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The Study of Optimal Inventory Model for Deteriorating Item with Partial Backlogging and Inflation Under Trade Credit

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ABSTRACT

A very significant portion of the business's commercial effects, such as inventory, is greatly influenced by how the market reacts to various events, especially inflation. Today, inflation is a widespread problem. As a situation of disequilibrium where increased buying power either tends to cause or is a result of rising prices, inflation might be characterized as such. The economic, political, social, and moral fabric of society is disrupted during a protracted, persistent, and continuous era of inflation. An increase in the money supply and an increase in price levels are considered forms of inflation. The majority of the time, when we hear the term "inflation," we hear about a rise in prices relative to some standard. It is usually only a matter of time until increased price levels indicate that the money supply has been expanded. Because of inflation, companies must spend a lot of time and effort valuing their goods. The effects of inflation were not taken into account in the majority of earlier studies on inventory models. This was because inflation was thought not to impact the inventory policy significantly. We develop a mathematical model for the item inventory that degrades but does not do so immediately after being stocked. Additionally, the model considers that customers are now permitted a certain amount of time inside commercial activities to express their satisfaction with the delivered items. Within the trade credit period, they might use the money they ha've earned from trades of the goods they have supplied to gain interest. They only incur interest charges when they do not pay the supplier what they owe by the deadline. This idea is unfounded because an enterprise's resources and return on investment have a strong correlation. When making long-term investments and forecasts, it is especially important to consider inflation. We find maximizing profit and minimizing the loss of deteriorating products with partial backlogging and inflation under trade credit.

1. Introduction

The research on time-dependent demand rates has gained popularity in recent years. It has been noted that the pace of demand for recently released goods like electronics, mobile phones, and trendy clothing grows over time and becomes constant. In corporate situations, it is impossible to prevent product deterioration. Most of the time, as time goes on, demand for commodities rises, and items that are kept for later use always lose some of their worth. This tendency is referred to as item deterioration in inventory. Some goods, including steel, glass, toys, and hardware, degrade relatively slowly. Many inventory systems cannot disregard the influence of deterioration of physical products since items including medication, vegetables, liquor, harmful compounds, and food grains degrade quickly finished time. The control of decaying things becomes a major concern in every inventory

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system because it is a realistic miracle in many inventory systems. Any inventory system that experiences deterioration experiences the issue of shortages, which is defined as a fraction of inventory that is not accessible to meet consumer demand in a specific time frame. Generally speaking. Ghare and Sharder initially looked into the rotting inventory issue. (1963), who created an EOQ model with a steady degradation rate. Covert and Philip (1973) extended this work by creating an EOQ model for a variable degradation rate. In real-world positions, however, inventory cases deteriorate due to partial backlogging or other factors. Shah (1977) generalized Ghare and Shrader's work. Manjusri Basu and Sudipta Sinha extended the Yan & Cheng model for time-dependent backlogging rate in 2007. In a supply chain management system, "Rau *et al.*" (2004) took into account an inventory model to decide on an economical ordering strategy for degrading commodities. Teng and Chang (2005) used an inventory model to establish an economic production amount used for deteriorating items. Regarding deteriorating items with a time-proportionate demand and no scarcity, Dave and Patel (1983) created an inventory model in conjunction by an instantaneous replenishment policy. A finite rate of replenishment order level inventory model that allows for lacks was developed by Roy and Chaudhary (1983).

2. Literature Review

In their models, An inventory model for degrading products under preservation technologies with time-dependent and quality-dependent demand was created by S. Sindhuja and P. Arathi (2023).

