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MULTI OBJECTIVE OPTIMIZATION TECHNIQUES IN STOCHASTIC SYSTEM MAINTENANCE ALLOCATION PROBLEM

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ABSTRACT

In this paper, the problem of optimum compromise allocation of repairable and replaceable components in a system is formulated as a multi-objective all integer non linear programming problem ($AINLPP$). A solution procedure using the five different approaches are considered namely the goal programming, $\in$ - constraint, Distance-based, D_1 - distance and value function to obtain the compromise allocation for a system. A numerical example is also presented to illustrate the computational details.

1. INTRODUCTION

Life inevitably involves decision making, choices and searching for compromises. It is only natural to want all of these to be as good as possible, in other words, optimal. The difficulty here lies in the (at least partial) conflict between our various objectives and goals. Most everyday decisions and compromises are made on the basis of intuition, common sense, chance or all of these. However, there are areas where mathematical modeling and programming are needed, such as engineering, economics, sample surveys etc.

In many real-life cases multi-objective optimizations techniques are required to be considered for determining an optimal policy. Some authors have discussed the multi-objective optimization formulations such as Busacca *et al.* (2001), Fu and Diwekar (2004), Panda *et al.* (2005), Diaz-Garcia and Ulloa (2006, 2008), Wang *et al.* (2009), Khowaja *et al.* (2012), Ghufran *et al.* (2012) Ali *et al.* (2011), Ali *et al.* (2013), Raghav *et al.* (2014) and Khan *et al.* (2003) suggested new compromise criterion for determining an optimal policy.

In many industrial environments, systems are required to perform a sequence of operations (or missions) with finite breaks between each operation. During these breaks, it may be advantageous to perform repair and replacement on some of the system components. However, it may be impossible to perform all desirable maintenance activities prior to the beginning of the next mission due to limitations on maintenance resources. A repairable system is a system in which the failed or deteriorated components can be repaired to operate normally. Many authors have discussed the allocation problem of repairable components. Among them are Rice *et al.* (1998), Schneider and Cassady (2004), Rajagopalan and Cassady (2006), Iyoob *et al.* (2006), Schneider et al. (2009), Ali *et al.* (2013a, 2011a, 2011b), Ali and Hasan (2013, 2013a) and many others.

In this paper we work out the compromise allocation of repairable and replaceable components in a system using the compromise criterion to maximize the system reliability subject to the cost and time constraints. The problem is formulated as multi-objective all integer non linear programming problem ($AINLPP$). Using the initial knowledge about the failed components within the subsystem; this problem is solved by the techniques namely the goal programming, $\in$ - constraint, Distance-based, D_1 - distance and value function to obtain the compromise allocation of replaceable and repairable components. A numerical example is worked out to illustrate the computational details of the techniques. The numerical solution is obtained through the optimization software LINGO.

LINGO is a user's friendly package for constrained optimization developed by LINDO Systems Inc. A user's guide- LINGO User's Guide (2001) is also available. For more information one can visit the site http://www.lindo.com.

2. FORMULATION OF THE PROBLEM AND NOTATIONS

We assume that the system comprises subsystems of two types of characteristics. One is the characteristics of subsystems in which the components are very sensitive to the functioning of the whole system and, therefore, on deterioration these should be replaced by new ones. Let these subsystems range from 1 to s. The second characteristic of subsystems is those in which the components after deterioration can be repaired and then replaced. Let such subsystems range from $s+1$ to m. In fig. 1 the group X consists of the s subsystems with sensitive components which on failure are replaced by new ones and Y the remaining $(m-s)$ subsystems in which the components can be repaired.

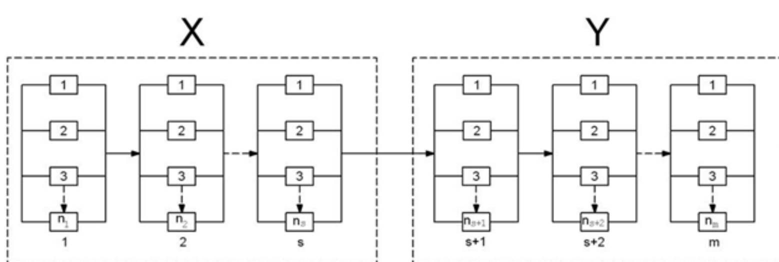

Figure 1-Parallel components in Repairable and Replaceable Subsystems

In fig.1 the group X is a series arrangement of the subsystems (subsystem 1, subsystem 2… subsystem s); its reliability can be defined as

$$R(d_{ij}) = \left\{ \prod_{i=1}^{s} \left[1 - \prod_{j=1}^{n_i} \left(1 - r_{ij}\right)^{n_i} \right] \right\} \tag{1}$$

