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A LEXICOGRAPHIC GOAL PROGRAMMING APPROACH FOR A COMPROMISE ALLOCATION OF REPAIRABLE COMPONENTS

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ABSTRACT

In this paper, we have formulated the problem of allocation of repairable components within subsystems of a series-parallel system as a multi-objective non linear programming problem. A solution procedure using D_1-distances in the lexicographic goal programming approach is proposed. The solution corresponding to the minimum D_1-distances gives the best compromise solution. A numerical example is given to illustrate the procedure.

1. INTRODUCTION

We consider a system which requires performing a sequence of identical production runs after every given (fixed) period. A production run in the system consists of several subsystems where each subsystem can work properly if at least one of its components is operational. The following assumptions are also made:

(I) All the components can be repaired if deteriorated or failed

(II) All component states are independent

(III) Reliability of each component in a subsystem is same

(IV) Cost spent and time taken on repairing each component within a subsystem are same

The reliability of a system can be increased by proper selection of the number of repairable components to be repaired from its subsystems. The system is virtually repaired under the limitations on some parameters such as cost spent and time taken in repairing the failed components. A repairable system is a system in which the failed or deteriorated components can be repaired to operate normally. Some examples of repairable systems are a computer network, a manufacturing system, a power plant or a fire prevention system. A number of military and industrial organizations also depend upon the efficient utilization of repairable systems (vehicles, machinery, computers, etc.) for the successful operation of their organization. Many authors have discussed the allocation problem of repairable components. Among them are Rice *et al.* (1998), Cassady *et al.* (2001a, 2001b), Schneider and Cassady (2004), Rajaopalan and Cassady (2006), Iyoob *et al.* (2006), Schneider et al. (2009), Ali *et al.* (2011a, 2011b) and many others. Generally, the authors have considered a single criterion for the objective function in allocation problem of repairable components. However, in many real-life cases multiple criteria are required to be considered for determining an optimal policy. For example, the success of designing a four wheeler is determined by such parameters as its cost (to be minimized), reliability (to be maximized), energy consumption (to be minimized), weight (to be minimized), volume (to be minimized) and speed (to be maximized) etc. Some authors have discussed the multi-objective optimization formulations such as Busacca *et al.* (2001), Fu and Diwekar (2004), Panda *et al.* (2005), Wang *et al.* (2009), Ali *et al.* (2011c, 2011d).

In this paper, we have formulated the problem of determining the number of repairable components in a system as a multi-objective non linear programming problem (NLPP) which is solved by lexicographic goal programming technique with "Minimum D_1-distances". Software package lingo is used for solving the NLPP.

2. ALLOCATION PROBLEM OF REPAIRABLE COMPONENTS IN A PARALLEL–SERIES SYSTEM

Consider a system in which m subsystems are connected in series, the $i-th$ subsystem consisting of n_i components connected in parallel ($i=1,\cdots,m$). Let r_i be the reliability of each component in the $i-th$ subsystem.

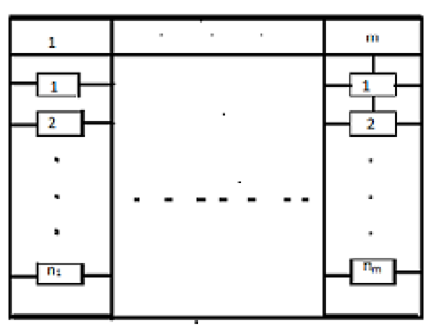

Fig. 1 Parallel-Series System

In fig.1 the system for a fixed period is a series arrangement of the subsystems (subsystem1, subsystem 2 subsystem m), its reliability can be defined as

$$R = \prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i} \right\} \tag{2.1}$$

At the completion of a particular production run, each component in a subsystem is either functioning or failed. Ideally all the failed components in the subsystems should be repaired and then replaced back prior to the beginning of the next production run. However, due to the constraints on the time and cost, it may not be possible to repair all the failed components in the subsystems. Let a_i be the total number of failed components in $i-th$ subsystem.

The time required for repairing and then replacing back all the failed components in the system is given by

$$T = \sum_{i=1}^{m} t_i \left[a_i + \exp(\theta_i a_i) \right] \tag{2.2}$$

where t_i is the time required to repair a component in $i-th$ subsystem and $\exp(\theta_i\, a_i)$ is the additional time spent due to the interconnection between parallel components (Wang *et al.* (2009)).

The maintenance time available for repairing and then replacing back the failed components between two production runs is T_0 units. if $T_0 < T$, then all failed components can not be repaired and then replaced back prior to beginning of the next production run.