He assumed the rate of deterioration of individual commodities is based on deteriorating items. Snigdha Benerji and Swati Agrawal (2017) created an inventory model for deteriorating items with freshness and price dependence and assumed that a major part of the retail industry deals with items resulting in lower demand at the same price. Narges Khanlarzade and Babak Yousefi (2022) presented A Strategic inventory of deteriorating products with demand and disruption to obtain the retailer's strategic inventory management that can convince the supplier to present lower wholesale prices. Mansuri Basu and Shinha (2007) extended the Yan Cheng model for time-dependent backlogging rate. Rau *et al.* (2004) considered an inventory model for determining an economic ordering policy for deteriorating items in a supply chain management system. Teng and Chang (2005) used an inventory model for degrading items to establish an economic production amount. Dev and Chaudhuri (1986) and Mishra (1975) relied on the theory that the pace of deterioration varied with time. Raafat (l991) provided a widespread overview of this. The challenges of suitable experimental records to measure distributions were covered by Berrotoni (1962). A manufacturing inventory ideal for degrading goods with demand based on stock and a consistent rate of construction was created by Mandal and Phaujdar(1989).

Padmanabhan and Vrat (1995) have also contributed to the field in this way. By assuming a constant rate of inflation, "Chandra and Bahner (1988)", Jesse *et al.* (1983), and Mishra (1979) constructed their representations and illustrate the impact of price increases on list models. "For an inventory model of decaying items under inflation", Liao *et al.* (2000) analyze the impact of an allowable payment delay. A model for EOQ that accounts for price-dependent inflation was created by Bhahmbhatt (1982). An EOQ model with scarcities below the influence of price rises and time concession was considered by Ray and Chaudhuri (1997). Goal (1985) created an EOQ model under the circumstances of a legal payment delay; an ideal replenishment strategy for the EOQ model was taken into consideration by Chung *et al.* (2002) and Hung (2003) under the allowable payment delay. A time-dependent demand and partial backlog inventory model for degrading goods was created by

V. Kumar Mishra and L. S. Singh (2010). To supplement the conventional EOQ model, Vinod Kumar Mishra (2013) made an inventory model with an adjustable decline rate. An inventory model for randomly degrading objects with time-dependent requests and fractional backlog was established by Mandal (2013). There is a complete backlog of unmet demand, and through a deficiency, moreover, all consumers gap for the succeeding demand to arrive (which would result in a complete backlog), or wholly trades abandon the scheme (completely lost). The primary determinant of whether or not a backlog is accepted is the duration of the replenishment wait time.

Table 1: Contribution table.

Author's	**deteriorating**	**Partial backlogging**	**Inflation**	**Trade credits**
Lio *et al.* (2000)	Considered	Not considered	Considered	Considered
Chung *et al.* (2002)	Not considered	Not considered	Not considered	Considered
Hung *et al.* (2003)	Not considered	Not considered	Not considered	considered
Rau *et al.* (2004)	considered	Not considered	Not considered	Not considered
Teng & Chang (2005)	considered	Not considered	Not considered	Not considered
Mansuri basu (2007)	considered	Not considered	Not considered	Not considered
Vinod *et al.* (2010)	considered	Considered	Not considered	Not considered
Mandal *et al.* (2013)	considered	Considered	Not considered	Not considered
Snigdha Benerji and Swati Agrawal (2017)	considered	Not considered	Not considered	Not considered
Narges Khanlarzade and Babak yousefi (2022)	considered	Not considered	Not considered	Not considered
S. Sindhuja and P. Arathi (2023)	considered	Not considered	Not considered	Not considered

The following presumptions form the foundation of the mathematical model described in this paper:

3. Assumption

i) There is believed to be only one vendor and a single buyer per item.
ii) A single item with the current inventory's constant depreciation rate is considered.
iii) The holding expense varies as an escalating stage task of storage time.
iv) Shortages are acceptable with some backlog storage.
v) A stationary policy that makes use of the same lot size assumption.
vi) Demand varies over time.
vii) Inflation is measured.
viii) The amount of allowable payment delay is measured.

