Aligarh Journal of Statistics
Vol. 32 (2012), 85-96

ON LOWER GENERALIZED ORDER STATISTICS FROM INVERSE p^{th} ORDER EXPONENTIAL DISTRIBUTION AND ITS CHARACTERIZATION

MINIMOL, S. and P. YAGEEN THOMAS

ABSTRACT

In this paper, we derive some recurrence relations satisfied by single and product moments of lower generalized order statistics arising from inverse p^{th} order exponential distribution. Further, we have obtained a characterization of inverse p^{th} order exponential distribution based on the recurrence relation for single moments of lower generalized order statistics.

1. INTRODUCTION

The concept of generalized order statistics (gos) was introduced by Kamps (1995). It provides a general frame work for models of ordered random variables. Several known results in submodels can be subsumed, generalized and integrated within n general frame work. For an extensive study in this area one may refer to the works of Kamps and Gather (1997), Keseling (1999), Cramer and Kamps (2000), Kamps and Cramer (2001) and Pawlas and Szynal (2001a).

Let $X(1,n,m,k), X(2,n,m,k),....., X(n,n,m,k)$, be n gos arising from an absolutely continuous distribution function $F(x)$ and probability density function $f(x)$, ($n>1$, m and k are real numbers and $k>0$). Then the joint pdf of $X(1,n,m,k), X(2,n,m,k),....., X(n,n,m,k)$ is given by (Kamps, 1995).

$$f_{1,...,n}(x_1, x_2, ..., x_n) = k\left(\prod_{j=1}^{n-1}\gamma_j\right)\left(\prod_{i=1}^{n-1}[1-F(x_i)]^{m_i} f(x_i)\right)[1-F(x_n)]^{k-1} f(x_n) \tag{1.1}$$

on the cone $F^{-1}(0^{+}) < x_1 \le x_2 \le ... \le x_n < F^{-1}(1)$ of $\Re^n$, where $\overline{F}(x) = 1 - F(x)$ and $\gamma_j = k + (n - j)(m + 1)$.

By appropriate choice of the parameters we can deduce the forms of (1.1) for ordinary order statistics, k^{th} record values, sequential order statistics and progressive *type II* censored order statistics.

For $m_1 = ... = m_{n-1} = 0, k = 1$ and $m_1 = ... = m_{n-1} = -1, k \in N$, (1.1) reduces to that of ordinary order statistics and k^{th} record values respectively. However, when F is an inverse distribution function the concept of *gos* is inapplicable. Hence in such situations the concept of lower (dual) generalized order statistics becomes essential.

The concept of lower generalized order statistics $(\lg os)$ was introduced by Pawlas and Szynal (2001b). Let $n \in N, k \ge 1, \tilde{m} = (m_1, m_2, ..., m_{n-1}) \in \Re^{n-1}$ be the parameters such that $\gamma_r = k + (n - r) + M_r > 0$

where

$$M_r = \sum_{j=r}^{n-1} m_j \text{ for all } r, 1 \le r \le n-1.$$

Then $X'(r, n, \tilde{m}, k), r = 1, 2, ..., n$ are called lower *gos* if their joint *pdf* is given by

$$f_{1,...,n}(x_1, x_2, ..., x_n) = k\left(\prod_{j=1}^{n-1}\gamma_j\right)\left(\prod_{i=1}^{n-1}[F(x_i)]^{m_i} f(x_i)\right)[F(x_n)]^{k-1} f(x_n), \tag{1.2}$$

for $F^{-1}(1) > x_1 \ge x_2 \ge ... \ge x_n > F^{-1}(0)$ of $\Re^n$.

Here we may consider two cases.

(i) $m_1 = m_2 = ... = m_{n-1} = m$

(ii) $\gamma_i \ne \gamma_j, i \ne j, i, j = 1, 2, ..., n-1$

For case (i), the *pdf* of r^{th} lower *gos* based on a random sample from an absolutely continuous distribution function F is of the form

$$f_{X'(r,n,m,k)}(x) = \frac{C_{r-1}}{(r-1)!}\left[F(x)\right]^{\gamma_r - 1} f(x)\left[g_m(F(x))\right]^{r-1} \tag{1.3}$$

and the joint *pdf* of r^{th} and s^{th} lower *gos*, $1 \le r < s \le n$ is given by

$$f_{X'(r,n,m,k),X'(s,n,m,k)}(x,y) = \frac{C_{s-1}}{(r-1)!(s-r-1)!}[F(x)]^m [g_m(F(x))]^{r-1}$$
$$\times [h_m(F(y)) - h_m(F(x))]^{s-r-1}[F(y)]^{\gamma_s - 1} f(x) f(y), x > y \tag{1.4}$$

where, $C_{r-1} = \prod_{j=1}^{r} \gamma_j$,

$$h_m(x) = \begin{cases} -\dfrac{x^{m+1}}{m+1}, m \ne -1 \\ -\ln x, m = -1 \end{cases}$$

and $g_m(x) = h_m(x) - h_m(1), x \in [0,1)$.