The cost required for replacing the failed components after repairs in the system is given by

$$C = \sum_{i=1}^{m} c_i \left[a_i + \exp(\beta_i a_i) \right] \tag{2.3}$$

where $\exp(\beta_i\, a_i)$ is the additional cost spent due to the interconnection between parallel components (Wang *et al.* (2009)).

The maintenance cost available for repairing and then replacing back the failed components between two production runs is C_0 units. If $C_0 < C$, then all failed components can not be repaired and replaced back prior to beginning of the next production run.

In such cases, a method is needed to decide how many failed components should be repaired and replaced back prior to the next production run and the rest be left in a failed condition. This process is referred to as selective maintenance (see Rice *et al.* (1998)). In the selective maintenance the number of components available for the next production run in the $i-th$ subsystem will be

$$(n_i - a_i) + d_i\,, \qquad i = 1, 2, ..., m \tag{2.4}$$

where d_i is the number of repaired components in subsystem i prior to the next production run respectively and n_i is the total number of components available in parallel in the $i-th$ subsystem. We have assumed that the repair time and repair cost of each failed components in a subsystem are same. The reliability of the system for a production run is given by

$$R = \prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - a_i + d_i} \right\} \tag{2.5}$$

The repair time constraint for the system is given as

$$\sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right] \le T_0 \tag{2.6}$$

and the repair cost constraint for the system is given as

$$\sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i) \right] \le C_0 \tag{2.7}$$

A single objective selection problem of repairable component is to find the number of repairable components d_i for $i-th$ subsystem to meet the time and cost constraints (2.6) and (2.7) while the reliability (2.5) of the system is maximized

$$\left.\begin{aligned} P_1: \quad & \text{Max. } R = \prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - a_i + d_i} \right\} \\ & \text{subject to} \\ & \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right] \le T_0 \\ & \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i\, d_i) \right] \le C_0 \\ & \text{and} \quad 0 \le d_i \le a_i\,, i = 1, 2, ..., m \text{ and integers} \end{aligned}\right\} \tag{2.8}$$

The other formulations of single objective allocation problems are:

$$P_2: \quad \text{Min.} C = \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i) \right]$$

subject to

$$\prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - a_i + d_i} \right\} \geq R$$

$$\sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right] \leq T_0$$

and $0 \leq d_i \leq a_i, i = 1,2,..., m$ and integers (2.9)

and

$$P_3: \quad \text{Min.} T = \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right]$$

subject to

$$\prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - a_i + d_i} \right\} \geq R$$

$$\sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i) \right] \leq C_0$$

and $0 \leq d_i \leq a_i, i = 1,2,..., m$ and integers (2.10)

3. SOLUTION USING LEXICOGRAPHIC GOAL PROGRAMMING APPROACH

The problems P_1, P_2 and P_3 are the maintenance allocation problems with single objectives. However, in the event when the reliability of the system, the available repair cost and the available repair time are all of equally serious concern, we may apply the following lexicographic goal programming approach. Let us consider for instance the following multi-objective problem:

$$\left.\begin{array}{ll}
Min\, C = \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i)\right] & (i) \\
and & \\
Min T = \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] & (ii) \\
\text{subject to} & \\
\prod_{i=1}^{m} 1 - (1 - r_i)^{n_i - a_i + d_i} \geq R^* & (iii) \\
\text{and } \; 0 \leq d_i \leq a_i, i = 1,2,..., m \text{ and integers} & (iv)
\end{array}\right\} \quad (3.1)$$

In the problem (3.1), let us first consider the repairing cost as more important than the repairing time. Then we solve the problem (3.1) by minimizing (*i*) subject to (*iii*) and (*iv*) (*i.e.* We neglect the objective (*ii*)).

Let the minimum of the NLPP (3.1), while neglecting the second objective be C_0^*. Next we solve the following NLPP:

$$\left.\begin{array}{l}
\text{Min } T = \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] + \delta_1 \\
\text{subject to} \\
\sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i)\right] - \delta_1 \leq C_0^* \\
\prod_{i=1}^{m} \left\{1 - (1 - r_i)^{n_i - a_i + d_i}\right\} \geq R^* \\
\delta_1 \geq 0 \\
\text{and } \quad 0 \leq d_i \leq a_i, i = 1,2,..., m \text{ and integers}
\end{array}\right\} \quad (3.2)$$

Where δ_1 is the deviational variable.

By solving the NLPP (3.2) let the optimum repairing time obtained be T_0^*.

The following lexicographic goal programming problem is then solved:

$$\left.\begin{array}{l} Min\ \delta = \sum_{i=1}^{2} \delta_i \\ \\ \text{subject to} \\ \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i)\right] - \delta_1 \le C_0^* \\ \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] - \delta_2 \le T_0^* \\ \prod_{i=1}^{m} \left\{1 - (1 - r_i)^{n_i - a_i + d_i}\right\} \ge R^* \\ \qquad \delta_1,\ \delta_2 \ge 0 \\ \text{and} \quad 0 \le d_i \le a_i ,\, i = 1,2,...,m \text{ and integers} \end{array}\right\} \quad (3.3)$$

Let the solution of the NLPP (3.3) obtained be $(d_1^{(1)}, \ldots, d_m^{(1)})$.