4. Notation

The vendor-related parameters are;

$I_{v1}(t)$: Retailer inventory level while t falls among 0 and T_1

$I_{v2}(t)$: Retailer inventory level while t falls among 0 and T_2

I_{mv} : Extreme stock on behalf of the vendor

P_v : The vendor's portion production cost

F_{vj} : Keeping the vendor's fee of the article in period j.

$F_v(t)$: retaining the item's cost at time t, $F_v(t) = F_{vj}$ if $t_{j=1} < t < t_j$

C_{SV} : The vendor's setup costs for each cycle of manufacturing.

VC : Overall vendor price for each item

The buyer-related parameters are:

$I_b(t)$: The buyer's catalog level.

l_{mb} : Maximum stock for buyer

Pb : The cost to the customer of each unit.

F_{bj} : Holding the rate of the article for the customer in period j.

$F_b(t)$: Holding price of the article at time t, $F_b(t) = F_{bj}$ if $F_b(t) = F_{bj}$

C_{sb} : Setup fee for each order for the buyer.

C_2 : For backlogged things, there is an item shortage cost.

C_3 : Cost per unit of lost sales.

B(t) : Indicate the portion, t being the amount of time until the next replenishment.

We take, $B(t) = 1/(1+\delta t)$ where a positive constant is the backlogging parameter.

B_c : total cost to the buyer per item

These other pertinent variables are listed below:

a+ct : Demand rate in which the constants a and c are positive

$T = T_1 + T_2$

and I(t) is the inventory level at time t. T Time span of every cycle, where

θ	:	Decline rate
T_1	:	The amount of time spent producing during each cycle of production:
T_2	:	The period of time spent not producing
P	:	The annual production rate
n	:	There are numerous diverse timeframes with various holding cost rates.
r	:	The rate of inflation
TC	:	The combined price of the seller and every consumer, per item
M	:	Amount of time that accounts settings may be delayed.
$t_{1'}$	:	The point in time when a scarcity occurs.

5. Mathematical Model and Description

5.1. System for Vendor Inventory

In this approach, manufacturing for the vendor begins at time zero at an unbroken amount P at the beginning of every cycle.

i.e., at t=0. Inventory levels rise with production, drop with demand, and eventually reach their highest point of value owing to decline. up to time T_1 and touches at extreme value I_{mv} as shown in Fig. l.

$$I_{V1}'(t)+\theta I_{V1}(t)=P-[a+ct],\ 0\le t\le T_1 \tag{1}$$

Demand and deterioration cause the inventory level to decline after time T_1 until time T_2, when it hits zero.

$$I_{V2}'(t)+\theta I_{V2}(t)=-[a+ct],\ 0\le t\le T_2 \tag{2}$$

with boundaries

$$I_{v1}(0)=0, I_{v2}(T_2)=0$$

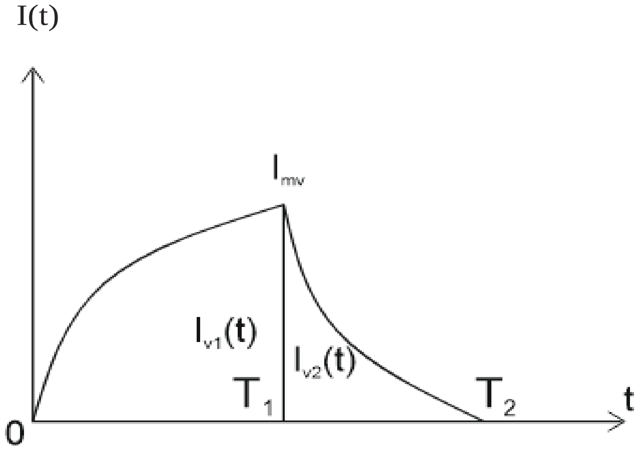

Fig 1: Vendr inventory control system.

