - bp)}{(\theta\omega(\xi) + (1 - k)\beta)} \right) [1 - e^{-(\theta\omega(\xi) + (1 - k)\beta)t}], \quad 0 \leq t \leq t_1 \tag{3}$$

$$I_2(t) = \left(\frac{(a - bp)}{(\theta\omega(\xi) + \beta)} \right) [e^{(\theta\omega(\xi) + \beta)(T - t_1)} - 1], \quad t_1 \leq t \leq T \tag{4}$$

Maximum level of inventory,

$$Q = \left[\frac{(k - 1)(a - bp)}{(\theta\omega(\xi) + (1 - k)\beta)} \right] [1 - e^{-(\theta\omega(\xi) + (1 - k)\beta)t_1}] \tag{5}$$

Applying the continuity condition at $t = t_1$

$$t_1 = \frac{e^{(\theta\omega(\xi) + \beta)T}}{[k(\theta\omega(\xi) + \beta) + (\theta\omega(\xi) + \beta)^2T]} \tag{6}$$

The various costs associated with the model are as follows:

Production Cost

$$\begin{aligned} PC &= C_p \int_0^{t_1} k[(a - bp) + \beta I_1(t)]e^{-rt} dt \\ &= C_p k \left[\frac{(a - bp)}{r} (1 - e^{-rt_1}) + \frac{\beta(k - 1)(a - bp)}{\theta\omega(\xi) + (1 - k)} \left(\frac{1 - e^{-rt_1}}{r} + \frac{e^{-(\theta\omega(\xi) + (1 - k)\beta + r)t_1} - 1}{\theta\omega(\xi) + (1 - k)\beta + r} \right) \right] \end{aligned} \tag{7}$$

Setup Cost

$$SC = A \tag{8}$$

Holding cost

$$\begin{aligned} HC &= C_h [\int_0^{t_1} e^{-rt} I_1(t) dt + \int_{t_1}^T e^{-rt} I_2(t) dt] \\ &= C_h \left\{ \left(\frac{(a - bp)(k - 1)}{\theta\omega(\xi) + (1 - k)\beta} \right) \left[\frac{1 - e^{-rt_1}}{r} + \frac{e^{-(\theta\omega(\xi) + (1 - k)\beta + r)t_1} - 1}{\theta\omega(\xi) + (1 - k)\beta + r} \right] \right. \\ &\quad \left. + \left(\frac{a - bp}{\theta\omega(\xi) + \beta} \right) \left[\frac{e^{(\theta\omega(\xi) + \beta)T}}{\theta\omega(\xi) + \beta + r} (e^{-(\theta\omega(\xi) + \beta + r)t_1} - e^{-(\theta\omega(\xi) + \beta + r)T}) + \frac{e^{-rT} - e^{-rt_1}}{r} \right] \right\} \end{aligned} \tag{9}$$

Deterioration cost

$$\begin{aligned} DC &= C_d [\int_0^{t_1} \theta\omega(\xi) e^{-rt} I_1(t) dt + \int_{t_1}^T \theta\omega(\xi) e^{-rt} I_2(t) dt] \\ &= C_d \theta\omega(\xi) \left\{ \left(\frac{(a - bp)(k - 1)}{\theta\omega(\xi) + (1 - k)\beta} \right) \left[\frac{1 - e^{-rt_1}}{r} + \frac{e^{-(\theta\omega(\xi) + (1 - k)\beta + r)t_1} - 1}{\theta\omega(\xi) + (1 - k)\beta + r} \right] \right. \\ &\quad \left. + \left(\frac{a - bp}{\theta\omega(\xi) + \beta} \right) \left[\frac{e^{(\theta\omega(\xi) + \beta)T}}{\theta\omega(\xi) + \beta + r} (e^{-(\theta\omega(\xi) + \beta + r)t_1} - e^{-(\theta\omega(\xi) + \beta + r)T}) + \frac{e^{-rT} - e^{-rt_1}}{r} \right] \right\} \end{aligned} \tag{10}$$

