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# Economic Order Quantity Inventory Model with Price Dependent Expected Demand under Probabilistic Deterioration and Shortages

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## ABSTRACT

The economic order quantity (EOQ) model is the most suitable and applicable inventory model for a supply chain consisting of a single retailer. Generally, an inventory model with a constant demand rate for a perishable item is inappropriate for customers' changing habits. In the competitive world, market demand always changes with time and plays a crucial role in optimizing the supply chain. Initially, the demand rate was slow, but after some time, it gradually increased. Hence, in this paper, the price-dependent uniform demand rate is assumed to be used to develop the EOQ inventory model. Demand during the scheduling period occurs uniformly, and it varies with the selling price of the perishable item. The formulated model assumes two probabilistic deterioration rates (exponential and Weibull) at two separate time points in the inventory cycle. Next, the proposed EOQ inventory model is enriched with time-dependent linear holding costs under shortages, which are completely backlogged. The optimum order quantity is obtained by maximizing the profit expression of the inventory during the scheduling period, and the concavity of the model is checked and validated using a graphical technique. For the model, analytical solutions are difficult to obtain using the classical approach since the model is highly nonlinear. Hence, the genetic algorithm is a class of evolutionary algorithms that solves the formulated model. Lastly, sensitivity analysis of the model is discussed at different parameter values.

## 1. Introduction

The inventory models with the constant demand for a perishable item have been studied by several authors in literature. But in reality, the demand for a perishable item is not constant. It is affected by several variables such as seasonality, selling price of the decaying item, availability etc. In inventory, deterioration of items is a realistic phenomenon, and it is described as decay, spoilage, damage, etc. of items. During the normal storage period, several items, such as pharmaceutical items, food items, perfumes, blood etc., decrease their quality under deterioration, and their life is fixed. Hence, when deciding the optimum inventory level for a single-tier supply chain, the marginal loss due to a decline in their use quality cannot go unnoticed (Dye *et al.* (2007)). The main task in inventory optimization is determining the optimal inventory policy.

Price of a perishable good is an important factor that affects its demand rate. Price decision is one of the main fundamental problems a retailer faces in the supplier-retailer supply chain (Panda *et al.* (2009)). Pricing is a significant criteria for almost all retailers in the supermarket to increase sales. Therefore, price reduction affects the demand of the perishable item. Several authors were studied stock and price sensitive demand phenomena while developing the inventory model (Alfares and Ghaithan (2016), Begum *et al.* (2012) and Khanra *et al.* (2010)). Hence, uniform demand during the scheduling period as a function of selling price is assumed in the proposed model. The expected demand rate is computed using a continuous uniform distribution over the parameters $a > 0$ and

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$b > 0$. The objective of the paper is to develop an economic order quantity (EOQ) inventory model under probabilistic deterioration, linear time dependent holding cost and selling price dependent uniformly distributed demand rate. In which, the demand rate is assumed to be uniformly dependent on the selling price. It is the first time that, two different probabilistic deterioration rates viz. exponential and Weibull with price varying uniform demand rate is considered while the development of the EOQ inventory model under linear time dependent holding cost. Such models have wide applications in food industry. The main contributions of this paper as: developing a new probabilistic EOQ inventory model. To find out the analytical solution if any, a solution methodology based on evolutionary algorithm is formulated.

The rest of the paper is organized as follows. The exhaustive review of literature will be explained in Section 2. In Section 3, formulation of mathematical model with assumptions and notations are presented. The solution methodology based on genetic algorithm is explained in Section 4. Section 5 presents the numerical illustrations. The impact of various parameters is checked using sensitivity analysis and the results are presented in Section 6. Lastly, Section 7 concludes the study and presents further developments of the study.

## 2. Literature Review

A deterministic EOQ stock dependent demand rate inventory model for a single item was proposed by Goh (1994). Strictly differentiable with a concave polynomial functional form of the demand rate, nonlinear holding cost as a function of on hand inventory was assumed in the developed model. Deterministic inventory model for perishable items under-price dependent demand was formulated by Dye *et al.* (2007). Shortages were allowed in the model and they were partially backlogged. Panda *et al.* (2009) developed an EOQ inventory model with selling price and stock dependent demand rate. Khanra *et al.* (2010) formulated an EOQ inventory model for the deteriorated item. Stock as well as price dependent demand rate was assumed in the model. Heaviside function was used to represent the deterioration rate of the item and it starts after a time $'t'$ where $t \sim exp(\theta)$. Sarkar *et al.* (2013) developed an EOQ inventory model for finite replenishment rate and time dependent increasing demand for a perishable item. A fuzzy EOQ inventory model under selling price and promotional effort dependent demand rate have developed by De and Sana (2015). Triangular fuzzy number was used to represent the fuzziness among the parameters of the model.