The aforementioned differential equations' solutions are now

$$I_{V1}(t)=\left(1-e^{-(\theta)t}\right)\left(\frac{P-a}{\theta}+\frac{c}{(\theta)^2}\right)-\frac{ct}{\theta}\quad 0\le t\le T_1 \tag{3}$$

$$I_{V2}(t)=\left[\frac{a}{\theta}-\frac{c}{(\theta)^2}\right]\left(e^{(\theta)(T_2-t)}-1\right)+\frac{c}{(\theta)}\left(T_2e^{(\theta)(T_2-t)}-t\right)\quad 0\le t\le T_2 \tag{4}$$

5.2. System for Customer's Inventory:

Customer rotation starts with the extreme inventory I_{mb} at t = 0 and this inventory slowly reduces to zero at time t_1' due to the call and decline effects coexisting simultaneously, as seen in Fig. 1.2.

$$I_{b1}'(t)+\theta I_{b1}(t)=-[a+ct]\quad 0\le t\le t_1' \tag{5}$$

After time t_1' the following differential equation controls the change in the inventory when there is partial backlogging.

$$I_{b2}'(t)=-\frac{[a+ct]}{1\delta\left(\frac{T}{n}-1\right)}\quad t_1'<t\le T/n \tag{6}$$

Boundary condition is $I_{b1}(t_1')=0=I_{b2}(t_2')$

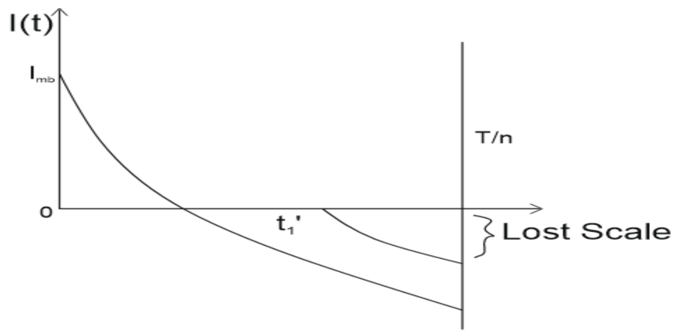

Fig 2: Inventories for buyers when shortages are permitted.

The aforementioned differential equations' solutions are now

$$I_{b1}(t)=\left(\frac{a}{\theta}-\frac{c}{(\theta)^2}\right)\left(e^{(\theta)(t_1'-t)}-1\right)+\frac{c}{(\theta)}\left(t_1'e^{(\theta)(t_1'-t)}-t\right)\qquad 0\le t\le t_1' \tag{7}$$

$$I_{b2}(t) = -\frac{1}{\delta}\left[a + \frac{c}{\delta}\left(1+\delta\frac{T}{n}\right)\right]\log\left\{\frac{\left(1+\delta\left(\frac{T}{n}-t_1\right)\right)}{\left(1+\delta\left(\frac{T}{n}-t\right)\right)}\right\} + \frac{c}{\delta}(t-t_1) \quad t_1^{'} \le t \le T/n \tag{8}$$

By using the boundary condition $I_{mv} = I_{v2}(0)$ and $I_{mb} = I_{b1}(0)$, we have

$$I_{mv} = T_2(a + cT_2) + \frac{T_2^2}{2}\left[a(\theta) - c + cT_2(\theta)\right] \tag{9}$$

$$I_{mb} = t_1^{'}\left(a + ct_1^{'}\right) + \frac{t_1^{'2}}{2}\left[a(\theta) - c + ct_1^{'}(\theta)\right] \tag{10}$$

with boundaries $I_{v1}(T_1) = I_{v2}(0)$, we can solve the following equation:

$$(p-a)\left(T_1 - \frac{(\theta)T_1^2}{2}\right) - \frac{cT_1^2}{2} = T_2(a + cT_2) + \frac{T_2^2}{2}\left[a(\theta) - c + cT_2(\theta)\right]$$

With the help of Taylor's series expansion

$$T_1 = \frac{(a + cT_2)T_2}{(p-a)} + \frac{T_2^2}{2(p-a)}\left[(\theta)(a + cT_2) - c\right] \tag{11}$$