Price discount

$$PD = C_p r_1 \left[\int_0^{t_1} ((a - bp) + \beta I_1(t)) e^{-rt} dt + \int_{t_1}^T ((a - bp) + \beta I_2(t)) e^{-rt} dt \right]$$

$$PD = C_p r_1 \left\{ (a - bp) \left(\frac{1 - e^{-rT}}{r} \right) + \beta \left[\frac{(a - bp)(k - 1)}{\theta\omega(\xi) + (1 - k)\beta} \left(\frac{1 - e^{-rt_1}}{r} + \frac{e^{-(\theta\omega(\xi) + (1 - k)\beta + r)t_1} - 1}{\theta\omega(\xi) + (1 - k)\beta + r} \right) + \frac{(a - bp)}{\theta\omega(\xi) + \beta} \left(\frac{e^{(\theta\omega(\xi) + \beta)T}}{\theta\omega(\xi) + \beta + r} \left(e^{-(\theta\omega(\xi) + \beta + r)t_1} - e^{-(\theta\omega(\xi) + \beta + r)T} \right) + \frac{e^{-rT} - e^{-rt_1}}{r} \right) \right] \right\} \quad (11)$$

Preservation technology cost

$$PTC = \xi T \quad (12)$$

$$ATC(t_1, T) = \frac{1}{T} (PC + SC + HC + DC + PD + PTC)$$

$$\begin{aligned} &= \frac{1}{T} \left[C_p k \left(\frac{a - bp}{r} (1 - e^{-rt_1}) \right) + \frac{\beta(k - 1)(a - bp)}{\theta\omega(\xi) + (1 - k)\beta} \left[\frac{1 - e^{-rt_1}}{r} + \frac{e^{-(\theta\omega(\xi) + (1 - k)\beta + r)t_1} - 1}{\theta\omega(\xi) + (1 - k)\beta + r} \right] \right. \\ &\quad + A + \frac{(a - bp)(k - 1)}{\theta\omega(\xi) + (1 - k)\beta} \left[\frac{1 - e^{-rt_1}}{r} + \frac{e^{-(\theta\omega(\xi) + (1 - k)\beta + r)t_1} - 1}{\theta\omega(\xi) + (1 - k)\beta + r} \right] \\ &\quad + \frac{a - bp}{\theta\omega(\xi) + \beta} \left[\frac{e^{(\theta\omega(\xi) + \beta)T}}{\theta\omega(\xi) + \beta + r} \left(e^{-(\theta\omega(\xi) + \beta + r)t_1} - e^{-(\theta\omega(\xi) + \beta + r)T} \right) + \frac{e^{-rT} - e^{-rt_1}}{r} \right] \\ &\quad \left. + [C_h + C_d\theta\omega(\xi) + C_p r_1 \beta] + \left[C_p r_1 (a - bp) \frac{1 - e^{-rT}}{r} \right] + \xi T \right] \quad (13) \end{aligned}$$

5 Solution methodology

To minimize the average total cost and determine the optimal values of the decision variables t_1 and T , the following methodology is used .

- i. Find $\frac{\partial ATC}{\partial t_1}$ and $\frac{\partial ATC}{\partial T}$.
- ii. To find the value of t_1 and T , put $\frac{\partial ATC}{\partial t_1} = 0$ and $\frac{\partial ATC}{\partial T} = 0$.
- iii. For maxima and minima, find $\left(\frac{\partial^2 ATC}{\partial t_1^2} \right) \left(\frac{\partial^2 ATC}{\partial T^2} \right) - \left(\frac{\partial^2 ATC}{\partial t_1 \partial T} \right)^2 > 0$, and $\left(\frac{\partial^2 ATC}{\partial t_1^2} \right) > 0$.

6 Numerical illustration

Given Parameters (in appropriate units)

C_p (Production cost per unit) = 13.07, A (Setup cost per production cycle) = 15, p (Selling price per unit) = 7.67, a (Demand constant parameter) = 18, b (Price elasticity factor) = 0.31, r (Rate of inflation) = 0.33, C_h (Holding cost per unit per time) = 13, θ (Deterioration rate) = 0.01, v (Preservation technology factor) = 2, β (Stock dependent parameter) = 77, C_d (Deterioration cost per unit) = 0.06, ξ (Investment in preservation technology) = 103, r_1 (Per unit price discount) = 7, k (Production rate factor) = 0.99.

