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A CAPACITATED STOCHASTIC LINEAR TRANSSHIPMENT PROBLEM

Shakeel Javaid and M.M. Khalid

ABSTRACT

The present paper analyses a transshipment problem under uncertain demand with upper bounds restrictions on some or all routes. The objective is to maximize the net expected revenue, i.e., the total expected revenue minus the transportation and transshipment costs. The stochastic transshipment problem is reduced to an equivalent deterministic transportation problem for which an algorithm is developed and numerically illustrated.

1. INTRODUCTION

In a transportation problem, shipments are allowed only between source-sink pairs. In many applications, this assumption is too strong. For example, it is often the case that shipments may be allowed between sources and between sinks. Moreover, there may also exist points through which units of a product can be transshipped from a source to a sink. Models with these additional features are called transshipment problems. Interestingly, it turns out that any given transshipment problem can be converted easily into an equivalent transportation problem. The availability of such a conversion procedure significantly broadens the applicability of our algorithm for solving transportation problems. In this paper, the transshipment problem [Orden (1956)] is considered, treating the demands as a discrete random variable. The purpose of this paper is to study the stochastic transshipment problem (STSP) with upper bounds under prohibited routes. For dealing with uncertainty of demands, we have used the technique by Ferguson and Dantzig (1956) and Garvin (1963). An algorithm is developed in which additional upper bounds on the route capacities are imposed; the upper bound represents the upper limits on the amount that can be shipped over any given route

2. NOTATIONS AND THE FORMULATION OF THE PROBLEM

We consider a transshipment problem with m sources and n sinks numbered as $1,2,\cdots,m$ and n sinks numbered as $m+1,m+2,\cdots,m+n$.

Let

a_i : the quantity available at source $i = 1,2,\cdots,m$

b_j : the quantity demanded at sink $j = m+1, m+2,\cdots,m+n$

x_{ij} : the quantity shipped from station i to j $(i, j = 1,2,\cdots,m+n)$

c_{ij} : the per unit shipment cost from station i to j $(i, j = 1,2,\cdots,m+n)$

u_i : quantity transshipped at the station i $(i = 1,2,\cdots,m+n)$

c_i : per unit transshipment cost (including unloading, reloading, and storage etc.) at the station i $(i = 1,2,\cdots,m+n)$.

The problem is to determine x_{ij} so as to minimize the total cost of transportation and transshipment. It may be mathematically stated as under:

Problem P$_1$: Find x_{ij} so as to

$$\text{Minimize} \quad f = \sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{m+n}{}^{*} c_{ij}x_{ij} + \sum_{i=1}^{m+n} c_i u_i \tag{2.1}$$

subject to

$$\sum_{j=1}^{m+n}{}^{*} x_{ij} = \begin{cases} a_i + u_i & i = 1,2,\cdots,m \\ u_i & i = m+1,\cdots,m+n \end{cases} \tag{2.2}$$

$$\sum_{i=1}^{m+n}{}^{*} x_{ij} = \begin{cases} u_j & j = 1,2,\cdots,m \\ b_j + u_j & j = m+1,\cdots,.m+n \end{cases} \tag{2.3}$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j \tag{2.4}$$

$\sum_{j=1}^{m+n}{}^{*}$ indicates that the term $j = i$ is excluded from the sum. The constraints (2.2) implies that the total quantity that leaves the source i $(=1,2,\cdots,m)$ is equal to the quantity available plus the quantity transshipped and the total quantity that leaves the sink i $(= m+1, m+2,\cdots,m+n)$ is equal to the quantity transshipped. Similarly constraints (2.3) implies that the total quantity that arrives at a source i $(=1,2,\cdots,m)$ is equal to the quantity that source transships and the total arriving at a sink is equal to the demand at that sink plus the quantity that the sink transships. Constraints (2.4) are the usual nonnegative restrictions. Here u_i are unknown, so we impose an upper bound u_0 (say), on the amount that can be transshipped at any point, so that

$$u_i = u_0 - x_{ii}, \quad i = 1,2,\cdots,m, \tag{2.5}$$

where x_{ii} is a nonnegative slack. After substituting (2.5) in (2.1) to (2.3) and on simplifying the original transshipment problem P_1 is reduced to the following genuine transportation type linear programming problem.