Knowing $T = T_1 + T_2$

We can solve

$$T = T_2\left[1 + \frac{(a + cT_2)}{(p-a)} + \frac{T_2}{2(p-a)}\left((\theta)(a + cT_2) - c\right)\right] \tag{12}$$

Case I: Backdated holding cost increase

For every customer and vendor, the annual holding costs are

$$HC_b = \frac{F_{bj}n}{T}\int_0^{t_1^{'}} I_{b1}(t)e^{-n}dt$$

$$= \frac{F_{bj}n}{T}\left[\frac{\left(a + ct_1^{'}\right)t_1^{'2}}{2} - \frac{t_1^{'2}}{2r}\left\{(\theta)\left(a + ct_1^{'}\right) - c\right\}\right] \tag{13}$$

and

$$HC_v = \frac{F_{vj}}{T}\left[\int_0^{T_1} I_{v1}(t)e^{-rt}dt + e^{-rT_1}\int_0^{T_2} I_{v2}(t)e^{-rt}dt - n\int_0^{t_1'} I_{b1}(t)e^{-rt}dt\right]$$

$$= \frac{F_{vj}}{T}\left[\begin{array}{l}(p-a)\left\{\frac{T_1^2}{2} - \frac{rT_1^3}{2} + \frac{(\theta)rT_3^4}{4}\right\} + \frac{crT_1^4}{4} + e^{-rT_2}\left\{\frac{(a+cT_2)T_2^2}{2}\right\} \\ -n\left\{(a+ct_1')\frac{t_1'^2}{2} - \left(\frac{(\theta)(a+ct_1')-c}{2}\right)\frac{t_1'^2}{r}\right\}\end{array}\right] \tag{14}$$

Case II: Increases in holding gradual costs

$$HC_b = \frac{n}{T}\left[F_{b1}\int_0^{t_1} I_{b1}(t)e^{-rt}dt + F_{b2}\int_{t_1}^{t_2} I_{b1}(t)e^{-rt}dt + ... + F_{bc}\int_{t_{c-1}}^{t_c=t_1'} I_{b1}(t)e^{-rt}dt\right]$$

$$= \frac{n}{T}\sum_{j=1}^{c} F_{bj}\int_{t_{j-1}}^{t_j} I_{b1}(t)e^{-rt}dt$$

$$= \frac{n}{T}\sum_{j=1}^{c} F_{bj}\left[(a+ct_1')\left\{-(t_1'-t_j)\frac{e^{-rt_j}}{r} + \frac{e^{-rt_j}}{r^2} + (t_1'-t_{j-1})\frac{e^{-rt_{j-1}}}{r} - \frac{e^{-rt_{j-1}}}{r^2}\right\}\right.$$

$$\left. + \left\{\frac{a(\theta)-c+ct_1'(\theta)}{2}\right\}\left\{-(t_1'-t_1)^2\frac{e^{-rt_1}}{r} + 2(t_1'-t_j) - 2\frac{e^{-rt_j}}{r^3} + (t_1'-t_{j-1})^2\frac{e^{-rt_{j-1}}}{r} - \frac{2(t_1'-t_{j-1})e^{-rt_{j-1}}}{r^2} + \frac{2e^{-rt_{j-1}}}{r^3}\right\}\right] \tag{15}$$

$$HC_v = \frac{1}{T}\left[F_{v1}\int_0^{T_1} I_{v1}(t)e^{-rt}dt + e^{-rT_1}F_{v2}\int_0^{T_2} I_{v2}(t)e^{-rt}dt - n\left\{F_{b1}\int_0^{t_1} I_{b1}(t)e^{-rt}dt + F_{b2}\int_{t_1}^{t_2} I_{b1}(t)e^{-rt}dt + ... + F_{bc}\int_{t_{c-1}}^{t_c=t_1'} I_{b1}(t)e^{-rt}dt\right\}\right]$$