And in fig.1 the group Y is also a series arrangement of the subsystems (subsystem $s+1$, subsystem 2… subsystem m); its reliability can be defined as

$$R(d_{ij}) = \left\{ \prod_{i=S+1}^{m} \left[1 - \prod_{j=1}^{n_i} \left(1 - r_{ij}\right)^{n_i} \right] \right\} \tag{2}$$

At the completion of a particular production run, each component in a subsystem is either functioning or failed. Ideally all the failed components in the subsystems should be repaired or replaced by its new one prior to the beginning of the next production run. However, due to the constraints on the time and cost, it may not be possible to repair and replace all the failed components in the subsystems. Let a_{ij} be the total number $j-th$ type of failed components in $i-th$ subsystem.

The time required for repaired or replaced by its new one all the failed components in the system is given by

$$T = \sum_{i=1}^{m} \sum_{j=1}^{n_i} t_{ij} a_{ij} \tag{3}$$

where t_{ij} is the time required to repair and replace a $j-th$ type of failed component in $i-th$ subsystem. The maintenance time available for repairing and replacing the failed components between two production runs is T_0 units.

If $T_0 < T$, then all failed components cannot be repaired and replaced prior to beginning of the next production run.

The cost required for replacing and repairing the failed components in the system is given by

$$C = \sum_{i=1}^{m} \sum_{j=1}^{n_i} c_{ij} a_{ij} \tag{4}$$

The maintenance cost available for repairing and replacing the failed components between two production runs is C_0 units.

If $C_0 < C$, then all failed components cannot be repaired and replaced prior to beginning of the next production run.

In such cases, a method is needed to decide how many failed components should be repaired and replaced prior to the next production run and the rest be left in a failed condition.

This process is referred to as selective maintenance (See Rice *et al.* (1998)).

In the selective maintenance the number of $j-th$ type components available for the next production run in the $i-th$ subsystem will be

$$(n_i - a_{ij}) + d_{ij},\ i = 1,2,...,m \tag{5}$$

where d_{ij} is the number of repaired and replaced $j-th$ type components in subsystem i prior to the next production run respectively and n_i is the total number of components available in parallel in the $i-th$ subsystem.

We have assumed that the repair and replace time and cost of each failed components in a subsystem are same. The reliability of the subsystems range from 1 to s for a production run is given by

$$R(d_{ij}) = \left\{ \prod_{i=1}^{s} \left[1 - \prod_{j=1}^{n_i} (1 - r_{ij})^{n_i - a_{ij} + d_{ij}} \right] \right\} \tag{6}$$

And the reliability of the subsystems range from $s+1$ to m for a production run is given by

$$R(d_{ij}) = \left\{ \prod_{i=S+1}^{m} \left[1 - \prod_{j=1}^{n_i} (1 - r_{ij})^{n_i - a_{ij} + d_{ij}} \right] \right\} \tag{7}$$

The repair time constraint for the system is given as

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} t_{ij} d_{ij} \le T_0 \tag{8}$$

and the repair cost constraint for the system is given as

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} c_{ij} d_{ij} \le C_0 \tag{9}$$

However, in the event when the reliability of the subsystems of group X and group Y time are of equally serious concern.

Let us consider for instance the following multi-objective problem:

$$\left.\begin{aligned}
&Maximize\ R(d_{ij}) = \left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} && (i)\\
&and\ \ Maximize\ R(d_{ij}) = \left\{\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} && (ii)\\
&Subject\ to\ \sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij}d_{ij} \le T_0 && (iii)\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i} c_{ij}d_{ij} \le C_0 && (iv)\\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ \text{int}eger && (v)\\
&n_{ij} \ge a_{ij}, \quad i=1,2,\ldots,m\ and\ j=1,2,\ldots,n_i && (vi)
\end{aligned}\right\} \qquad (10)$$

In many practical situations the constraint equations (*iii*) and (*iv*) are not fixed and taken as probabilistic. Thus the above problem (10) can be written in the following chance constrained programming form as:

$$\left.\begin{aligned}
&Maximize\ R(d_{ij}) = \left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} && (i)\\
&and\\
&Maximize\ R(d_{ij}) = \left\{\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} && (ii)\\
&Subject\ to\\
&P\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij}d_{ij} \le T_0\right) \ge p_0 && (iii)\\
&P\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i} c_{ij}d_{ij} \le C_0\right) \ge p_0 && (iv)\\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ \text{int}eger && (v)\\
&n_{ij} \ge a_{ij}, \quad i=1,2,\ldots,m\ and\ j=1,2,\ldots,n_i && (vi)
\end{aligned}\right\} \qquad (11)$$

where p_0, $0 \le p_0 \le 1$ is a specified probability.