Wu *et al.* (2017) developed an EOQ inventory model with price sensitive trapezoidal type demand under partial backlogging. By assuming selling price, reference price, product freshness and stock level dependent demand rate inventory model was developed by Li and Teng (2018). An economic product quantity (EPQ) inventory model was formulated by Ruidas *et al.* (2019), in which stock and price dependent demand with stochastic production rate was assumed. Agi and Soni (2020) have formulated a deterministic inventory model for optimizing pricing and inventory control of a perishable item. Model was enriched under stock, selling price and freshness condition of the perishable product dependent demand rate. Next, price varying demand economic inventory model was proposed by Kurade (2021). Latpate *et al.* (2024) proposed new green supply chain models under random demand. Das *et al.* (2024) formulated and economic order quantity inventory model for the green products by assuming selling price dependent demand rate. For solving the developed inventory model, they have proposed new teaching learning based optimization algorithm.

A replenishment policy with price dependent demand and Weibull distributed deterioration is proposed by Wee (1997). Mondal *et al.* (2003) formulated an inventory model price dependent demand. Under no shortages, Weibull distributed amelioration rate and random deterioration rate was assumed in the model. Mukhopadhyay *et al.* (2005) developed an inventory replenishment policy by assuming selling price dependent demand and Weibull deterioration. An EOQ inventory model over an infinite time horizon with price dependent demand under partial backlogging for perishable items with time varying deterioration was proposed by Sana (2010). Begum *et al.* (2012) developed an inventory model under without backlogging with selling price dependent demand and time proportional deterioration. Various probabilistic supply chain models for perishable items were proposed by Kurade *et al.* (2022). They were assumed constant, Weibull and generalized exponential distributed deterioration rates. New solution methodology were proposed for solving purpose. Latpate *et al.* (2023) proposed new cold supply chain inventory models for agriculture products under stochastic deterioration rates. The necessity and sufficient conditions of the formulated models were proposed by using graphical way. They solved the formulated model using Matlab. Giri *et al.* (2024) proposed an imperfect economic product quality inventory model for perishable products. The various pollution costs under uncertain planning horizon were considered in the developed model.

Genetic algorithm a class of stochastic search optimization algorithm developed by Holland (1992). It works using three genetic operators viz., reproduction, crossover and mutation (Vose (1999), Deb (2001), Sivanandam *et al.* (2008)). During last decade, genetic algorithm is fused with various other optimization techniques for the development of new robust optimization algorithm (Deb (2001)). Latpate and Kurade (2017) developed a new optimization algorithm for solving uncertain optimization problems involved in the supply chain. Lambora *et al.* (2019) presented the detailed review on genetic algorithms for big data and machine learning. Kurade and Latpate (2020) developed EOQ inventory models under no, complete and partial shortages. They have developed new evolutionary algorithm for solving the model. Latpate and Kurade (2022) formulated a multi-objective multi-index multi–tier supply chain model for optimizing the crude oil network of India. Fuzzy and evolutionary based optimization algorithm was developed for solving the model.

Pando *et al.* (2013) formulated an economic lot size inventory model by considering nonlinear holding cost hinging on time and quantity. The demand rate was dependent level of inventory. Shortages were not allowed in the formulated model. Yang (2014) developed an inventory model with stock dependent demand rate as well as holding cost. The formulated model was enriched under shortages and they were partially backlogged. Demand rate as a power function of stock was considered in the model. Alfares and Ghaitan (2016) proposed an inventory model with price dependent demand under time varying storage dependent holding cost. In the model, demand rate as a linearly decreasing function of price, holding cost a linearly increasing function of the storage time, and purchase cost a decreasing function of the order size were considered. San-José et al (2019) formulated an inventory policy for price and quantity under power demand pattern with nonlinear holding cost. The time and selling price dependent demand rate was considered while development of the model.

**Table 1:** An overview of existing literature.

| Authors | Year | Probabilistic Deterioration | Demand Rate | Genetic Algorithm | Shortages | Holding Cost |
|---|---|---|---|---|---|---|
| Agi *et al.* | 2020 | Yes | Age, stock and price | No | No | - |
| Alfares and Ghaithan | 2016 | No | Price | No | No | Time varying |
| Begum *et al.* | 2012 | Yes | Price | No | No | - |
| Dari and Sani | 2020 | Yes | Quadratic | No | No | Linear |
| Das *et al.* | 2024 | Yes | Selling price and green level | No | No | - |
| De and Sana | 2015 | No | Promotional effort and selling price | No | Yes | - |
| Dye *et al.* | 2007 | Yes | Constant | No | Yes | - |
| Goh | 1994 | No | General | No | No | General |
| Khanra *et al.* | 2010 | Yes | Stock and price | No | No | - |

| | | | | | | |
|---|---|---|---|---|---|---|
| Kurade | 2021 | Yes | Price varying | Yes | No | Linear |
| Kurade *et al.* | 2020 | Yes | Constant | Yes | Yes | Constant |
| Kurade *et al.* | 2022 | Yes | Constant | Yes | Yes | Constant |
| Latpate *et al.* | 2023 | Yes | Constant | No | No | - |
| Li and Teng | 2018 | Yes | Selling price, reference price, product freshness, and displayed stocks | No | No | - |
| Pando *et al.* | 2013 | No | Constant | No | No | Nonlinear |
| Present paper | ----- | Yes | Price dependent uniform | Yes | Yes | Linear |