$$HC_v = \frac{1}{T}\left[(p-a)F_{v1}\left\{\frac{T_1^2}{2} - \frac{rT_1^3}{2} + (\theta+b)\frac{rT_1^4}{4}\right\} + \frac{F_{v1}crT_1^4}{4} + e^{-rT_1}\frac{F_{v2}(a+cT_2)T_2^2}{2}\right]$$

$$-n\sum_{j=1}^{c} F_{bj}(a+ct_1')\left\{\left\{-(t_1'-t_1)\frac{e^{-rt_j}}{r} + \frac{e^{-rt_j}}{r^2} + (t_1'-t_{j-1})\frac{e^{-rt_{j-1}}}{r} - \frac{e^{-rt_{j-1}}}{r^2}\right\}\right.$$

$$+ \left\{\frac{a(\theta)-c+ct_1'(\theta)}{2}\right\}\left\{-(t_1'-t_1)^2\frac{e^{-rt_1}}{r} + 2(t_1'-t_1) - 2\frac{e^{-rt_1}}{r^3}\right.$$

$$\left. + (t_1'-t_{j-1})^2\frac{e^{-rt_{j-1}}}{r} - \frac{2(t_1'-t_{j-1})e^{-rt_{j-1}}}{r^2} + \frac{2e^{-rt_{j-1}}}{r^3}\right\} \tag{16}$$

When $t_1^{'} > M$

Interest due each cycle for each unit of time is

$$IP = \frac{nP_b I_p}{T}\int_M^{t_1^{'}} I_{b1}(t) e^{-rt} dt$$

$$= \frac{nP_b I_p}{T}\int_M^{t_1^{'}} \left[\left(t_1^{'} - t\right)\left(a + ct_1^{'}\right) + \frac{\left(t_1^{'} - t\right)^2}{2}\left\{a(\theta) - c + ct_1^{'}(\theta)\right\} \right] e^{-rt} dt$$

$$= \frac{nP_b I_p}{T}\left[\left(a + ct_1^{'}\right)\left(t_1^{'} - M\right) + \left(t_1^{'} - \frac{M}{2} + \frac{rM^2}{2}\right) \right]$$

$$+ \left\{ \frac{(\theta)\left(a + ct_1^{'}\right) - c}{2} \right\}\left(t_1^{'} - M\right)\left\{ -Mt_1^{'} + \frac{M^2}{2}\left(t_1^{'} - M\right) \right\} \tag{17}$$

Interest is earned once every unit of time,

$$IE = \frac{nP_b I_e}{T}\int_0^{t_1^{'}} e^{-rt}(a + ct) dt$$

$$= \frac{nP_b I_e}{T}\left[\left(a + ct_1^{'}\right)\left(t_1^{'} - \frac{rt_1^{'2}}{2}\right) \right] \tag{18}$$

When $t_1^{'} \le M$

Interest earned up to time t_1’ is

$$IE = P_b I_e \int_0^{t_1^{'}} e^{-rt}(a + ct) dt$$

$$= P_b I_e \left[\left(a + ct_1^{'}\right)\left(t_1^{'} - \frac{rt_1^{'2}}{2}\right) \right]$$

Interest received through $\left(M - t_1^{'}\right)$ is $P_b I_e \left[\left(a + ct_1^{'}\right)\left(t_1^{'} - \frac{rt_1^{'2}}{2}\right) \right] \int_{t_1^{'}}^{M} e^{-rt} dt$

$$= P_b I_e \left[\left(a + ct_1^{'}\right)\left(t_1^{'} - \frac{rt_1^{'2}}{2}\right) \right]\left(\frac{e^{-rt_1^{'}}}{r} - \frac{e^{-rM}}{r} \right)$$