Problem P₂:

$$\text{Minimize} \quad f = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij}x_{ij} + \sum_{i=1}^{m+n} c_i u_i$$

subject to

$$\sum_{j=1}^{m+n} x_{ij} = \begin{cases} a_i + u_o & i = 1, 2, \cdots, m \\ u_o & i = m+1, \cdots, m+n \end{cases} \tag{2.6}$$

$$\sum_{i=1}^{m+n} x_{ij} = \begin{cases} u_o & j = 1, 2, \cdots, m \\ d_j + u_o & j = m+1, \cdots, m+n \end{cases} \tag{2.7}$$

$$x_{ij} \geq 0, \tag{2.8}$$

where $c_{ii} = -c_i$, and the asterisk (*) on the summations has now been disappeared. As $u_i \geq 0$, we must have $x_{ii} \leq u_0$, which is guaranteed by equations (2.6) – (2.7), because any x_{ii} will always appear in one equation that has u_0 on the right hand side. Here, the upper bound u_0 can interpreted as the size of a fictitious stockpile at each source and sink which is large enough to take care of all transshipments. Assume initially a value for u_0 which is sufficiently large to ensure that all x_{ii} will be in the optimal basis. Such a value can be easily found as the volume of goods transshipped at any point cannot exceed the total volume of goods produced (or received). Hence, set,

$$u_0 = \sum_{i=1}^{m} a_i \tag{2.9}$$

Which ensures that u_0 is not limiting. The unused stockpile at the station $i = 1, 2, \cdots, m+n$, if any, will be absorbed in the slack x_{ii}.

3. PROBLEM REFORMULATION UNDER STOCHASTIC ENVIRONMENT

So far we have treated the demands b_j as if they are fixed constraints. However, we assume b_j as independent discrete random variables with known probability distributions. To take care of the randomness of demands, instead of minimizing the total cost, we take our objective as the maximization of the net expected revenue, the net expected revenue being defined as total expected revenue minus loss in transshipment. Let s_j be the selling price of the product shipped to sink j. We also introduce the upper bounds d_{ij} that represents the upper limit on the amount that can be shipped over the route (i, j). The objective function can, therefore be written as:

$$\min_{x_{ij}} f = \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} c_{ij}x_{ij} + \sum_{i=1}^{m+n} l_i u_o - \sum_{j=1}^{m+n} f_j(s_j, y_j)$$

$$\text{or } \max_{x_{ij}} Z = \sum_{j=1}^{m+n} f_j(s_j, y_j) - \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} c_{ij}x_{ij} - \sum_{i=1}^{m+n} l_i u_o$$

where $f_j(s_j, y_j)$ is an unknown function that describes the expected revenue from destination j if a total of y_j unit is shipped to this destination.

The third term on the right hand viz. $\sum_{i=1}^{m+n} c_i u_0$ is a constant and therefore can be simply ignored. So our problem now becomes.

Problem P₃: $\max_{x_{ij}} Z = \sum_{j=1}^{m+n} f_j(s_j, y_j) - \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} c_{ij}x_{ij} + \text{constant}$ (3.1)

Subject to constraints (2.6) and (2.7) 3.2)

$$0 \le x_{ij} \le d_{ij} \quad (3.3)$$

4. THE EQUIVALENT DETERMINISTIC PROBLEM

Let the demand b_j's at various destinations be independent random variables and the probability distribution of b_j $(j = 1, 2, \cdots, n)$ be in increasing order as:

Demand b_j	$b_{1j} <$	$b_{2j} <$	$\cdots$	$b_{H_j j}$
$p(b_j = b_{hi}) = p_{hi}$	p_{1j}	p_{2j}	$\cdots$	$p_{H_j j}$
$p(b_j \ge b_{hj})$	$t_{1j} = \sum_{h=1}^{H_j} p_{hj}$	$t_{2j} = \sum_{h=2}^{H_j} p_{hj}$	$\cdots$	$t_{H_j j} = p_{H_j j}$

To determine the function $f_j(s_j, y_j)$ note that y_j, the net quantity shipped to sink j, can be any amount between the lowest value b_{1j} and the highest value $d_{H_j j}$ in the probability distribution of the demand b_j $(j = 1, 2, \cdots, n)$.

If $0 \le y_j \le b_{1j}$, then each of the y_j units shall be absorbed with probability $t_{1j}(=1)$. Hence, the expected revenue is $= s_j t_{1j} y_j$.

If $b_{1j} \le y_j \le b_{2j}$, then each unit upto b_{1j} shall be absorbed with probability t_{1j} and each of the additional units $(y_j - b_{1j})$ shall be absorbed with probability t_{2j}.