The holding cost which is a power function of amount of time in stock was considered. Dari and Sani (2020) formulated an economic product quantity model for delayed perishable items. The quadratic type demand rate and linear holding cost was assumed in the developed model. They investigated optimal set of production rates that minimizes the total inventory cost per unit time, the best cycle length and the economic production quantity. Kurade (2021) formulated an economic inventory model with linear holding cost. In which he developed new evolutionary optimization algorithm. The above exhaustive literature review hints that (see Table 1), there is a lack of research in the price dependent uniform demand with probabilistic deterioration. Also, there is a lack of research on what would happen if there were several deterioration rates across phases during the scheduling period, uniform selling price-dependent demand, solution methodology, and time-dependent holding costs. The present research highlights such issues that several authors have not handled in the past. Almost all authors assumed that price dependent demand over the scheduling period is deterministic i.e., fixed. Also, selling price dependent demand rate and time dependent linear holding cost is not assumed while developing the EOQ inventory models. Hence, in this paper, an EOQ inventory model under shortages is developed by assuming these realistic assumptions.

## 3. Mathematical Model

The objective of the study is to maximize the profit function $\pi(t_1,\ t_2)$ where $t_1$ and $t_2$ are the decision variables. Here, an EOQ inventory model by assuming single retailer for single tier supply chain is developed. The behaviour of inventory during the scheduling period is displayed in Figure 1. From figure 1, we see that at the beginning of the inventory cycle the inventory level reaches to $S$. As time passes it decreases due to deterioration and satisfying the market demand. During inventory cycle inventory decreases with rate of deterioration is constant up to time $t_1$ and after $t_1$ the time dependent deterioration is considered up to time $t_2$. In $[0,\ \ t_1]$ deterioration rate follows an exponential distribution and in $[t_1,\ t_2]$ it follows a Weibull distribution. During the time period $[t_2,\ T]$, the market demand of the perishable product is completely backlogged i.e., no lost sale in the scheduling period.

### 3.1 Assumptions:

To construct a probabilistic EOQ inventory model for the above defined situation, following assumptions during the scheduling period [0, T] are considered to develop the model:

i) Price dependent uniform demand rate i.e., demand rate$= P * f(t)$ where $f(t) \sim U(a, b)$. Thus, expected demand rate is, $D(P) = E[demand\ rate] = P * \left(\frac{a+b}{2}\right)$.
ii) Probabilistic deterioration – a deterioration of the perishable item follow an exponential distribution during [0, $t_1$] and a Weibull distribution during $[t_1,\ t_2]$.
iii) Linear time dependent holding cost i.e., $H(t) = \alpha + \beta t$.
iv) Replenishment rate is infinite but its size is finite and negligible lead time.

v) Shortages are allowed and they are completely backlogged.
vi) A single perishable item is considered.