The total interest received each unit period is

$$IE = \frac{nP_b I_e}{T}\left[\left(a+ct_1^{'}\right)\left(t_1^{'}-\frac{rt_1^{'2}}{2}\right)\right]\left(1+\frac{e^{-rt_1^{'}}}{r}-\frac{e^{-rM}}{r}\right) \tag{19}$$

Customer pays no interest in this instance. is

For every customer and seller, the annual depreciated costs are:

$$DC_b = \frac{nP_b}{T}\left[I_{Mb} - \int_0^{t_1^{'}} (a+ct)e^{-rt}dt\right]$$

$$= \frac{nP_b}{T} t_1^{'2}\left[\left(\frac{a(\theta)-c}{2}\right)+\frac{ar}{2}\right] \tag{20}$$

$$DC_v = \frac{P_v}{T}\left[PT_1 - nl_{Mb}\right]$$

$$= \frac{P_v}{T}\left[PT_1 - n\left\{t_1^{'}\left(a+ct_1^{'}\right)+\frac{t_1^{'2}}{2}\left((\theta)\left(a+ct_1^{'}\right)-c\right)\right\}\right] \tag{21}$$

The annual setup fees for every buyer and vendor are:

$$SC_b = \frac{nC_{sb}}{T} \tag{22}$$

$$SC_v = \frac{nC_{sv}}{T} \tag{23}$$

Cost of shortages to the buyer per cycle

$$AS_b = \frac{nC_2}{T}\int_{t_1^{'}}^{\frac{T}{n}} -I_{b2}(t)e^{-rt}dt$$

$$= -\frac{nC_2}{\delta T}\left[\left\{a+\frac{c}{\delta}\left(1-\delta\frac{T}{n}\right)\right\}\log\left\{1+\delta\left(\frac{T}{n}-t_1^{'}\right)\right\}\left\{\frac{T}{n}-t_1^{'}-\frac{1}{\delta}\left(1-\frac{r}{2\delta}\left(1+\delta\frac{T}{n}\right)\right)\right.\right.$$
$$\left.\left(1+\delta\frac{T}{n}\right)+\left(t_1^{'}-\frac{rt_1^{'2}}{2}\right)\right\}+\frac{c}{\delta r}\left(\frac{T}{n}-t_1^{'}\right)\left(1-e^{-r\frac{T}{n}}\right)-\left\{a+\frac{c}{\delta}\left(1+\delta\frac{T}{n}\right)\right\}$$
$$\left.\left(\frac{T}{n}-t_1^{'}\right)\left\{\frac{r}{4}\left(\frac{T}{n}+t_1^{'}\right)-\left(1-\frac{r}{2\delta}\left(1+\delta\frac{T}{n}\right)\right)\right\}\right] \tag{24}$$

Cycle opportunity cost resulting from lost sales

$$OC = \frac{nC_3}{T}\int_{t_1^{'}}^{\frac{T}{n}}\left[1-\frac{1}{1+\delta\left(\frac{T}{n}-t\right)}\right](a+ct)e^{-rt}dt$$

$$=\frac{nC_3}{T}\left[\left(e^{\frac{-rT}{n}}-e^{-rt_1^{'}}\right)\left\{-\frac{c}{r^3}-\frac{c}{r^2}-\frac{2\delta c}{r^4}-\frac{\delta a}{r^3}\right\}-\frac{\delta cT}{r^3 n}e^{\frac{-rT}{n}}\right.$$

$$\left.+\frac{\delta ct_1^{'}}{r^3}e^{-rt_1^{'}}-\frac{c\delta}{r^3}\left(\frac{T}{n}-t_1^{'}\right)e^{-rt_1^{'}}\right] \quad (25)$$