For case (ii), the *pdf* of r^{th} lower *gos* is given by

$$f_{X'(r,n,\tilde{m},k)}(x) = C_{r-1} f(x) \sum_{u=1}^{r} a_u(r) [F(x)]^{\gamma_u - 1} \tag{1.5}$$

and the joint *pdf* of r^{th} and s^{th} lower *gos*, $1 \le r < s \le n$ is given by

$$f_{X'(r,n,\tilde{m},k),X'(s,n,\tilde{m},k)}(x,y) = C_{s-1} \sum_{u=r+1}^{s} a_u^{(r)}(s) \left[\frac{F(y)}{F(x)}\right]^{\gamma_u} \sum_{u=1}^{r} a_u(r) [F(x)]^{\gamma_u}$$
$$\times \frac{f(x)}{F(x)} \frac{f(y)}{F(y)}, x > y \tag{1.6}$$

where,

$$a_u(r) = \prod_{v=1}^{r} \frac{1}{(\gamma_v - \gamma_u)}, 1 \le u \le r \le n$$

and

$$a_u^{(r)}(s) = \prod_{v=r+1}^{s} \frac{1}{(\gamma_v - \gamma_u)}, \; r+1 \le u \le s \le n$$

For a detailed account on lower *gos* one may refer to Burkschat et al. (2003), Ahsanullah (2004) and Khan et al. (2008). Nain (2010) has obtained recurrence relations for the single and product moments of ordinary order statistics arising from p^{th} order exponential distribution. In our present work we introduce inverse p^{th} order exponential distribution and discuss some distributional properties of this distribution using lgos. In section 2, we derive recurrence relations for single and product moments of lower *gos* arising from inverse p^{th} order exponential distribution In section 3, we obtain a characterization result based on the recurrence relation for the considered family of distributions.

2. RECURRENCE RELATIONS FOR SINGLE AND PRODUCT MOMENTS OF LOWER GENERALIZED ORDER STATISTICS

A random variable X is said to have inverse p^{th} order exponential distribution if its probability density function is of the form

$$f(x)=\left(\frac{a_0}{x^2}+\frac{a_1}{x^3}+\frac{a_2}{x^4}+\cdots+\frac{a_p}{x^{p+2}}\right)e^{-\left(\frac{a_0}{x}+\frac{a_1}{2x^2}+\frac{a_2}{3x^3}+\cdots+\frac{a_p}{(p+1)x^{p+1}}\right)}, \tag{2.1}$$

$a_p > 0, x > 0$ and p is some positive integer.

The *cdf* corresponding to (2.1) is given by

$$F(x)=e^{-\left(\frac{a_0}{x}+\frac{a_1}{2x^2}+\frac{a_2}{3x^3}+\cdots+\frac{a_p}{(p+1)x^{p+1}}\right)}. \tag{2.2}$$

It can be seen that

$$f(x)=\sum_{i=0}^{p}\frac{a_i}{x^{i+2}}F(x). \tag{2.3}$$

It should be noted that if Y follows a p^{th} order exponential distribution, then $X = 1/Y$ follows the inverse p^{th} order exponential distribution defined by the *pdf* (2.1). Thus (2.1) is a generalized class of models which includes inverse exponential, inverse Rayleigh, inverse Weibull distributions and so on. Hence any result generated to this generalized class of distribution provides a unified like results which are being enjoyed by a very large class of distributions.