In the above problem (11), let us assume that t_{ij} and c_{ij} are independently normally distributed random variables. The equivalent deterministic non-linear programming problem (11) to the stochastic programming problem is given by

$$\left.\begin{array}{ll}
Maximize\ R(d_{ij}) = \left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} & (i) \\
and & \\
Maximize\ R(d_{ij}) = \left\{\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} & (ii) \\
Subject\ to & \\
\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij} + K_\alpha \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \le T_0 & (iii) \\
\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij} + K_\alpha \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \le C_0 & (iv) \\
0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ \mathrm{int}eger & (v) \\
n_{ij} \ge a_{ij}, \quad i = 1,2,\ldots,m\ and\ j = 1, 2,\ldots,n_i & (vi)
\end{array}\right\} \quad (12)$$

3. OPTIMIZATION METHODS FOR SOLVING MULTI-OBJECTIVE PROGRAMMING PROBLEM

The design parameters involved in reliability allocation problem has usually been taken to be precise values. This means that every probability involved is perfectly determinable. In this case, it is usually assumed that there exist some complete probabilistic information about the system and the component behavior. However, in real life situations, there are not sufficient statistical data available in most of the cases where the system is new or exists only as a project. It is not always possible to observe the stability from the statistical point of view. This means that only some partial information about the system components is known. The various methods proposed to solve the multi-objective programming problem of stochastic reliability allocation problem can be classified according to the available information about the system.

3.1 The Goal Programming Technique

This method is used when we have considered the problem of more than one objective. In reliability optimization, we have considered system has several subsystems and these subsystems have several components. Each and every subsystem has distinct characteristic components itself. The system which has been described and formulated as multi-objective programming problem (see equation12) in previous section.

Now the formulated problem must know the importance of the characteristics that is all the information about the characteristics is given. Now the problem (equation 12) may be stated separately for p characteristics components within a subsystem are used. The solution procedure for solving p characteristics components by using goal programming are given as:

$$\left.\begin{array}{l} Maximize\ R(d_{ij}) = \left(R_1(d_{ij}), R_2(d_{ij}), ..., R_p(d_{ij})\right), \\ Subject\ to \\ \sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2} \le T_0 \\ \sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2} \le C_0 \\ 0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ integer \\ n_{ij} \ge a_{ij}, \quad i = 1,2,...,m\ and\ j = 1, 2,...,n_i \end{array}\right\} \quad (13)$$

where

$$R_k(d_{ij}) = \left\{\prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n_i}\left(1 - r_{ij}\right)^{n_i - a_{ij} + d_{ij}}\right]\right\}, \ k = 1,2,....,p.$$

Let $R_k^*(d_{ij})$ be the optimum value of $R_k(d_{ij})$ obtained by solving the following non linear integer programming problem

$$\left.\begin{array}{l} Maximize\ R_k(d_{ij}), \qquad k = 1,2,..., p \\ Subject\ to \\ \sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2} \le T_0 \\ \sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2} \le C_0 \\ 0 \le d_{ij} \le a_{ij}, \ \forall\ d_{ij}\ are\ integer \\ n_{ij} \ge a_{ij}, \quad i = 1,2,...,m\ and\ j = 1, 2,...,n_i \end{array}\right\} \quad (14)$$

Further let

$$\tilde{R}_k(d_{ij}) = \tilde{R}_k(d_{1k}, d_{2k}, \ldots, d_{mk}) = \left\{ \prod_{i=1}^{m} \left[1 - \prod_{j=1}^{n_i} \left(1 - r_{ij}\right)^{n_i - a_{ij} + d_{ij}} \right] \right\}, \quad k = 1, 2, \ldots, p. \tag{15}$$

(15) denote the reliability of the system under the compromise allocation, where d_{ij} are to be worked out.

Obviously $\tilde{R}_k(d_{ij}) \le R_k^*(d_{ij})$ and $R_k^*(d_{ij}) - \tilde{R}_k(d_{ij}) \ge 0; k = 1, 2, \ldots, p$ will give the decrease in the system reliability due to not using the individual optimum allocation for $k-th$ characteristic components.