Optimal Values:

- ATC^* (Average total cost) = 18.448,
- t_1^* (Optimal time for decision variable t_1) = 3.74886
- T^{**} (Optimal replenishment cycle time) = 4.46888

7 Convexity

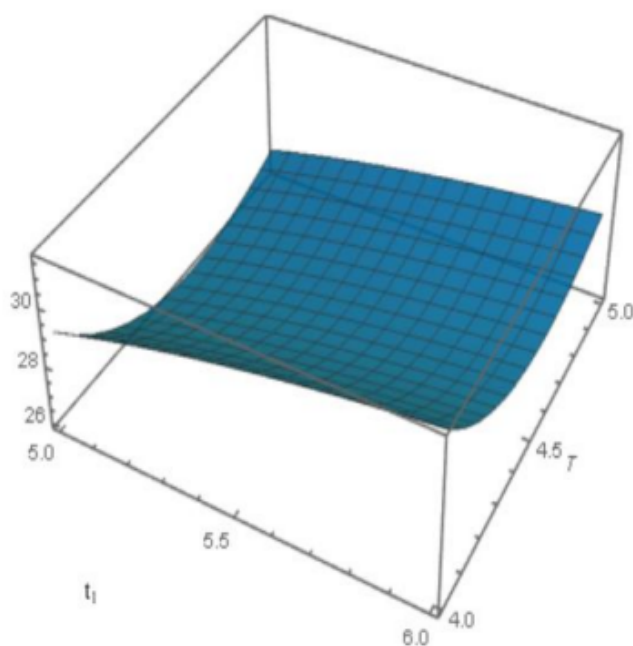


Figure 2: Convexity among the production stopping time t_1 , cycle length T , and the average total cost.

8 Sensitivity analysis

The effects of variation (from -20 % to 20 %) of various parameters are shown in table 8.1.

8.1 Table

Parameter	%change	t_1	T	ATC
A	20	3.74886	4.47865	19.1186
	10	3.74886	4.47422	18.7835
	-10	3.74886	4.46426	18.1122
	-20	3.74886	4.45767	17.7761
p	20	3.74886	4.60249	31.4235
	10	3.74886	4.5335	25.0426
	-10	3.74886	4.40851	11.6404
	-20	3.74886	4.35166	4.62023
b	20	3.74886	4.61624	32.6206
	10	3.74886	4.53994	25.6626
	-10	3.74886	4.40284	10.9777
	-20	3.74886	4.34124	3.25216
C_h	20	3.74886	4.46599	18.2057
	10	3.74886	4.46751	18.3269
	-10	3.74886	4.47111	18.5971
	-20	3.74886	4.4721	18.6903
C_p	20	4.16793	3.50055	-6.11913
	10	3.61482	4.30639	6.32152
	-10	2.90909	4.69703	30.2348
	-20	4.10526	4.90297	41.6221
r	20	3.63217	3.62792	47.1585
	10	3.74726	4.17917	34.6943
	-10	3.75239	4.79884	-3.69333
	-20	3.75186	5.16567	-30.6812
β	20	3.12627	4.89485	52.4021
	10	3.40937	4.67078	37.1847
	-10	3.91245	3.90779	-5.13712
	-20	3.74255	3.73663	-38.8
ξ	20	3.74886	4.46888	39.048
	10	3.74886	4.46888	28.748
	-10	3.74886	4.46904	8.14805
	-20	3.74886	4.46904	-2.15195
r_1	20	3.49569	3.49116	-66.435
	10	3.74886	4.10496	-22.3872
	-10	3.7518	5.0291	56.1667
	-20	3.74886	6.67706	88.9914
k	20	-	0.0164925	1267.59
	10	-	0.0164925	1267.59
	-10	2.37987	2.3762	-281.642
	-20	2.20558	2.20144	-350.687