Hence, the expected revenue is $= s_j t_{1j} b_{1j} + s_j t_{2j}(y_j - b_{1j})$.

So, in general, if $b_{hj} \le y_j \le b_{h+1j}$, then the expected revenue is

$s_j\{t_{1j}b_{1j} + t_{2j}(b_{2j} - b_{1j}) + \cdots + t_{h+1j}(y_j - b_{hj})\}$.

Let us now break y_j into incremental units y_{hj} $(h = 1, 2, \cdots, H_j)$ as:

$$y_j = y_{1j} + y_{2j} + \cdots + y_{hi} + \cdots + y_{H_j j}, \tag{4.1}$$

$$\text{where} \quad \left.\begin{array}{ll} 0 \le y_{1j} \le b_{1j} & = M_{1j} \\ 0 \le y_{2j} \le b_{2j} - b_{1j} & = M_{2j} \\ \vdots \quad \vdots \quad \vdots & \vdots \\ 0 \le y_{H_j j} \le b_{H_j j} - b_{H_j -1, j} & = M_{H_j j} \end{array}\right\} \tag{4.2}$$

Relation (4.1) makes physical sense only if there exists some $h = h_j$ (say) such that all intervals below the h_j – th interval are filled to capacity and all intervals above it are empty i.e.

$$\left.\begin{array}{ll} y_{hj} = M_{hj} & (h = 1, 2, \cdots, h_j - 1) \\ y_{hj} \le M_{hj} & (h = h_j) \\ y_{hj} = 0 & (h = h_j + 1, \cdots, H_j) \end{array}\right\} \tag{4.3}$$

Assuming for the time being that the conditions (4.3) hold, the total expected revenue from sink j is:

$$f_j(s_j, y_j) = \sum_{h=1}^{H_j} s_j t_{hj} y_{hj} \tag{4.4}$$

Substituting the value of $f_j(s_j, y_j)$ from (4.4) in (3.1) and treating both x_{ij} and y_{hj} as decision variables, the deterministic equivalent to Problem P_3, is given as Problem P_4.

Problem P_4:

$$\text{Max} \quad Z = \sum_{i=1}^{m+n} \sum_{h=1}^{H_j} F_{hj} y_{hj} - \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} , \tag{4.5}$$

where $(F_{hj} = s_j t_{hj})$.

Subject to constraints (2.6) and (2.7a) (4.6)

$$\sum_{i=1}^{m+n} x_{ij} - \sum_{h=1}^{H_j} y_{hj} = u_0 , j = m+1, \cdots, m+n \tag{4.7}$$

$$x_{ij} , y_{hj} \geq 0 \qquad (\forall i, j, h) \tag{4.8}$$

$$x_{ij} \leq d_{ij} \qquad (\forall i, j) \tag{4.9}$$

$$y_{hj} \leq M_{hj} \qquad (\forall j, h) \tag{4.10}$$

and subject to the additional stipulation that the constraints (4.3) are also satisfied.

Fortunately, it turns out that (4.3) do not restrict our choice of optimum solution in any way. This can be handled by the theorem as given by Javaid et al. (1998).

5. PRELIMINARIES TO THE SOLUTION OF PROBLEM P_4

1. It is assumed that the set of all feasible solutions of Problem P_4 is regular (i.e. non- empty and bounded) and that the denominator of the objective function is positive for all feasible solution, Bela Martos (1968) and Cooper (1962).
2. Problem P_4 is a transportation type linear fractional programming problem with upper bound restrictions on some variables, therefore its global maximum may exist/exists at a basic feasible solution of its constraints, Swarup (1970)
3. We shall, hereinafter, call the constraints (4.6) through (4.8) as the original system and the constraints (4.6) through (4.10) as the capacitated system.
4. As none of the constraints in the original system is redundant, a basic feasible solution to the original system shall contain (m+n) basic variables. For the capacitated system also, a basic feasible solution shall contain (m+n) basic variables and the same may be found by working on the original system provided that some of the non-basic variables are allowed to take their upper bound values, Swarup (1970), Garwin (1963).
5. The special structure of Problem P_4, permits us to arrange it into an array as shown in Table (5.1).