Now consider the following goal:

"Find d_{ij} such that the decrease in the reliability of the system for each components characteristic due to the use of compromise allocation, d_{ij} instead of individual optimum allocation, should not greater than $\delta_k\ (k = 1, 2, \ldots, p)$".

Where $\delta_k \ge 0,\ k = 1, 2, \ldots, p$ are the unknown goal variables.

To achieve these goals d_{ij} must satisfy

$$R_k^*(d_{ij}) - \tilde{R}_k(d_{ij}) \le \delta_k\ ; k = 1, 2, \ldots, p \tag{16}$$

or

$$\tilde{R}_k(d_{ij}) + \delta_k \ge R_k^*(d_{ij}); k = 1, 2, \ldots, p$$

$$\left\{ \prod_{i=1}^{m} \left[1 - \prod_{j=1}^{n_i} \left(1 - r_{ij}\right)^{n_i - a_{ij} + d_{ij}} \right] \right\} + \delta_k \ge R_k^*(d_{ij}) \quad ; k = 1, 2, \ldots, p \tag{17}$$

The value $\sum_{k=1}^{p} \delta_k$ will give us the total decreases in reliability of the system by not using the individual optimum allocations.

This suggests the following integer goal programming problem ($IGPP$) to solve:

$$\left.\begin{aligned}
&\text{Minimize } \sum\nolimits_{k=1}^{p} \delta_k, \\
&\text{Subject to} \\
&\left\{\prod_{i=1}^{m}\left[1-\prod_{j=1}^{n_i}\left(1-r_{ij}\right)^{n_i-a_{ij}+d_{ij}}\right]\right\}+\delta_k \geq R_k^*(d_{ij});\, k=1,2,\ldots,p, \\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \leq T_0 \\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \leq C_0 \\
&0 \leq d_{ij} \leq a_{ij}, \forall\, d_{ij} \text{ are integer} \\
&n_{ij} \geq a_{ij}, \quad i=1,2,\ldots,m \text{ and } j=1,2,\ldots,n_i
\end{aligned}\right\} \tag{18}$$

3.2 The $\in$ - Constraint Technique

This method is used when only partial information is available. For using this method first we investigate the most important characteristics. In $\in$-constraint method one of the objective functions is selected to be optimized and remaining objective functions are converted into constraints by setting an upper bound to each them, See S. Rios et al. (1989), Miettinen (1999).

Let us assume that the $l-th$ characteristic $l \in \{1,2,\ldots,p\}$ be the most important characteristic in the study. Under this technique problem (12) for obtaining the optimum solution can be restated as:

$$\left.\begin{aligned}
&\text{Maximize} \qquad R_k(d_{ij})_l, \\
&\text{Subject to } R_k(d_{ij})_r \geq \theta_r, r \neq l,\ r=1,2,\ldots,p, \\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \leq T_0 \\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \leq C_0 \\
&0 \leq d_{ij} \leq a_{ij}, \forall\, d_{ij} \text{ are integer} \\
&n_{ij} \geq a_{ij}, \quad i=1,2,\ldots,m \text{ and } j=1,2,\ldots,n_i
\end{aligned}\right\} \tag{19}$$

where, θ_r is pre established bound for each of the $p-1$ remaining reliability of the system, which are given as constraints.

In practice, θ_r can be defined as the minimum individual values of the following problems:

$$\left.\begin{aligned}
&\textit{Maximize} \quad R_k(d_{ij})_r, \\
&\textit{Subject to} \\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \le T_0 \\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \le C_0 \\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij} \text{ are integer} \\
&n_{ij} \ge a_{ij}, \quad i = 1,2,\ldots,m \\
&\text{and } j = 1, 2, \ldots, n_i
\end{aligned}\right\} r = 1,2,\ldots,p\,; r \ne l \tag{20}$$

3.3 Distance based Technique

In many situations, sufficient information about the components is not available, or it is difficult to decide which is the most important subsystem within the system. In such situations, distance based method is very useful, See S. Rios et al. (1989), and R. E. Steuer (1986).