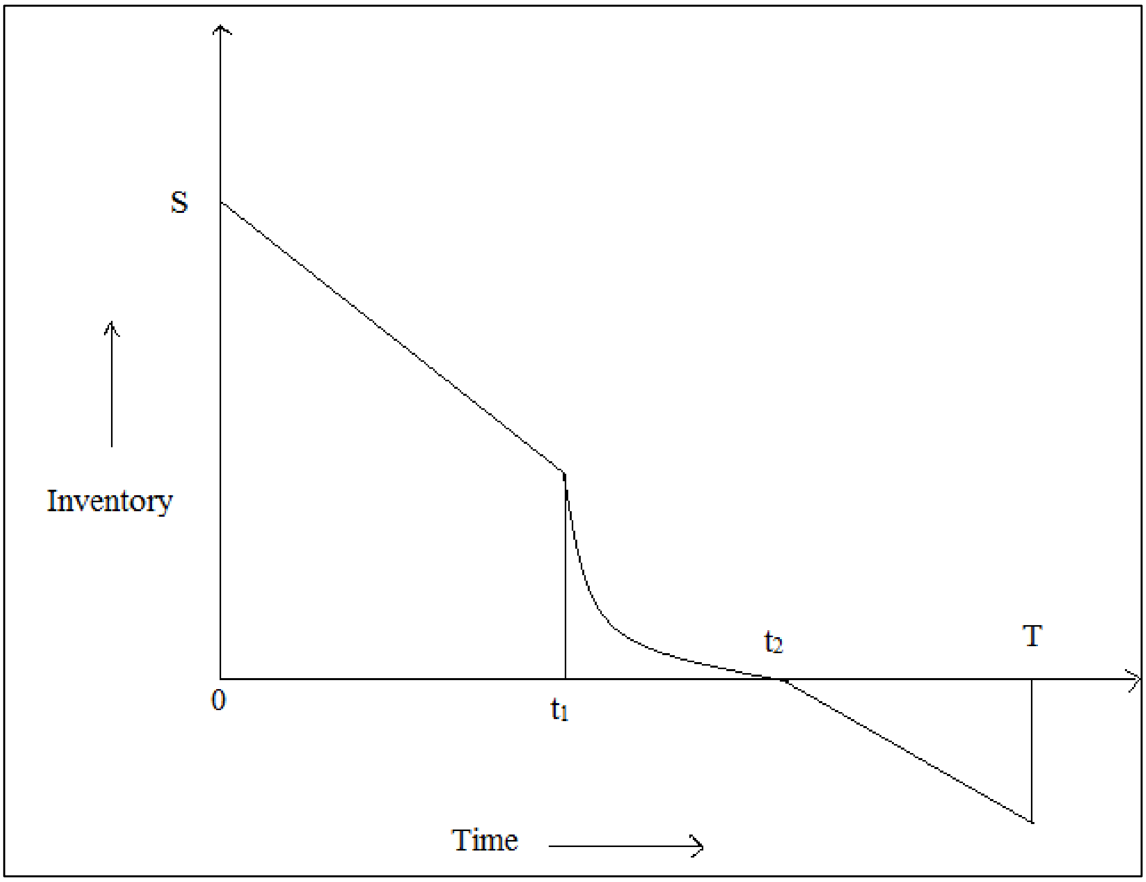


**Fig. 1:** Behaviour of inventory in complete backlogging considering two probabilistic deterioration rates.

**3.2 Notations:**

**3.2.1 Decision Variables:**

- $t_1$ : time point up to which perishable item follow an exponential deterioration.
- $t_2$ : time point after which shortages will start.

**3.2.2 Parameters:**

- $I(t)$ : level of inventory at time t.
- T : fixed scheduling period or inventory cycle.
- $\theta_1(t)$ : deterioration rate up to $t_1$.
- $\theta_2(t)$ : deterioration rate after time $t_1$ and up to $t_2$.
- $H(t)$ : time dependent holding cost.
- P : per unit selling cost ($).
- C : per unit procurement cost ($).
- $C_2$ : per unit shortage cost in [$t_2$, T] ($).
- $C_3$ : per unit decay cost ($).
- $D(P)$ : demand rate dependent on per unit selling cost.
- $C_{ord}$ : order cost in [0, T] ($).

- $\theta$ : parameter of an exponential distribution with p.d.f.
  $$f(x) = \theta e^{-\theta x};\ x > 0, \theta > 0.$$
  The deterioration rate of the inventory which follows an exponential distribution is, $\theta_1(t) = \theta$
- $\gamma, \xi$ : parameters of the Weibull distribution with p.d.f.,
  $$f(x) = \gamma \xi x^{\xi - 1}\, e^{-\gamma x^{\xi}};\ x > 0, \gamma, \xi > 0.$$
  The deterioration rate of the inventory which follows a Weibull distribution is, $\theta_2(t) = \gamma \xi t^{\xi - 1}$ $(\gamma > 0, \xi > 0)$
- $\alpha, \beta$ : holding cost parameters, $\alpha,\ \beta > 0$.
- $a, b$ : demand rate parameters $a, b > 0$.
- $\pi(t_1,\ t_2)$: the profit per unit time during the scheduling period [0, T].
- $Q$ : economic order quantity (EOQ).
- $S$ : maximum level of inventory at time t=0.

The behaviour of inventory level is represented by the following linear differential equations:

$$\frac{dI(t)}{dt} + \theta_1(t)\, I(t) = -D(P);\ 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} + \theta_2(t)\, I(t) = -D(P);\ t_1 \le t \le t_2 \tag{2}$$

$$\frac{dI(t)}{dt} = -D(P);\ t_2 \le t \le T \tag{3}$$

Boundary conditions, $I(0) = S$; and $I(t_2) = 0$

Thus, at the above boundary conditions the solutions of equations (1), (2) and (3) are,

$$I(t) = Se^{-\theta t} - D(P)e^{-\theta t} \int_0^t e^{\theta x} dx \quad ; \quad 0 \le t \le t_1 \tag{4}$$

$$I(t) = D(P)\, e^{-\gamma t^{\xi}} \int_t^{t_2} e^{\gamma x^{\xi}} dx\ ;\ t_1 \le t \le t_2 \tag{5}$$

$$I(t) = -D(P) \int_{t_2}^{t} dt\, ; \quad t_2 \le t \le T \tag{6}$$

In equation (5) put $t = t_2$ and using boundary conditions,

$$S = \int_0^{t_1} D(P)\, e^{\theta x} dx + e^{-\gamma t_1^{\xi} + \theta t_1} \int_{t_1}^{t_2} D(P)\ e^{\gamma x^{\xi}} dx$$

$$= P\left(\frac{a+b}{2}\right)\left[\frac{1}{\theta}\left(e^{\theta t_1} - 1\right) + \frac{e^{-\gamma t_1^{\xi} + \theta t_1}}{\gamma \xi}\left(\frac{e^{\gamma t_2^{\xi}}}{t_2^{\xi - 1}} - \frac{e^{\gamma t_1^{\xi}}}{t_1^{\xi - 1}}\right)\right] \tag{7}$$