Table 5.1: Special Structure of Problem P_4

x_{11} d_{11} 0	…	x_{m1} d_{m1} c_{m1}	$x_{1\,m+1}$ $d_{1\,m+1}$ $c_{1\,m+1}$	…	$x_{1\,m+n}$ $d_{1\,m+n}$ $c_{1\,m+n}$	a_1+u_o
…….	…	…….	…….	…	…….	…….
…….	…	…….	…….	…	…….	…….
x_{m1} d_{m1} c_{m1}	…	…….	…….	…	…….	a_m+u_o
$x_{m+1\,1}$ $d_{m+1\,1}$ $c_{m+1\,1}$	…	…….	…….	…	…….	u_o
…….	…	…….	…….	…	…….	…….
…….	…	…….	…….	…	…….	…….
$x_{m+n\,1}$ $d_{m+n\,1}$ $c_{m+n\,1}$	…	…….	x_{m+nm+1} d_{m+nm+1} $c_{m+n\,m+1}$	…	x_{m+nm+1} d_{m+nm+n} $c_{m+n\,m+n}$	u_o
t_o	…	t_o	(shaded)	(shaded)	(shaded)	
			$y_{1\,m+1}$ $M_{1\,m+1}$ $F_{1\,m+1}$	…	…….	
			…….	…	…….	
			t_o	…	t_o	

In the above table, there are $(m+n)$ rows in columns $j=1,2,\cdots,m$ and $(m+n+H)$ rows in columns $j=m+1,\cdots,m+n$. Here $H=\max.H_j$, so that there shall be some empty boxes near the bottom of the table in columns $j=m+1,\cdots,m+n$. These empty boxes shall be crossed out.

Absence of the row totals for y_{hj}'s in the table indicates that there are no row equations for y_{hj} variables. Besides, to obtain the column equations (4.8), each y_{hj} has to be multiplied by (-1). We have omitted (-1) from y_{hj} boxes for convenience.

6. INITIAL BASIC FEASIBLE SOLUTION AND OPTIMALITY CRITERIA

To start with, we fix the demands b_j's approximately equal to their expected values such that

$$\sum_{j=m+1}^{m+n} b_j = \sum_{i=1}^{m} d_{ij} \text{ and } \sum_{j=m+1}^{m+n} b_j = \sum_{i=1}^{m} a_i$$

and also such that for all j except $j = j^*$, each b_j falls at the upper end of one of the intervals y_{hj} into which b_j has been divided, i.e.,

$$b_j = \sum_{h=1}^{h'_j} M_{hj}$$

for some $h'_j \le H_j$ and for all j except $j = j^*$ (the b_j can always be so chosen that it is done).

With these fixed demands the upper portion of the Table 5.1 resembles a $(m+n)\times(m+n)$ standard transportation problem for which an initial basic feasible solution with $\{2(m+n)-1\}$ basic variables may be obtained as follows.

Ignore the upper bounds on x_{ij}'s and write down the basic feasible solution by the North-West Corner Rule or any other method for standard transportation. If this solution satisfies the upper bound constraints, we hit the target. If it violates these constraints, however, then we divide the basic variables into two groups.

a) the infeasible variables which violate their upper bounds and
b) the feasible variables which do not violate them.

Now, we discard temporarily the upper bounds on the infeasible variables and replace the original objective function by one that minimizes the sum of the infeasible variables. The existing solution now acts as the initial basic feasible solution for the artificial problem we have just created, and we begin the iterations, keeping in mind the upper bounds on the feasible variables.

As we proceed, some infeasible variables will increase while others will decrease, but their general level decreases because we are decreasing their sum. At certain iteration, as soon as some of the originally infeasible variables dip below or become equal to their upper bounds, these variables join the group of feasible variables, become upper bounded and are removed from the objective function. We continue this till

a) all the infeasible variables disappear or
b) the objective function cannot be further improved while some infeasible variables still remain.

The later indicates that no feasible solution of the capacitated system exists while the former indicates that a basic feasible solution has been found.

After a basic feasible solution with $\{2(m+n)-1\}$ variables has been found for the transportation problem (represented in the upper portion of Table (5.1)), we enter in each column of the lower portion of the Table (5.1), non basic y_{hj}'s at their upper bounds in turn $h = 1,2,...$ until we have entered enough non basic y_{hj} so that their sum over h is equal to b_j (fixed earlier).

Obviously, we shall never have to enter y_{hj} below its upper bound except in column $j = j^*$, where the last nonzero entry will be $y_{hj^*} \le M_{hj^*}$. This last entry and the $\{2(m+n)-1\}$ basic x_{ij} found earlier, constitute the required initial basic feasible solution with $2(m+n)$ basic variables.