Under this technique, problem (12) may be expressed as follows,

Let the p - component vector of targets ξ is

$$\xi = \begin{pmatrix} \xi_1 \\ \vdots \\ \xi_p \end{pmatrix},$$

where ξ_k be the ideal point or goal for the objective $R_k(d_{ij});\ k = 1,2,\ldots,p.$

This vector of target ξ can be computed by maximizing each objective $R_k(d_{ij});\ k = 1,2,\ldots,p.$ separately. Thus, ξ is the vector of individual constrained minima, which can be obtained by solving the following *AINLPP* :

$$\left.\begin{aligned}
&\textit{Maximize} \quad R_k(d_{ij}),\\
&\textit{Subject to}\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2}\le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2}\le C_0\\
&0\le d_{ij}\le a_{ij},\ \forall\ d_{ij}\ are\ \text{integer}\\
&n_{ij}\ge a_{ij},\quad i=1,2,\ldots,m\\
&and\ j=1,\ 2,\ldots,n_i
\end{aligned}\right\}\ k=1,2,\ldots,p. \tag{21}$$

After computing ξ, the optimization problem is formulated as

$$\left.\begin{aligned}
&\textit{Maximize}\ D\left[R_k(d_{ij}),\xi_k\right],\\
&\textit{Subject to}\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2}\le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2}\le C_0\\
&0\le d_{ij}\le a_{ij},\ \forall\ d_{ij}\ are\ \text{integer}\\
&n_{ij}\ge a_{ij},\quad i=1,2,\ldots,m\ and\ j=1,\ 2,\ldots,n_i
\end{aligned}\right\}\ k=1,2,\ldots,p. \tag{22}$$

where

$D\left[R_k(d_{ij}),\xi_k\right]$ may be expressed as $\sqrt{\sum_{k=1}^{p}\left[R_k(d_{ij})-\xi_k\right]^2}$. Because minimization of $\sqrt{\sum_{k=1}^{p}\left[R_k(d_{ij})-\xi_k\right]^2}$ is equivalent to minimize $\sum_{k=1}^{p}\left[R_k(d_{ij})-\xi_k\right]^2$, the *AINLPP* (22) becomes

$$\left.\begin{aligned}
&Minimize\ \sum_{k=1}^{p}\left[R_k(d_{ij})-\xi_k\right]^2,\\
&Subject\ to\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2}\ \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2}\ \le C_0\\
&0\le d_{ij}\le a_{ij},\forall\ d_{ij}\ are\ \text{integer}\\
&n_{ij}\ge a_{ij},\quad i=1,2,\ldots,m\ and\ j=1,2,\ldots,n_i
\end{aligned}\right\}\tag{23}$$

Alternatively, Khuri & Cornell (1986) proposed another distance given by

$$\left.\begin{aligned}
&Minimize\ \sum_{k=1}^{p}\frac{\left[R_k(d_{ij})-\xi_k\right]^2}{\xi_k^2},\\
&Subject\ to\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2}\ \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2}\ \le C_0\\
&0\le d_{ij}\le a_{ij},\forall\ d_{ij}\ are\ \text{integer}\\
&n_{ij}\ge a_{ij},\quad i=1,2,\ldots,m\ and\ j=1,2,\ldots,n_i
\end{aligned}\right\}\tag{24}$$

3.4 D_1-Distance Method of Lexicographic Goal Programming

In the problem (12), let us first consider the reliability of the replacement components more important than the reliability of the repairing component within the system. Then we solve the problem (12) by maximizing (*i*) subject to (*iii*) to (*vi*) (*i.e.* we neglect the objective (*ii*)).

Let the maximum of the $NLPP$ (12), while neglecting the second objective be $R(d_{ij})^*_{replacement}$. Next we solve the following $NLPP$:

$$\left.\begin{aligned}
&\text{Maximize } R(d_{ij}) = \left\{\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} - \delta_1\\
&\text{Subject to}\\
&\left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} + \delta_1 \ge R(d_{ij})^*_{replacement}\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \le C_0\\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij} \text{ are integer}\\
&n_{ij} \ge a_{ij}, \quad i = 1,2,\ldots,m \text{ and } j = 1, 2,\ldots,n_i\\
&\delta_1 \ge 0
\end{aligned}\right\} \tag{25}$$

where δ_1 is the deviational variable. By solving the $NLPP$ (25) let the optimum reliability of the repairing component obtained be $R(d_{ij})^*_{repairing}$. The following lexicographic goal programming problem is then solved:

$$\left.\begin{aligned}
&\text{Minimize } \delta_1 + \delta_2\\
&\text{Subject to}\\
&\left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} + \delta_1 \ge R(d_{ij})^*_{replacement}\\
&\left\{\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}(1-r_{ij})^{n_i-a_{ij}+d_{ij}}\right]\right\} + \delta_2 \ge R(d_{ij})^*_{repairing}\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \le C_0\\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij} \text{ are integer}\\
&n_{ij} \ge a_{ij}, \quad i = 1,2,\ldots,m \text{ and } j = 1, 2,\ldots,n_i\\
&\delta_1, \delta_2 \ge 0
\end{aligned}\right\} \tag{26}$$

Let the solution of the $NLPP$ (26) obtained be ($d_1^{(1)},...,d_m^{(1)}$).