Case II: **Retroactive holding cost increase**

(i) When $t_1{'} > M$

Total Cost $(TC_{1a}) = VC + BC$

$= HC_b + DC_b + SC_b + AS_b + OC + HC_v + DC_v + SC_v +$ Interest payable $-$ Interest earned Where $HC_b, DC_b, SC_b, AS_b, OC, HC_v, DC_v, SC_v$ Interest payable and interest earned are (13), (20), (22), (24), (25), (14), (21), (23), (17) and (18) respectively. The following are the required and sufficient requirements for the reduction of the total relevant cost per unit of time:

$$\frac{\partial TC_{1a}\left(T_2,t_1^{'}\right)}{\partial T_2}=0 \quad \frac{\partial TC_{1a}\left(T_2,t_1^{'}\right)}{\partial t_1^{'}}=0$$

$$\frac{\partial^2 TC_{1a}\left(T_2,t_1^{'}\right)}{\partial T_2^2}>0, \quad \frac{\partial^2 TC_{1a}\left(T_2,t_1^{'}\right)}{\partial t_1^{'2}}>0$$

and $$\left(\frac{\partial^2 TC_{1a}\left(T_2,t_1^{'}\right)}{\partial T_2^2}\right)\left(\frac{\partial^2 TC_{1a}\left(T_2,t_1^{'}\right)}{\partial t_1^{'2}}\right)-\left(\frac{\partial^2 TC_{1a}\left(T_2,t_1^{'}\right)}{\partial t_1^{'}\partial T_2}\right)>0$$

When $t_1^{'} \le M$

Total Cost $(TC_{1b}) = VC + BC$

$= HC_b + DC_b + SC_b + AS_b + HC_v + DC_v + SC_v -$ Interest earned

Where $HC_b, DC_b, SC_b, AS_b, HC_v, DC_v, SC_v$ and Interest earned and interest earned are (13), (20), (22), (24), (25), (14), (21), (23) and (19) respectively.

Example; A numerical example is given with the help of following input parameters a =51 unit, b = 0.5 and s =31 rs/unit, θ =0.8, c_2 = 10rs/unit, c_p = 17rs/unit, p= 16rs/unit, k =0.005, w =100, d =16 units, h_r =0.07rs/unit,

h_0 = 0.06rs/unit corresponding to these input values, the optimal values of v and T are 50.7338daus and 61.6414 days respectively and with the minimum value is 733.838 rs,

6. Managerial Insights

To help the decision-maker determine the best course of action for the company, the following managerial insights can be obtained:

i) Consumers are becoming more environmentally conscious. Retailers must, therefore, consider inflation to take advantage of this possibility. The investment has led to an increase in requests for these goods. Consequently, it also affects the retailer's profit.
ii) The study suggests that merchants should plan their inventory procedures with inflation in mind to avoid receiving false information about their financial status.
iii) The study found that since the selling price affects the clients' decision, the retailer must balance The selling price of the goods and the system's profit. By raising the holding fee, retailers can get better storage facilities for losing money on their inventory.
iv) Retailers can utilize order amounts to cut damaged units, which harm their income. Consequently, to maximize earnings, the retailer must make the best choice. The retailer must decide which option is better for him in the meantime.
v) The study found that because scarcity prices, accumulation criteria, and missed deal expenses harm earnings, merchants must plan for various policies to grow their base of devoted customers.

7. Conclusion

This chapter examines an inventory model under trade credit for decaying items with partial backlogs and inflation.. In this model, we assume that the backlog rate is a function that decreases over time. For degraded products, a supply chain inventory model was created that accounts for shortages due to inflation and period cost of money. The backlog rate was considered to be a decreasing function of time until the subsequent replenishment to be more accurate. It is correct that estimating the stock-out is incredibly challenging. The suggested model can be expanded in several ways. For instance, it is possible to think of demand as a more all-encompassing pattern that varies in response to stochastic, price, and stock. According to the analysis, considering a reasonable delay benefits the business financially and contributes to a sustainable environment. The results also showed that high prices do not always translate into a large profit. Setting the selling price with careful planning will enable you to do this. According to the study, venders can acquire a precise commercial condition by estimating the inventory problem while considering inflation and permissible delay.

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