Next we assume that the repairing time is more important than the repairing cost. Then we solve the NLPP (3.1) by considering the time objective and neglecting the cost objective. Let the minimum so obtained be T_0^*. In the next step solve the following NLPP for optimum repairing cost

$$\left.\begin{array}{l} \text{Min}\ C = \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i)\right] + \delta_1 \\ \text{subject to} \\ \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] - \delta_1 \le T_0^* \\ \prod_{i=1}^{m} \left\{1 - (1 - r_i)^{n_i - a_i + d_i}\right\} \ge R^* \\ \qquad \delta_1 \ge 0 \\ \text{and} \quad 0 \le d_i \le a_i ,\, i = 1,2,...,m \text{ and integers} \end{array}\right\} \quad (3.4)$$

Let the minimum cost obtain be C_0^*.

The following lexicographic goal programming problem is then solved:

$$\left.\begin{array}{l} Min\ \delta = \sum_{i=1}^{2} \delta_i \\ \text{subject to} \\ \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] - \delta_1 \le T_0^* \\ \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i)\right] - \delta_2 \le C_0^* \\ \prod_{i=1}^{m} \left\{1 - (1 - r_i)^{n_i - a_i + d_i}\right\} \ge R^* \\ \delta_1,\ \delta_2 \ge 0 \\ \text{and} \quad 0 \le d_i \le a_i\ , i = 1,2,...,\ m \text{ and integers} \end{array}\right\} \quad (3.5)$$

Let the solution of the NLPP (3.5) obtained be $(d_1^{(2)}, \ldots, d_m^{(2)})$.

In this way the priorities are given to the objectives one after the other and a set of solutions is obtained. Out of these solutions, an ideal solution is identified as follows:
$d_i^* = \left\{\max(d_1^{(1)}, d_1^{(2)}), \max(d_2^{(1)}, d_2^{(2)}),..., \max(d_m^{(1)}, n_m^{(2)})\right\} = \left\{d_1^*, d_2^*,..., d_m^*\right\}$, say.

The D_1 –distances of different solutions from the ideal solution defined in (3.7) below are then calculated. The solution corresponding to the minimum D_1 –distance gives the best compromise solution.

A general procedure with *p* objectives is the following. As explained above, we will obtain $P!$ (factorial) different solutions by solving the $P!$ problems arising for $P!$ different priority structures.

Let $d_i^{(r)} = \left\{d_1^{(r)}, d_2^{(r)},..., d_m^{(r)}\right\}, 1 \le r \le P!$ be the $P!$ number of solutions obtained by giving priorities to P objective functions.

Let $(d_1^*, d_2^*,..., d_m^*)$ be the ideal solution. But in practice ideal solution can never be achieved. The solution, which is closest to the ideal solution, is acceptable as the best compromise solution, and the corresponding priority structure is identified as most appropriate priority structure in the planning context. To obtain the best compromise solution, following goal programming problem is to be solved.

$$\left.\begin{aligned}
&\underset{1\le r\le P!}{Min}\ \sum_{i=1}^{m}\varepsilon_{ir}\\
&\text{subject to}\\
&d_i^* - d_i^{(r)} - \varepsilon_{ir} = 0\\
&and\ \ \varepsilon_{ir} \ge 0,\ 1 \le r \le P\,!\\
&0 \le d_i^r \le a,\ i = 1,\ldots,m \text{ and integer}
\end{aligned}\right\} \tag{3.6}$$

Where ε_{ir} are the deviational variables.

Now,

$$(D_1)^r = \sum_{i=1}^{m}\left|d_i^* - d_i^{(r)}\right| \tag{3.7}$$

is defined as the D_1 – distance from the ideal solution $\{d_1^*, d_2^*, ..., d_m^*\}$, to the $r-th$ solution $\{d_1^{(r)}, d_2^{(r)}, ..., d_m^{(r)}\}$, $1 \le r \le P!$

Therefore,

$$(D_1)_{opt} = \underset{1\le r\le P!}{Min}(D_1)^r$$

$$(D_1)_{opt} = \underset{1\le r\le P!}{Min}\sum_{i=1}^{m}\left|d_i^* - d_i^{(r)}\right| \tag{3.8}$$

$$= \underset{1\le r\le P!}{Min}\sum_{i=1}^{m}\varepsilon_{ir} \tag{3.9}$$

Let the minimum be attained for $r = p$

Then

$\{d_1^{(p)}, d_2^{(p)}, ..., d_m^{(p)}\}$ is the best compromise solution of the problem.