Thus, the optimum order quantity Q is,

$$Q = S + \int_{t_2}^{T} D(P)\, dx$$

$$= P\left(\frac{a+b}{2}\right)\left[\frac{1}{\theta}\left(e^{\theta t_1} - 1\right) + \frac{e^{-\gamma t_1^{\xi} + \theta t_1}}{\gamma \xi}\left(\frac{e^{\gamma t_2^{\xi}}}{t_2^{\xi - 1}} - \frac{e^{\gamma t_1^{\xi}}}{t_1^{\xi - 1}}\right) + (T - t_2)\right] \tag{8}$$

Therefore profit per unit time is,

$$\pi(t_1,t_2)=\frac{1}{T}\begin{pmatrix}Sales\ Revenue-Deterioration\ cost-Holding\ cost\\ -Shortage\ cost-Order\ cost\end{pmatrix} \tag{9}$$

, where $(Sales\ Revenue)\ =\ P*\left[\int_0^{t_1}D(P)dx+\int_{t_1}^{t_2}D(P)dx+\int_{t_2}^{T}D(P)dx\right]$

$$=P^2T\left[\frac{a+b}{2}\right] \tag{10}$$

$D_T$ =Total number of deteriorated items during [0, $t_2$]

$$=S-\int_0^{t_2}D(P)dx$$

$$=P\left(\frac{a+b}{2}\right)\left[\frac{1}{\theta}\left(e^{\theta t_1}-1\right)+\frac{e^{-\gamma t_1^{\xi}+\theta t_1}}{\gamma\xi}\left(\frac{e^{\gamma t_2^{\xi}}}{t_2^{\xi-1}}-\frac{e^{\gamma t_1^{\xi}}}{t_1^{\xi-1}}\right)-t_2\right]$$

$(Deterioration\ cost)\ =\ C_3*D_T$

$$=C_3P\left(\frac{a+b}{2}\right)\left[\frac{1}{\theta}\left(e^{\theta t_1}-1\right)+\frac{e^{-\gamma t_1^{\xi}+\theta t_1}}{\gamma\xi}\left(\frac{e^{\gamma t_2^{\xi}}}{t_2^{\xi-1}}-\frac{e^{\gamma t_1^{\xi}}}{t_1^{\xi-1}}\right)-t_2\right] \tag{11}$$

$(Holding\ cost)\ =\ C\left[\int_0^{t_1}(\alpha+\beta t)I(t)dt+\int_{t_1}^{t_2}(\alpha+\beta t)I(t)\,dt\right]$

$$=C\begin{bmatrix}\int_0^{t_1}(\alpha+\beta t)\left[Se^{-\theta t}-D(P)e^{-\theta t}\int_0^{t}e^{\theta x}dx\right]dt\\ +\int_{t_1}^{t_2}(\alpha+\beta t)\left[D(P)\,e^{-\gamma t^{\xi}}\int_t^{t_2}e^{\gamma x^{\xi}}dx\right]\,dt\end{bmatrix} \tag{12}$$

$(Shortage\ cost)\ =\ -C_2\int_{t_2}^{T}I(t)dt$

$$=-C_2\int_{t_2}^{T}\left[-\int_{t_2}^{t}D(P)\,dx\right]dt$$

$$=C_2P\left(\frac{a+b}{2}\right)\left[\frac{1}{2}(T^2-t_2^2)-t_2(T-t_2)\right] \tag{13}$$

By substituting equation (10), (11), (12) and (13) in (9), we get the profit per unit time is,

$$\pi(t_1,t_2)=\frac{1}{T}\begin{pmatrix}P^2T\left[\frac{a+b}{2}\right]-C_3P\left(\frac{a+b}{2}\right)\left[\frac{1}{\theta}\left(e^{\theta t_1}-1\right)+\frac{e^{-\gamma t_1^{\xi}+\theta t_1}}{\gamma\xi}\left(\frac{e^{\gamma t_2^{\xi}}}{t_2^{\xi-1}}-\frac{e^{\gamma t_1^{\xi}}}{t_1^{\xi-1}}\right)-t_2\right]\\ -C\begin{bmatrix}\int_0^{t_1}(\alpha+\beta t)\left[Se^{-\theta t}-D(P)e^{-\theta t}\int_0^{t}e^{\theta x}dx\right]dt\\ +\int_{t_1}^{t_2}(\alpha+\beta t)\left[D(P)\,e^{-\gamma t^{\xi}}\int_t^{t_2}e^{\gamma x^{\xi}}dx\right]\,dt\end{bmatrix}\\ -C_2P\left(\frac{a+b}{2}\right)\left[\frac{1}{2}(T^2-t_2^2)-t_2(T-t_2)\right]-C_{Ord}\end{pmatrix} \tag{14}$$

## 4. Genetic Algorithm

For solving the proposed EOQ inventory model having two probabilistic deterioration rates, price dependent demand rate with time dependent holding cost, a genetic algorithm (GA) a class of stochastic search algorithm is utilized. It's a biologically inspired stochastic search procedure and optimization technique works using three genetic operators such as: reproduction/selection, crossover and mutation. It has two variants viz., real coded genetic algorithm (RCGA) and binary coded genetic algorithm (BCGA).