Table 2: Sensitivity analysis of various parameters

8.2 Figure

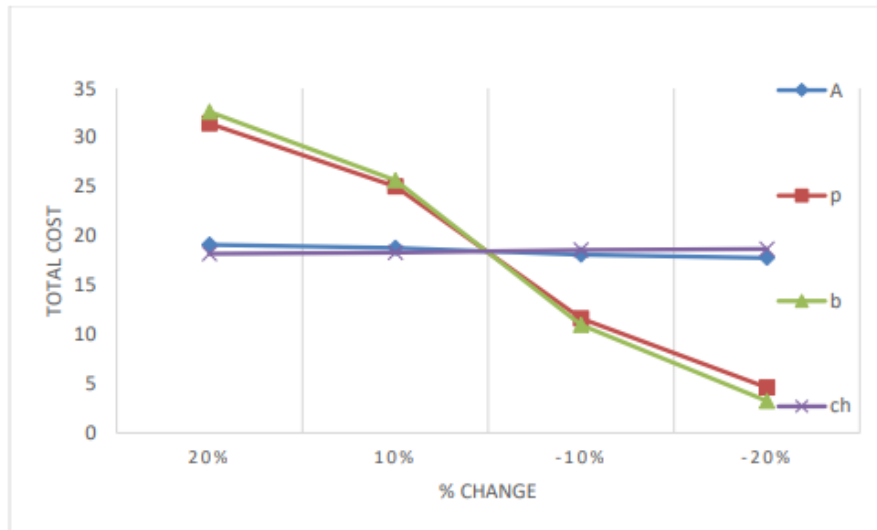


Figure 3: Graphical representation of variation in total cost with respect to parameters 'A, p, b and C_h .

8.3 Figure



Figure 4: Graphical representation of variation in total cost with respect to parameters $\theta, C_p, a,$ and r .

8.4 Figure

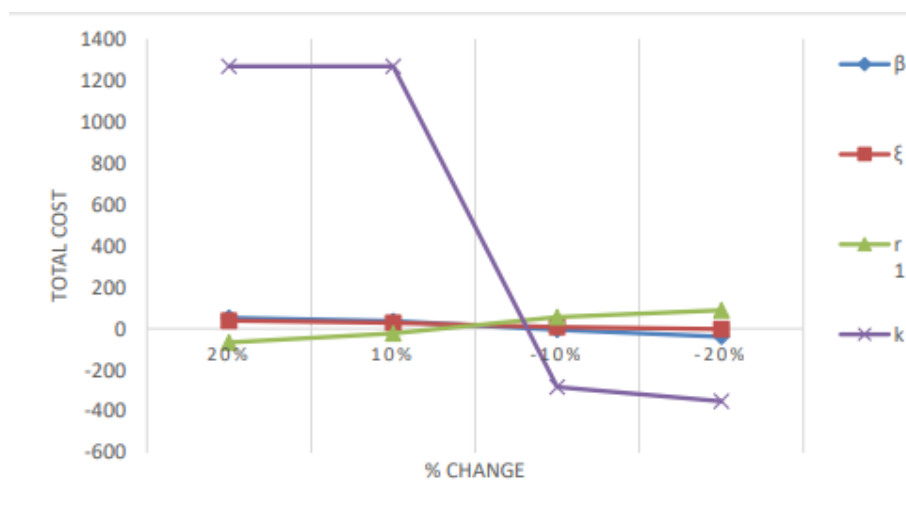


Figure 5: Graphical representation of variation in total cost with respect to parameters β , ξ , r_1 , and k .

9 Observations from the sensitivity analysis table

- i. When setup cost (A) increases from -20% to 20%, then T and ATC (average total cost) also increase but t_1 remains fixed.
- ii. When selling price (p) increases from -20% to 20%, then T and ATC (average total cost) increase but t_1 remains fixed.
- iii. When price elasticity factor (b) increases from 20% to 20%, then T and ATC (average total cost) increase but t_1 remains fixed.
- iv. When holding cost (C_h) increases from -20% to 20%, then T and ATC (average total cost) both slightly decrease but t_1 remains fixed.
- v. When production cost per unit (C_p) increases from -20% to 20%, then t_1 decreases and after this it increases, but T and ATC (average total cost) decrease.
- vi. When rate of inflation (r) increases from -20% to 20%, then t_1 slightly decreases and then increases, while T decreases and ATC (average total cost) increases.
- vii. When stock dependent parameter (β) increases from -20% to 20%, then t_1 increases and then decreases. T and ATC (average total cost) both increase.
- viii. When investment in preservation technology (ξ) increases from -20% to 20%, then t_1 remains fixed. T remains fixed initially, then slightly decreases, and remains fixed again. ATC (average total cost) increases.
- ix. When per unit price discount (r_1) increases from -20% to 20%, then t_1 slightly increases initially, then slightly decreases. T and ATC (average total cost) decrease.
- x. When production rate factor (k) increases from -20% to 20%, then t_1 increases slightly at first and after this it fails to converge. T increases slightly at first, then decreases and remains fixed. ATC (average total cost) increases and then remains fixed.