4. NUMERICAL ILLUSTRATION

Consider a system consisting of 5 subsystems. The reliability requirement for the given system is 0.99. The other parameters for the various subsystems are given in table 4.1.

Table 4.1: The parameters for the numerical example

Subsystems	**1**	**2**	**3**	**4**	**5**
n_i	4	6	8	5	7
a_i	2	4	6	3	5
r_i	0.9	0.75	0.65	0.70	0.75
c_i	12	7	6	8	9
t_i	3	4	7	6	5
θ_i	0.10	0.10	0.10	0.10	0.10
β_i	0.15	0.15	0.15	0.15	0.15

Solution by using lexicographic goal programming approach

The NLPP (3.1) is solved by giving priority to the cost objective using the values given in table 4.1. The optimal values obtained from (3.1) and (3.2) are $C_0^* = 159.40$ and $T_0^* = 115.74$. Next we solve the following NLPP corresponding to (3.3):

$$\left.\begin{aligned}
&\textit{Minimize} \quad \delta_1 + \delta_2 \\
&\textit{subject to} \\
&\sum_{i=1}^{m} c_i\left[d_i + \exp(\beta_i d_i)\right] - \delta_1 = 159.40 \\
&\sum_{i=1}^{m} t_i\left[d_i + \exp(\theta_i\, d_i)\right] - \delta_2 = 115.74 \\
&\prod_{i=1}^{m} 1 - (1 - r_i)^{n_i - a_i + d_i} \geq 0.99 \\
&\qquad \delta_1,\ \delta_2 \geq 0 \\
&\text{and} \quad 0 \leq d_i \leq a_i\,,\ i = 1,2,\ldots,m \text{ and integers}
\end{aligned}\right\} \qquad (4.1)$$

The solution to the NLPP (4.1) using software Lingo is

$$d_1^* = 1,\ d_2^* = 3,\ d_3^* = 4,\ d_4^* = 3,\ d_5^* = 3 \text{ with } \delta_1 = 3.20 \text{ and } \delta_2 = 0.025$$

In similar manner, NLPP (3.1) is solved by giving priority to the time objective and we obtain the values $T_0^* = 110.55$ and $C_0^* = 164.60$. Next solve the following NLPP corresponding to (3.5):

$$\left.\begin{array}{l} \textit{Minimize} \quad \delta_1 + \delta_2 \\ \textit{subject to} \\ \sum_{i=1}^{m} t_i\left[d_i + \exp(\theta_i d_i)\right] - \delta_1 = 110.55 \\ \sum_{i=1}^{m} c_i\left[d_i + \exp(\beta_i d_i)\right] - \delta_2 = 164.60 \\ \prod_{i=1}^{m} 1 - (1 - r_i)^{n_i - a_i + d_i} \geq 0.99 \\ \delta_1,\ \delta_2 \geq 0 \\ \text{and} \quad 0 \leq d_i \leq a_i,\ i = 1,2,\ldots,m \text{ and integers} \end{array}\right\} \quad (4.2)$$

The optimum allocation which is the solution to the NLPP (4.2) is

$$d_1^* = 2,\ d_2^* = 3,\ d_3^* = 4,\ d_4^* = 3,\ d_5^* = 2 \text{ with } \delta_1 = 2.68 \text{ and } \delta_2 = 1.25$$

Table 4.2: (solutions)

Run	Priorities	d_1	d_2	d_3	d_4	d_5
	$C^{(1)}, T^{(2)}$	1	**3**	**4**	**3**	**3**
	$T^{(1)}, C^{(2)}$	**2**	**3**	**4**	**3**	2
Ideal solution (d_i^*)		2	3	4	3	3

The symbols C and T correspond to the system repairing cost and repairing time.

Table 4.3: the D_1 – distance from the ideal solutions

Priority to	d_1	d_2	d_3	d_4	d_5	$(D_1)^r$
Repair Cost	1	0	0	0	0	**1**
Repair Time	0	0	0	0	1	**1**

In table 4.3, the D_1 – distances of all possible solutions from the ideal solution are calculated. From table 4.3, it is clear that the minimum of the D_1 – distances of the two priority structure solutions from the ideal solutions are equal to 1. Therefore the tie occurs. Thus, we may choose any one of the two priority structures.

5. CONCLUSION

This paper has suggested a compromise allocation of repairable components for a parallel series system. We propose the goal programming technique with "minimum D_1 – distances" for finding a compromise allocation of repairable components. The solution which is corresponding to minimum – D_1 distance is the best compromise solution. In the numerical example solved a tie occurs.

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