Next we assume that the reliability of the repairing components is more important than the reliability of the replacement of the components.

Then we solve the $NLPP$ (12) by considering reliability of the repairing components and neglecting the reliability of the replacement of the components.

Let the maximum so obtained be $R(d_{ij})^*_{repairing}$.

In the next step solve the following $NLPP$ for optimum reliability of the replacement of the components

$$\left.\begin{array}{l} Maximize\ R(d_{ij}) = \left\{ \prod_{i=1}^{s} \left[1 - \prod_{j=1}^{n_i} (1 - r_{ij})^{n_i - a_{ij} + d_{ij}} \right] \right\} - \delta_1 \\ Subject\ to \\ \left\{ \prod_{i=s+1}^{m} \left[1 - \prod_{j=1}^{n_i} (1 - r_{ij})^{n_i - a_{ij} + d_{ij}} \right] \right\} + \delta_1 \ge R(d_{ij})^*_{repairing} \\ \sum_{i=1}^{m} \sum_{j=1}^{n_i} \overline{t_{ij}}\, d_{ij} + K_\alpha \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{t_{ij}}^2} \le T_0 \\ \sum_{i=1}^{m} \sum_{j=1}^{n_i} \overline{c_{ij}}\, d_{ij} + K_\alpha \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n_i} d_{ij}^2\, \overline{\sigma}_{c_{ij}}^2} \le C_0 \\ 0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ \text{int}eger \\ n_{ij} \ge a_{ij}, \quad i = 1, 2, ..., m\ and\ j = 1, 2, ..., n_i \\ \delta_1 \ge 0 \end{array} \right\} \quad (27)$$

Let the maximum reliability obtain be $R(d_{ij})^*_{replacement}$.

The following lexicographic goal programming problem is then solved:

$$\left.\begin{aligned}
&Minimize\ \delta_1+\delta_2\\
&Subject\ to\ \left\{\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}\left(1-r_{ij}\right)^{n_i-a_{ij}+d_{ij}}\right]\right\}+\delta_1\ge R(d_{ij})^{*}_{repairing}\\
&\left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}\left(1-r_{ij}\right)^{n_i-a_{ij}+d_{ij}}\right]\right\}+\delta_2\ge R(d_{ij})^{*}_{replacement}\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}^2_{t_{ij}}}\ \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}^2_{c_{ij}}}\ \le C_0\\
&0\le d_{ij}\le a_{ij},\ \forall\ d_{ij}\ are\ \text{int}eger\\
&n_{ij}\ge a_{ij},\quad i=1,2,\dots,m\,and\ j=1,2,\dots,n_i\\
&\delta_1,\delta_2\ge 0
\end{aligned}\right\}\qquad(28)$$

Let the solution of the $NLPP$ (28) obtained be ($d_1^{(2)},\dots,d_m^{(2)}$). In this way the priorities are given to the objectives one after the other and a set of solutions is obtained. Out of these solutions, an ideal solution is identified as follows:

$$d_1^* = \left\{\max\left(d_1^{(1)},d_2^{(1)}\right),\max\left(d_2^{(1)},d_2^{(2)}\right),\dots,\max\left(d_m^{(1)},d_m^{(2)}\right)\right\}=\left\{d_1^*,d_2^*,\dots,d_m^*\right\}\text{, say.}$$

The D_1-distances of different solutions from the ideal solution defined in (30) below are then calculated. The solution corresponding to the minimum D_1-distance gives the best compromise solution. A general procedure with P objectives is the following. As explained above, we will obtain $P!$ (Factorial) different solutions by solving the $P!$ problems arising for $P!$ different priority structures. Let $d_i^{(\pi)} = \left\{d_1^{(\pi)},d_2^{(\pi)},\dots,d_m^{(\pi)}\right\}, 1\le\pi\le P!$ be the $P!$ number of solutions obtained by giving priorities to P objective functions. Let $\left(d_1^*,d_2^*,\dots,d_m^*\right)$ be the ideal solution. But in practice ideal solution can never be achieved.