Now, we derive recurrence relations for single and product moments of lgos arising from inverse p^{th} order exponential distribution with *pdf* (2.1). The single moment $\mu_{r,n,m,k}^{(j)} = E[X'^{j}(r,n,m,k)],\ j = 1,2,....$ of lower gos arising from an arbitrary continuous distribution with distribution function $F(x)$ and pdf $f(x)$ is given by

$$\mu_{r,n,m,k}^{(j)} = \frac{C_{r-1}}{(r-1)!}\int x^{j}\left[F(x)\right]^{\gamma_r - 1} f(x)\left[g_m(F(x))\right]^{r-1} dx, \tag{2.4}$$

and the product moment

$\mu_{r,s,n,m,k}^{(j,l)} = E[X'^{j}(r,n,m,k)Y'^{l}(s,n,m,k)],\ j = 1,2,....$ is given by

$$\mu_{r,s,n,m,k}^{(j,l)} = \frac{C_{s-1}}{(r-1)!(s-r-1)!}\iint x^{j} y^{l}[F(x)]^{m}[g_m(F(x))]^{r-1} \times[h_m(F(y)) - h_m(F(x))]^{s-r-1}[F(y)]^{\gamma_s - 1} f(x)f(y)dydx. \tag{2.5}$$

Case (i): $m_1 = m_2 = ... = m_{n-1} = m$

For case (i), we have the following theorem for single moments.

Theorem 2.1: Suppose X follows inverse p^{th} order exponential distribution with pdf (2.1), then

$$\mu_{r,n,m,k}^{(j)} = \gamma_{\mathrm{r}} \sum_{\mathrm{i}=0}^{\mathrm{p}} \frac{\mathrm{a_i}}{(\mathrm{j-i-1})}\left\{\mu_{r-1,n,m,k}^{(j-i-1)} - \mu_{r,n,m,k}^{(j-i-1)}\right\} \tag{2.6}$$

for $j = 1,2,...$ and $r \geq 2$.

Proof : From (2.4) and (2.3) we have,

$$\mu_{r,n,m,k}^{(j)} = \frac{C_{r-1}}{(r-1)!}\sum_{i=0}^{p} a_i \int_0^{\infty} x^{j-i-2}\left[F(x)\right]^{\gamma_r}\left[g_m(F(x))\right]^{r-1} dx.$$

Now integrating by parts the right hand side of the above equation, treating x^{j-i-2} for integration and the rest of the integrand for differentiation we get

$$\begin{aligned}\mu_{r,n,m,k}^{(j)} = {} & \frac{C_{r-1}}{(r-2)!}\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\int_0^{\infty} x^{j-i-1}\left[F(x)\right]^{\gamma_r + m}\left[g_m(F(x))\right]^{r-2} f(x)dx \\ & - \gamma_r \frac{C_{r-1}}{(r-1)!}\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\int_0^{\infty} x^{j-i-1}\left[F(x)\right]^{\gamma_r - 1}\left[g_m(F(x))\right]^{r-1} f(x)dx.\end{aligned} \tag{2.7}$$

On further simplification of (2.7) we get relation (2.6).

Remark 2.1: On putting $m=-1$, $k \geq 1$, in (2.6) we obtain the recurrence relation for single moments of the $k-$lower record values from inverse p^{th} order exponential distribution as follows:

$$\mu_{r;k}^{(j)} = k\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\left\{\mu_{r-1;k}^{(j-i-1)} - \mu_{r;k}^{(j-i-1)}\right\}. \tag{2.8}$$

On putting $k=1$ in (2.8), we get the result for the usual lower record values.

Remark 2.2: On putting $m=0$, $k=1$ in (2.6) we obtain the recurrence relation for single moments of order statistics from inverse p^{th} order exponential distribution as follows:

$$\mu_{n-r+1:n}^{(j)} = (n-r+1)\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\left\{\mu_{n-r+2:n}^{(j-i-1)} - \mu_{n-r+1:n}^{(j-i-1)}\right\} \tag{2.9}$$

Remark 2.3: By putting $a_i = 0$, $i \geq 2$ in (2.6) the result for inverse linear exponential distribution can be deduced.

We now establish the following theorem on the recurrence relation for the product moments of lower gos.

Theorem 2.2: Suppose X follows inverse p^{th} order exponential distribution with pdf (2.1). Then for $1 \leq r < s \leq n$,

$$\mu_{r,s,n,m,k}^{(j,l)} = \gamma_s\sum_{i=0}^{p}\frac{a_i}{(l-i-1)}\left\{\mu_{r,s-1,n,m,k}^{(j,l-i-1)} - \mu_{r,s,n,m,k}^{(j,l-i-1)}\right\} \tag{2.10}$$

for $j,l = 0,1,2,\ldots$. Also we have,

$$\mu_{r,r+1,n,m,k}^{(j,l)} = \gamma_{r+1}\sum_{i=0}^{p}\frac{a_i}{(l-i-1)}\left\{\mu_{r,n,m,k}^{(j+l-i-1)} - \mu_{r,r+1,n,m,k}^{(j,l-i-1)}\right\} \tag{2.11}$$