10 Managerial insights

The following insights are drawn from the perspective of a firm and its decision-makers:

- i. Increased demand necessitates longer production cycles, which in turn raises total costs because of higher inventory levels. Keeping the decision point constant suggests that the firm's original inventory strategy is resilient enough to handle fluctuations in demand.
- ii. If production rates are increased without aligning with actual demand, it can result in longer production cycles and potentially higher overall costs. To avoid this, decision-makers must carefully adjust production levels based on current and projected demand. This helps maintain optimal inventory levels and prevents unnecessary cost escalation.
- iii. Higher backorder costs encourage longer production cycles to avoid shortages, leading to increased total costs. Reducing backorder situations through improved demand forecasting can minimize these costs.
- iv. High holding costs encourage firms to reduce inventory levels and adopt shorter production cycles, which can help lower total costs. Decision-makers should focus on minimizing holding costs by improving storage efficiency and evaluating whether safety stock levels can be safely reduced without affecting service quality.
- v. The model shows strong resilience to changes in deterioration rate, suggesting that the firm's current inventory strategy is effective in minimizing the negative effects of product deterioration. Decision-makers should continue to monitor this factor to ensure long-term efficiency and sustainability.
- vi. The firm's investment rate in green technology does not appear to have a direct impact on inventory cycles or overall costs. This suggests that external factors—such as carbon taxes or government incentives—may play a more significant role in encouraging green investments. Decision-makers should consider these influences when planning sustainability strategies.
- vii. The cost associated with the carbon cap-and-trade policy may not significantly influence production cycle decisions directly. However, decision-makers should consider integrating carbon emissions reduction strategies, as these can offer substantial long-term economic and environmental advantages for the firm.
- viii. Initially, a firm's production manager may be encouraged to shorten the production cycle to minimize expenses associated with carbon emissions. If the carbon tax rises further, the manager might need to adjust the production strategy to keep overall costs under control.
- ix. A firm's decision maker can achieve substantial reductions in total costs and production cycle time by initially adopting green production practices. However, as the green production factor continues to rise, the additional benefits gradually diminish, prompting the manager to reconsider the production strategy.
- x. Higher emission rates drive efforts to reduce production cycle time, potentially leading to cost increases due to regulatory penalties. Managers should focus on emission reduction techniques to manage these costs.
- xi. Greater effectiveness in carbon reduction can lead to increased total costs, possibly due to investments in carbon-reducing technologies. Managers should balance carbon reduction efforts with cost-control measures.
- xii. Although higher setup costs may not immediately influence the initial production decision, they can contribute to higher overall costs. A firm's production manager can mitigate these increases by reducing the frequency of setups or optimizing batch sizes.

- xiii. Carbon credits incentivize a firm's production manager to slightly extend the production decision point while lowering both total cycle time and overall costs. Managers can strategically leverage these credits to minimize carbon-related expenses effectively.
- xiv. The cost elasticity associated with green technology influences production cycles and total costs in a non-linear manner. While early investments may initially increase costs, subsequent adjustments based on elasticity allow managers to shorten production cycles and stabilize overall expenditures.

11 Conclusion

The COVID-19 pandemic has highlighted the critical need for effective management of deteriorating products, such as medicines with limited shelf life, especially under inflationary conditions. In the proposed model, demand depends on both price and stock levels, capturing customer behavior accurately. Based on the numerical example solved using Mathematica 12.0, the optimal values are $ATC^* = 18.448$ (minimum average total cost), $t_1^* = 3.74886$ (optimal production decision timing), $T^{**} = 4.46888$ (optimal replenishment cycle). These results demonstrate that integrating price- and stock-dependent demand with preservation technology and inflation effects can substantially reduce total costs and improve production planning. Managers can utilize these insights to decide when to produce, how much to stock, and how to invest in preservation technologies to minimize losses. Future research could extend the model to multiple products and consider fully backlogged shortages, further enhancing its applicability in real-world inventory management during crises like COVID-19.

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13 Conflicts of Interest

The authors declare no conflict of interest.

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