The solution, which is closest to the ideal solution, is acceptable as the best compromise solution, and the corresponding priority structure is identified as most appropriate priority structure in the planning context. To obtain the best compromise solution, following goal programming problem is to be solved.

$$\left.\begin{array}{l} \underset{1\le\pi\le P!}{Min} \sum_{i=1}^{m} \varepsilon_{i\pi} \\ \text{subject to} \\ d_i^* - d_i^{(\pi)} - \varepsilon_{i\pi} = 0 \\ and\ \ \varepsilon_{i\pi} \ge 0, 1 \le \pi \le P\,! \\ 0 \le d_i^{\pi} \le a,\ i = 1,\ldots,m\ and\ \text{int}\,eger \end{array}\right\} \tag{29}$$

where $\varepsilon_{i\pi\backslash}$ are the deviational variables.

Now, $$(D_1)^{\pi} = \sum_{i=1}^{m} \left| d_i^* - d_i^{(\pi)} \right| \tag{30}$$

is defined as the D_1-distance from the ideal solution $\{d_1^*, d_2^*, ..., d_m^*\}$, to the $\pi - th$ solution $\{d_1^{(\pi)}, d_2^{(\pi)}, ..., d_m^{(\pi)}\}$, $1 \le \pi \le P!$

Therefore,

$$(D_1)_{opt} = \underset{1\le\pi\le P!}{Min} (D_1)^{\pi} = \underset{1\le\pi\le P!}{Min} \sum_{i=1}^{m} \left| d_i^* - d_i^{(\pi)} \right| \tag{31}$$

$$= \underset{1\le\pi\le P!}{Min} \sum_{i=1}^{m} \varepsilon_{i\pi} \tag{32}$$

Let the minimum be attained for $\pi = p$ .Then $\{d_1^{(p)}, d_2^{(p)}, ..., d_m^{(p)}\}$ is the best compromise solution of the problem.

3.5 The Value Function Technique

The $MNLPP$ (12) under the value fu**n**ction technique are

$$\left.\begin{aligned}
&Maximize \quad \psi\left(\sum\nolimits_{k=1}^{p} R_k\left(d_{ij}\right)\right),\\
&Subject\ to\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2} \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2} \le C_0\\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ \text{int}eger\\
&n_{ij} \ge a_{ij}, \quad i = 1,2,...,m\, and\ j = 1,\ 2,...,n_i
\end{aligned}\right\} \tag{33}$$

where $\psi(.)$ is a scalar function that summarizes the importance of each of the subsystem reliability of the p characteristics failed components. Usually, $\psi(.)$ is taken as weighted sum of p characteristics operational components within the subsystem. Under this property equation (33) becomes:

$$\left.\begin{aligned}
&Maximize \quad \sum\nolimits_{k=1}^{p} \alpha_k\ R_k\left(d_{ij}\right),\\
&Subject\ to\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2} \le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\, d_{ij} + K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i} d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2} \le C_0\\
&0 \le d_{ij} \le a_{ij}, \forall\ d_{ij}\ are\ \text{int}eger\\
&n_{ij} \ge a_{ij}, \quad i = 1,2,...,m\, and\ j = 1,\ 2,...,n_i
\end{aligned}\right\} \tag{34}$$

where $\sum\nolimits_{k=1}^{p}\alpha_k = 1,\ \alpha_k \ge 0,\ k = 1,2,...,p,\quad \alpha_k$ are the weights according to the relative importance, that how many components is operational within the subsystem. When complete information about the failed and operational components within the subsystem is available, the weights may be decided according to some measures of the relative importance of the operational components within the subsystems (see Ali and Hasan 2013). For example, weights α_k may be taken as

$$\alpha_k \propto \sum_{i=1}^{m}\sum_{j=1}^{n_i}(n_{kij} - a_{kij});\; k = 1,2,\ldots,p$$

or

$$\alpha_k = \beta \sum_{i=1}^{m}\sum_{j=1}^{n_i}(n_{kij} - a_{kij}).$$

where β is the constant of proportionality.