Proof: From (2.5), for $1 \leq r < s \leq n$ we obtain

$$\mu_{r,s,n,m,k}^{(j,l)} = \frac{C_{s-1}}{(r-1)!(s-r-1)!}\int_x x^j\left[F(x)\right]^m\left[g_m(F(x))\right]^{r-1} f(x)I(x)dx \tag{2.12}$$

where,

$$I(x) = \int_0^x y^l[h_m(F(y)) - h_m(F(x))]^{s-r-1}[F(y)]^{\gamma_s-1} f(y)dy. \tag{2.13}$$

Using (2.3) in (2.13) we get

$$I(x) = \sum_{i=0}^{p} a_i \int_0^x y^{l-i-2} [h_m(F(y)) - h_m(F(x))]^{s-r-1} [F(y)]^{\gamma_s} dy. \tag{2.14}$$

Integrating (2.14) by parts, treating y^{1-i-2} for integration and the rest of the integrand for differentiation, we get for $s > r+1$,

$$\begin{aligned} I(x) = {} & (s-r-1) \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \int_0^x y^{l-i-1} [h_m(F(y)) - h_m(F(x))]^{s-r-2} [F(y)]^{\gamma_s+m} f(y) dy \\ & - \gamma_s \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \int_0^x y^{l-i-1} [h_m(F(y)) - h_m(F(x))]^{s-r-1} [F(y)]^{\gamma_s-1} f(y) dy. \end{aligned} \tag{2.15}$$

Substituting (2.15) in (2.12) we get

$$\begin{aligned} \mu_{r,s,n,m,k}^{(j,l)} = {} & \frac{C_{s-1}}{(r-1)!(s-r-2)!} \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \int_0^\infty \int_0^x x^j y^{l-i-1} [g_m(F(x))]^{r-1} \\ & \times [h_m(F(y)) - h_m(F(x))]^{s-r-2} [F(x)]^m [F(y)]^{\gamma_{s-1}-1} f(x) f(y) dy dx \\ & - \frac{C_s}{(r-1)!(s-r-1)!} \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \int_0^\infty \int_0^x x^j y^{l-i-1} [g_m(F(x))]^{r-1} \\ & \times [h_m(F(y)) - h_m(F(x))]^{s-r-1} [F(x)]^m [F(y)]^{\gamma_s-1} f(x) f(y) dy dx \end{aligned} \tag{2.16}$$

which on further simplification leads to (2.10).

Further, for $s = r+1$, we have

$$I(x) = \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \left[x^{l-i-1} [F(y)]^{\gamma_{r+1}} - \gamma_{r+1} \int_0^\infty y^{l-i-1} [F(y)]^{\gamma_{r+1}-1} f(y) dy \right] \tag{2.17}$$

Substituting (2.17) in (2.12) and on further simplification we get (2.11).

Remark 2.4: Theorem 2.2 reduces to the result of single moments established in theorem 2.1 at $j = 0$.

Remark 2.5: On putting $m = -1, k \geq 1$, in (2.10) and (2.11) we obtain the recurrence relation for product moments of the k-lower record values arising from inverse p^{th} order exponential distribution.

For $k=1$, we get the relation for product moments of classical lower record values arising from inverse p^{th} order exponential distribution.

Remark 2.6: Putting $m=0, k=1$, in (2.10), the recurrence relation for product moments of order statistics from inverse p^{th} order exponential distribution is obtained as follows:

$$\mu_{n-r+1,n-s+1:n}^{(j,l)} = (n-s+1)\sum_{i=0}^{p}\frac{a_i}{(l-i-1)}\left\{\mu_{n-r+1,n-s+2:n}^{(j,l-i-1)} - \mu_{n-r+1,n-s+1:n}^{(j,l-i-1)}\right\} \tag{2.18}$$

Remark 2.7: By putting $a_i = 0, i \geq 2$ in (2.10) and (2.11) we get the recurrence relation for product moments of lower gos from inverse linear exponential distribution.

Case (ii): $\gamma_i \neq \gamma_j, i \neq j, i, j = 1,2,\ldots,n-1$.

Now we establish the following theorems based on recurrence relations for single and product moments arising from inverse p^{th} order exponential distribution.