The RCGA works more efficiently with real numbers and when the dimension of the problem is sufficiently large. However, when the dimension (number of decision variables) is too small, for example, when the number of decision variables of the optimization problem is 1 or 2, RCGA is inefficient. It stuck in local minima or maxima. For smaller set of decision variables BCGA works more efficiently than RCGA (Deb (2001) and Latpate and Kurade (2020)). Hence, in the present study optimal solution of the formulated profit expression in (14) is obtained using BCGA, since the formulated EOQ inventory model has only two decision variables.

For applicability of GA, first we have to consider its various parameters. Then by repeated applications of the genetic operator's viz., selection, crossover and mutation, GA provides the optimum solution. It stops when the maximum iteration reaches or there is no any significant improvement in the objective function of the optimization problem. The reproduction operator generates duplicate copies of the best strings in the current iteration. Crossover and mutation operator performs probabilistically. Crossover operator generates two parent strings using two randomly selected offspring by exchanging the information between them.

**Table 2**: Parameters of BCGA.

| Sr. No. | Genetic Operator | Value |
|---|---|---|
| 1 | Reproduction | Tournament Selection |
| 2 | Crossover | One point crossover |
| 3 | Mutation | Bit wise mutation |
| 4 | Crossover rate ($P_c$) | 0.8 |
| 5 | Mutation rate ($P_M$) | 0.2 |
| 6 | Population size | 200 |
| 7 | String length | 100 |
| 8 | Number of iterations | 100 |

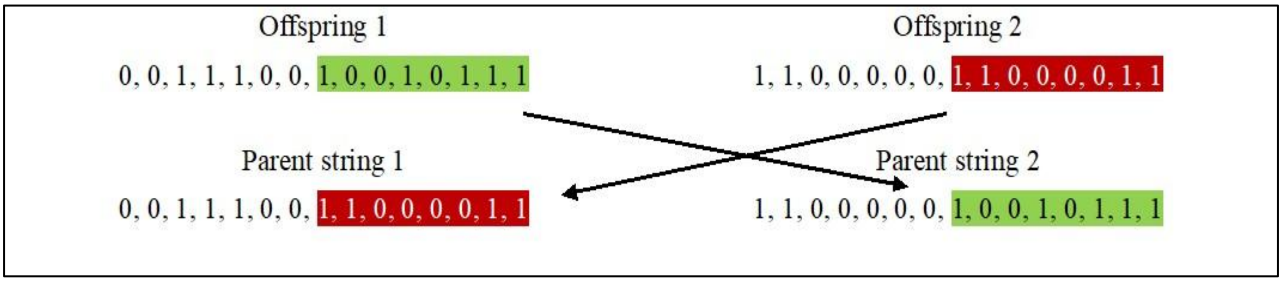


**Fig. 2:** One point crossover with randomly selected cut point is 7**.**

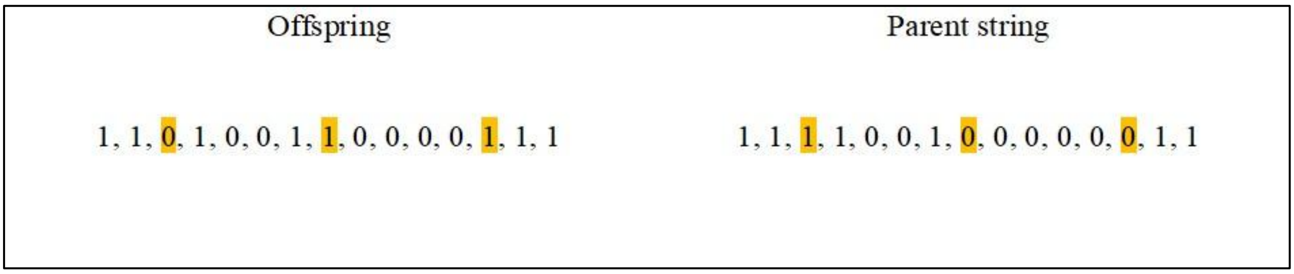


**Fig. 3:** Bit wise mutation with randomly selected genes are 3, 8, and 13.

**Algorithm 1- Flowchart for finding the optimum profit of the system.**

1. *Decide the parameters of the genetic algorithm, viz., string length, reproduction operator, crossover operator, mutation operator, crossover rate, mutation rate, size of the population and termination criteria.*
2. *During the scheduling period [0, T] decide the values of parameters (see section 3.2.2).*
3. *Based on these values, compute sales revenue using Eq. 10, deterioration cost using Eq. 11, holding cost using Eq. 12, and shortage cost using Eq. 13.*
4. *By considering all the above expressions, find the inventory system's total profit per unit time using Eq. 14 and genetic algorithm.*
5. *Stop the process, if suitable termination criteria's is met otherwise repeat above steps from 1 to 4.*

For performing the crossover operation, a random number is chosen from continuous uniform distribution oven 1 to N where N is the length of string. Then, after this point all genes were exchanged between the two selected offspring's. For crossover, only $(P_c * M)$ strings in the population selected for crossover operation where $P_c$, crossover probability and M, population size. The detail procedure of ope point crossover is shown in Figure 2. Mutation operator creates diversity in the population. For mutation, randomly $(P_M * M * N)$ genes are selected for creating the diversity in the population where $P_M$, the mutation probability. Its working procedure is displayed in Figure 3. Table 2 represents the various parameters of BCGA used in the present study.