Without loss of generality we can assume that $\sum_{k=1}^{p}\alpha_k = 1.$

Thus,

$$\sum_{k=1}^{p}\alpha_k = \beta\sum_{k=1}^{p}\sum_{i=1}^{m}\sum_{j=1}^{n_i}(n_{kij} - a_{kij}),$$

or

$$\beta = \frac{1}{\sum_{k=1}^{p}\sum_{i=1}^{m}\sum_{j=1}^{n_i}(n_{kij} - a_{kij})},$$

This provides

$$\alpha_k = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}(n_{kij} - a_{kij})}{\sum_{k=1}^{p}\sum_{i=1}^{m}\sum_{j=1}^{n_i}(n_{kij} - a_{kij})};\; k = 1,2,\ldots,p$$

Using (12) *AINLPP* (34) can be rewritten as

$$\left.\begin{aligned}
&\textit{Maximize}\quad \sum_{k=1}^{p}\alpha_k\left\{\prod_{i=1}^{s}\left[1-\prod_{j=1}^{n_i}\left(1-r_{ij}\right)^{n_i-a_{ij}+d_{ij}}\right]+\prod_{i=s+1}^{m}\left[1-\prod_{j=1}^{n_i}\left(1-r_{ij}\right)^{n_i-a_{ij}+d_{ij}}\right]\right\},\\
&\textit{Subject to}\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{t_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{t_{ij}}^2}\le T_0\\
&\sum_{i=1}^{m}\sum_{j=1}^{n_i}\overline{c_{ij}}\,d_{ij}+K_\alpha\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n_i}d_{ij}^2\,\overline{\sigma}_{c_{ij}}^2}\le C_0\\
&0\le d_{ij}\le a_{ij},\ \forall\ d_{ij}\ \textit{are integer}\\
&n_{ij}\ge a_{ij},\quad i=1,2,\ldots,m\ \textit{and}\ j=1,2,\ldots,n_i
\end{aligned}\right\}\tag{35}$$

4. NUMERICAL ILLUSTRATIONS AND DISCUSSION

Consider a system having the group X consisting of 3 subsystems and also the group Y Y consisting of 4 subsystems. The available time between two missions for repairing and replacing is 150 time units. The available cost of maintenance for repairing and replacing is for the next mission is 860 units. For the simplicity we have consider in the above numerical illustration; the reliability of each component in a subsystem is same, cost spent and time taken on replacing and repairing each component within a subsystem are same i.e. $i = j$. The remaining parameters for the various subsystems are given in table 1.

Table 1: The number of failed components and the respective cost and time etc. in the various subsystems

	Group X (Replaced)			**Group Y (Repair)**			
Subsystem	**1**	**2**	**3**	**4**	**5**	**6**	**7**
n_{ij}	6	5	10	7	9	12	10
r_{ij}	0.8	0.75	0.8	0.8	0.75	0.8	0.7
$\overline{t_{ij}}$	2	3	1	20	28	22	22
$\sigma^2_{t_{ij}}$	0.15	0.18	0.10	0.35	0.40	0.50	0.60
$\bar{c}_{ij}$	20	10	20	40	30	45	65
$\sigma^2_{c_{ij}}$	0	8	5	8	5	7	9
a_{ij}	3	3	6	5	7	9	7

The D_1 - distance solution tables are given as:

Table 2: (Solutions)

Run	**Priorities**	d_{11}	d_{22}	d_{33}	d_{44}	d_{55}	d_{66}	d_{77}
1	$\text{Re}\, place^{(1)}, \text{Re}\, pair^{(2)}$	2	3	1	1	0	0	1
2	$\text{Re}\, place^{(2)}, \text{Re}\, pair^{(1)}$	2	3	0	2	1	0	2
ideal solution (d^*_{ij})		**2**	**3**	**1**	**2**	**1**	**0**	**2**

Table 3: The $D_1 -$ distance from the ideal solutions

priority to	d_{11}	d_{22}	d_{33}	d_{44}	d_{55}	d_{66}	d_{77}	$(D_1)^r$
Replace	0	0	0	1	1	0	1	**3**
Repair	0	0	1	0	0	0	0	**1**

In Table 3 the D_1 - distance of all possible solutions from the ideal solution are calculated. From Table 3 it is clear that the minimum of the D_1 - distance of the two priority structure solutions from the ideal solutions are equal to **1**. Therefore, we choose repair components priority structure. The compromises solutions are obtained through the following techniques, the goal programming, $\in$ - constraint, Distance-based, D_1 - distance and value function. These techniques are applied to an example and the respective compromise allocations of components are summaries into a table 4.

Table 4: Optimum allocation of replaceable and repairable components under various Techniques

Techniques	d_{11}	d_{22}	d_{33}	d_{44}	d_{55}	d_{66}	d_{77}
Goal Programming	2	3	0	2	1	0	2
$\in$ -Constraint	2	1	0	2	1	0	3
Distance Based	2	3	0	1	2	0	2
Khuri & Cornell	2	3	0	1	2	0	2
D_1 - distance	2	3	0	2	1	0	2
Value Function	2	3	0	2	1	0	2

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