Theorem 2.3: For the distribution (2.2) and for $r \geq 2, j = 1,2,\ldots$

$$\mu_{r,n,\tilde{m},k}^{(j)} = -\sum_{i=0}^{p}\frac{a_i \gamma_u}{(j-i-1)}\mu_{r,n,\tilde{m},k}^{(j-i-1)}. \tag{2.19}$$

Proof: From (1.5) and (2.3) we have

$$\mu_{r,n,\tilde{m},k}^{(j)} = C_{r-1}\sum_{u=1}^{r} a_u(r)\sum_{i=0}^{p} a_i I(x), \tag{2.20}$$

where,

$$I(x) = \int_0^\infty x^{j-i-2}[F(x)]^{\gamma_u}\,dx. \tag{2.21}$$

Integrating (2.21) by parts, treating x^{j-i-2} for integration and the rest of the integrand for differentiation we get,

$$I(x) = -\frac{\gamma_u}{(j-i-1)}\int_0^\infty x^{j-i-1}[F(x)]^{\gamma_u - 1} f(x)\,dx.$$

Now, substituting $I(x)$ in (2.20) we get the result.

Remark 2.8: Theorem 2.1 can be deduced from theorem 2.3 by replacing $\tilde{m}$ with $m, m \neq -1$.

Theorem 2.4: For the inverse p^{th} order exponential distribution in (2.2) and for $1 \leq r < s \leq n$, $k = 1,2,\ldots$

$$\mu_{r,s,n,\tilde{m},k}^{(j,l)} = \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \left\{ \mu_{r,n,\tilde{m},k}^{(j+l-i-1)} - \gamma_u (\mu_{r,s,n,\tilde{m},k}^{(j,l-i-1)}) \right\}.$$

(2.22)

Proof: From (1.6) we have

$$\mu_{r,s,n,\tilde{m},k}^{(j,l)} = C_{s-1} \int_0^{\infty} x^j \sum_{u=r+1}^{s} a_u^{(r)}(s) \left[\frac{1}{F(x)} \right]^{\gamma_u} \left[\sum_{u=1}^{r} a_u(r) [F(x)]^{\gamma_u} \right] \frac{f(x)}{F(x)} I(x) dx. \tag{2.23}$$

where,

$$I(x) = \int_0^x y^l [F(y)]^{\gamma_u} \frac{f(y)}{F(y)} dy.$$

Using (2.3) we have,

$$I(x) = \sum_{i=0}^{p} a_i \int_0^x y^{l-i-2} [F(y)]^{\gamma_u} dy. \tag{2.24}$$

Integrating (2.24) by parts, treating y^{1-i-2} for integration and the rest of the integrand for differentiation we get,

$$I(x) = \sum_{i=0}^{p} \frac{a_i}{(l-i-1)} \left[x^{l-i-1} [F(x)]^{\gamma_u} - \gamma_u \int_0^x y^{l-i-1} [F(y)]^{\gamma_u - 1} f(y) dy \right]. \tag{2.25}$$

Substituting (2.25) in (2.23) and on further simplification we get the relation (2.22).

Remark 2.9: By replacing $\tilde{m}$ with m, we can deduce theorem 2.2 from theorem 2.4.

3. CHARACTERIZATION OF DISTRIBUTION USING PROPERTIES OF LOWER GENERALIZED ORDER STATISTICS

Now we consider the problem of characterization of inverse p^{th} order exponential distribution using the relation in theorem 2.1. For this we require the following result of Hwang and Lin (1984).

Proposition 3.1: Let $f(x)$ be a function absolutely continuous on (a,b) with $f(a)f(b)\geq 0,$ and let its derivative satisfy $f'(x)\neq 0$ *a.e.* on (a,b). Then under the assumption $\sum_{j=1}^{+\infty} n_j^{-1}=+\infty$ where $0<n_1<n_2<....$ the sequence $\{f^{n_j}(x), j\geq 1\}$ is complete on (a,b) if and only if the function $f(x)$ is monotone on (a,b).

Theorem 3.1: Let $X'(1,n,m,k), X'(2,n,m,k),..., X'(n,n,m,k)$ be n lower *gos* arising from an absolutely continuous distribution function $F(x)$. Then for $r, j\geq 1$ the following recurrence relation

$$\mu_{r,n,m,k}^{(j)}=\gamma_r\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\left[\mu_{r-1,n,m,k}^{(j-i-1)}-\mu_{r,n,m,k}^{(j-i-1)}\right] \tag{3.1}$$

is satisfied if and only if X follows inverse p^{th} order exponential distribution with *pdf* (2.1).