Thus, by considering above parameters of binary coded genetic algorithm (see Table 2 and fig. 2 and 3), profit of the EOQ inventory model is obtained. Since the formulated inventory model (see Eq. (14)) is highly nonlinear and not twice differentiable. Hence, obtaining the model's optimal solutions using an analytical approach such as Newton-Raphson is likely unfeasible. Hence, an optimization technique based on evolutionary approach is proposed here. The step-by-step procedure of the formulated BCGA is presented in Algorithm 1. In this flowchart, various steps of the proposed BCGA are presented. Starting from the deciding the values of parameters during the scheduling period, finding various functions and lastly finding the profit of the system. The flowchart stops when algorithm reaches the maximum iterations.

## 5. Numerical Example

**Example 1**: The EOQ inventory model is solved by considering following parameter values in appropriate units: $D(P) = P\left(\frac{10+20}{2}\right)$, a=10, b=20, T=1 year,$\theta$=0.5, $\gamma$=0.0001, $\xi$=1, $\alpha$=10, $\beta$=0, P=$15, C=$5, $C_{ord}$=$250, $C_2$=$7, $C_3$=$3. Here, the perishable item follows only exponential deterioration with constant holding cost during the period [0, $t_2$]. Thus, the present model gives the optimal result as, profit=$2635.37, EOQ=280, $t_2$=0.9531, $t_1$=0.7002, S=269.41, deterioration cost=$13.66, shortage cost=$223.96, holding cost=$252.016.

**Example 2**: The EOQ inventory model is solved by considering following parameter values in appropriate units: $D(P) = P\left(\frac{10+20}{2}\right)$, a=10, b=20, T=1 year,$\theta$=0.001, $\gamma$=0.1, $\xi$=1, $\alpha$=10, $\beta$=0, P=$15, C=\$5, $C_{ord}$ =\$250, $C_2$ =\$7, $C_3$ =\$3. Here, the perishable item follows only Weibull distributed deterioration with constant holding cost during the period [0, $t_2$]. Thus, the present model gives the optimal result as, profit=\$2643.85, EOQ=230, $t_2$=0.8277,$t_1$= 0.6685, S=187, deterioration cost=\$2.58, shortage cost=\$311.65, holding cost=\$166.91.

**Example 3**: The EOQ inventory model is solved by considering following parameter values in appropriate units: $D(P) = P\left(\frac{10+20}{2}\right)$, a=10, b=20,T=1 year,$\theta$=0.5, $\gamma$=0.1, $\xi$=1, $\alpha$=10, $\beta$=1.5, P=\$15, C=\$5, $C_{ord}$=\$250, $C_2$=\$7, $C_3$=\$3. Here, the perishable item follows exponential deterioration up to $t_1$ and Weibull deterioration after $t_1$ and up to $t_2$ during the inventory cycle.Also, holding cost is linearly dependent on time during the period [0, $t_2$]. Thus, the present model gives the optimal result as, profit=\$2642.64, EOQ=271, $t_2$ =0.9177, $t_1$ = 0.5849, S=248, deterioration cost=\$9.97, shortage cost=\$303.97, holding cost=\$168.42.

In example 1 and 2, holding cost is considered a constant; in example 3, holding cost is linearly dependent on time. From examples 1 and 2, we observed that we get the maximum profit for Weibull distributed deterioration than exponential deterioration when holding cost is constant. We get less EOQ, less deterioration cost, less holding cost, but high shortage cost in Weibull deterioration as compared with exponential deterioration. Also, in exponential deterioration, the inventory cycle duration is larger than the Weibull deterioration. The codes of the formulated binary coded genetic algorithm are written by R software and runs on 12th Gen Intel(R) Core(TM) i3-1215U @ 1.20 GHz and 8GB RAM.

## 6. Sensitivity Analysis

The impact of various parameters of probabilistic inventory model is analyzed through sensitivity analysis. Results are listed in following tables. In sensitivity analysis the impact of one parameter is checked on optimum profit by fixing other parameters.

**Table 3:** Impact of parameters on optimum profit.

(A)

| $\theta$ | Profit |
|---|---|
| 0.2 | 1143.49 |
| 0.6 | 1128.58 |
| 2.0 | 1064.81 |
| 2.5 | 1061.53 |
| 5.0 | 1043.04 |

(B)

| $\gamma$ | $\xi$ | Profit |
|---|---|---|
| 2 | 0.5 | 1082.56 |
| | 1.2 | 1147.85 |
| | 1.8 | 2291.69 |
| 3 | 0.5 | 1085.26 |
| | 1.2 | 1099.13 |
| | 1.8 | 1859.63 |
| 4 | 0.5 | 1081.81 |
| | 1.2 | 1094.81 |
| | 1.8 | 1598.69 |