Proof: The necessary part follows immediately from theorem 2.1.

Conversely, if the recurrence relation (3.1) is satisfied, then using (2.4) we have,

$$\begin{aligned}\frac{C_{r-1}}{(r-1)!}\int_0^\infty x^j[F(x)]^{\gamma_r-1}f(x)[g_m(F(x))]^{r-1}dx\\ =\gamma_r\frac{C_{r-2}}{(r-2)!}\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\int_0^\infty x^{j-i-1}[F(x)]^{\gamma_{r-1}-1}f(x)[g_m(F(x))]^{r-2}dx\\ -\gamma_r\frac{C_{r-1}}{(r-1)!}\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\int_0^\infty x^{j-i-1}[F(x)]^{\gamma_r-1}f(x)[g_m(F(x))]^{r-1}dx.\end{aligned} \tag{3.2}$$

Now, integrating by parts the second term on the RHS of (3.2) we get

$$\frac{C_{r-1}}{(r-1)!}\int_0^\infty x^j[F(x)]^{\gamma_r-1}f(x)[g_m(F(x))]^{r-1}\left\{f(x)-\left(\sum_{i=0}^{p}\frac{a_i}{x^{i+2}}\right)F(x)\right\}dx=0 \tag{3.3}$$

Then from proposition 3.1 it follows that

$$f(x)=\left(\sum_{i=0}^{p}\frac{a_i}{x^{i+2}}\right)F(x)$$

and consequently it follows from (2.3) that f(x) has the form (2.1).

Remark 3.1: For $m=-1, k\geq 1$ the following recurrence relation for single moments of the $k-$th lower record values

$$\mu_{r;k}^{(j)}=k\sum_{i=0}^{p}\frac{a_i}{(j-i-1)}\left\{\mu_{r-1;k}^{(j-i-1)}-\mu_{r;k}^{(j-i-1)}\right\} \tag{3.4}$$

becomes a characterization property of inverse p^{th} order exponential distribution.

Acknowledgements

The authors acknowledge their gratefulness to the learned referee for many of his constructive comments on an earlier version of this paper.

REFERENCES

Ahsanullah, M. (2004): A characterization of the uniform distribution by dual generalized order statistics. *Comm. Statist. Theory Methods*, **33**, 2921-2928.

Burkschat, M., Cramer, E., Kamps, U. (2003): Dual generalized order statistics. *Metron,* **LXI**, 13-26.

Cramer, E., Kamps, U. (2000): Relations for expectations of functions of generalized order statistics. *J. Statist. Plann. Inference,* **89**, 79-89.

Hwang, J. S., Lin, G. D. (1984): On a generalized moment problem II. *Proceedings of the American Mathematical Society,* **91**, 577-580.

Kamps, U. (1995): A concept of generalized order statistics. Teubner, Stuttgart.

Kamps, U., Cramer, E. (2001): On distributions of generalized order statistics. *Statistics,* **35**, 269-280.

Kamps, U., Gather, U. (1997): Characteristic property of generalized order statistics for exponential distributions. *Appl. Math. (Warsaw).,* **24**, 383-391.

Keseling, C. (1999): Conditional distributions of generalized order statistics and some characterizations. *Metrika*, **49**, 27-40.

Khan, R. U., Anwar, Z., Athar, H. (2008): Recurrence relations for single and product moments of generalized order statistics from exponentiated Weibull distribution. *Aligarh J. Statist.,* **28**, 37-45.

Nain, K. (2010): Recurrence relations for single and product moments of ordinary order statistics from p^{th} order exponential distribution. *Int. Math. Forum,* **5,** 1653-1662.

Pawlas, P., Szynal, D. (2001a): Recurrence relations for single and product moment of generalized order statistics from Pareto, generalized Pareto and Burr distributions. *Comm. Statist. Theory Methods,* **30**, 739-746.

Pawlas, P., Szynal, D. (2001b): Recurrence relations for single and product moment of lower generalized order statistics from the inverse Weibull distribution. *Demonstratio Math.,* **XXXIV**, 353-358.

Received : 16-01-2012

Revised : 06.06.2012

Minimol, S. and P.Yageen Thomas
Department of Statistics
University of Kerala, Trivandrum
E mail: minibhama@rediffmail.com
yageenthomas@gmail.com