(C)

| $\alpha$ | $\beta$ | Profit |
|---|---|---|
| 10 | 1.5 | 916.02 |
| | 10 | 905.42 |
| | 50 | 873.36 |
| 20 | 1.5 | 842.13 |
| | 10 | 841.83 |
| | 50 | 832.72 |
| 50 | 1.5 | 777.08 |
| | 10 | 776.92 |
| | 50 | 773.88 |

Table 3 shows the impact of various parameters on profit function. In this case, values of other parameters are fixed and those are for Table 3 (A) a=10, b=20, $\gamma$=0.1, $\xi$=2, $\alpha$=1, $\beta$=1.5, P=10, C=5, $C_{ord}$=250, $C_2$=7, $C_3$=3 for Table 3 (B) $\theta$=0.1 and other parameters of Table 3 (A) are same, for Table 3 (C) $\theta$=0.1, $\gamma$=0.5, $\xi$=2 and other parameters are same as in Table 3 (A) and Table 3 (B).

On the basis of the sensitivity analysis of different parameters, the following inferences are drawn (Table 3):

i) The smaller value of $\theta$ implies maximum profit.

ii) When the parameter $\theta$ increases, the total profit of the system is decreased.

iii) For small value of $\gamma$ and large value of $\xi$ implies maximum profit.

iv) The increasing value of $\gamma$ with fixed $\xi$ implies profit is decreased.

v) With fixed $\gamma$ and increasing $\xi$, optimum profit of the system increase.

vi) For small $\alpha$ and $\beta$ implies maximum profit.

vii) As $\alpha$ increases with fixed $\beta$, profit decreases.

viii) With fixed $\alpha$ and increasing $\beta$, optimum profit of the system decrease.

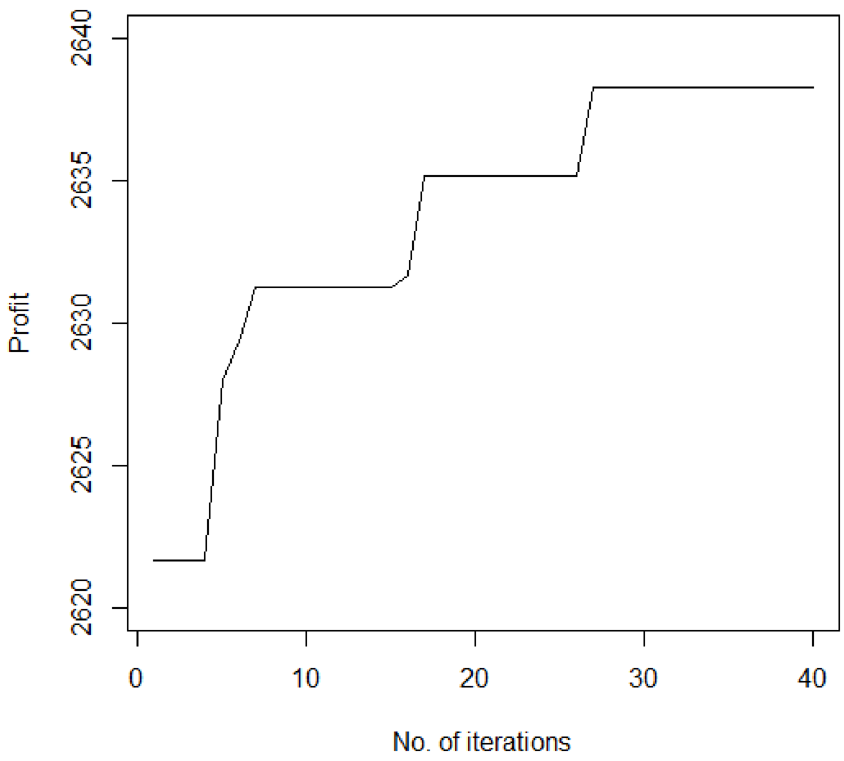


**Fig. 4:** Convergence plot for the profit function using BCGA.

The convergence plot of BCGA is obtained for the optimum profit and shown in Fig. 4. To reach the optimum solution; the algorithm takes smaller iterations. Hence, the computational complexity of the algorithm is much smaller. Since it reaches the optimum with smaller iterations.

## 7. Conclusion

In this paper, a probabilistic price dependent uniform demand rate EOQ inventory model is developed. Demand is uniformly dependent on the selling price of a perishable item in the formulated model. Time-dependent linear holding cost is considered when modeling the inventory system. Uniform demand and holding cost were considered as the continuously increasing selling price and time functions, respectively. From the results, we observed that, during the scheduling period [0, T] if a perishable item follows only a Weibull distribution, it gives the maximum profit. When an item follows an exponential distribution with time dependent holding cost, gave the maximum profit than fixed holding cost. The impact of various parameters of the model is checked using sensitivity analysis. In sensitivity analysis, the effect of one parameter is analyzed by fixing other parameters.

Sensitivity analysis shows that optimum profit is very sensitive to the parameters of the Weibull distribution. Such models have huge applications in the single-tier as well as multi-tier supply chains. According to the selling price of a perishable item, the supply chain manager fixes the selling price and then, using the methodology used in this paper, can obtain the supply chain's profit using a formulated solution methodology based on an evolutionary algorithm.

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