In case, the last non zero entry in column j^* is also at its upper bound, then we take the last y_{hj} entry of any column as or $2(m+n)$ – th basic variable.

Let the simplex multipliers corresponding to the objective function Z (Problem P_4) be u_i and v_j $(\forall i, j = 1,2,...,m+n)$.

These are determined by solving the following equations.

$$\left.\begin{aligned} c_{ij} + u_i + v_j &= 0 \quad \text{for basic } x_{ij} \\ F_{hj} - v_j &= 0 \quad \text{for basic } y_{hj} \end{aligned}\right\} \qquad (7.1)$$

These are $2(m+n)$ linear equations in as many unknowns u_i and v_j, and can be easily solved.

Let the relative cost coefficients corresponding to the variables x_{ij} and y_{hj} be C'_{ij} and F'_{hj}.

These are determined by solving the following equations.

$$\left.\begin{aligned} C'_{ij} &= u_i + v_j - c_{ij} \quad \text{for non basic } x_{ij} \\ F'_{hj} &= M_{hj} - v_j \quad \text{for non basic } y_{hj} \end{aligned}\right\} \qquad (7.2)$$

For a given b.f.s. (x_{ij}, y_{hj}), the value of Z of Problem P_4 is:

$$Z = \sum_{j=1}^{m+n}\sum_{h=1}^{H_j} F'_{hj} y_{hj} - \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} C'_{ij} x_{ij} + \left\{ \sum_{i=1}^{m+n} u_i (a_i + u_o) + \sum_{j=1}^{m+n} v_j u_o \right\} \qquad \ldots(7.3)$$

But, the relative cost coefficients for basic variables and the values of the non basic x_{ij} are zero. As regards the values of non basic y_{hj}'s-some are zero and the others at their upper bounds. Hence,

$$Z = \sum_{j=1}^{m+n}{}^{*}\sum_{h=1}^{H_j}{}^{*} F'_{hj} M_{hj} - \sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{m+n}{}^{*} C'_{ij} x_{ij} + \left\{ \sum_{i=1}^{m+n} u_i (a_i + u_o) + \sum_{j=1}^{m+n} v_j u_o \right\} \quad \ldots(7.4)$$

Where $\sum^{*}$ indicates the sum over those non basic y_{hj} which are at their upper bounds.

Differentiating (7.3) partially with respect to non basic x_{ij} and y_{hj}, we get,

$$\frac{\partial Z}{\partial x_{ij}} = -C'_{ij} \text{ and}$$

$$\frac{\partial Z}{\partial y_{hj}} = F'_{hj}$$

We observe that the value of Z can be improved in two possible ways:

- firstly by increasing the non-basic x_{ij} (or y_{hj}) whose C'_{ij} (or F'_{hj}) are positive.
- secondly by decreasing those non-basic x_{ij} (or y_{hj}) whose C'_{ij} (or F'_{hj}) are negative.

Thus a basic feasible solution is optimum iff

$$\left.\begin{array}{ll} C'_{ij} \le 0 & (\forall \text{ non basic } x_{ij} \textit{ at zero level}) \\ C'_{ij} \ge 0 & (\forall \text{ non basic } x_{ij} \textit{ at upper bounds}) \\ F'_{hj} \le 0 & (\forall \text{ non basic } y_{hj} \text{ at zero level}) \\ F'_{hj} \ge 0 & (\forall \text{ non basic } y_{hj} \text{ at upper bounds}) \end{array}\right\} \quad \ldots(7.5)$$

If any of the optimality criteria (7.5) is violated, the current solution can be improved. The non-basic variable which violates (7.5) most severely is selected to enter the basis. The values of the new basic variables are found in the usual manner by applying θ-adjustments. It should, however, be kept in mind that the coefficient of each y_{hj} in the column equations (4.7) is (-1).

The variable to leave the basis is the one that becomes either zero or equal to its upper bound. If two or more basic variables reach zero or their upper bounds simultaneously then only one of them becomes nonbasic. Should it happen that the entering variable itself attains upper or lower bound (zero) without simultaneously making any of the basic variables zero or equal to its upper bounds, the set of basic variables remains unaltered; only their values are changed to allow the so-called entering variable to be fixed at its upper or lower bound.

7. FINITENESS

The process is bound to terminate with a finite number of iterations as it involves movement from one basic feasible solution to another basic feasible solution, which is finite in number.

8. NUMERICAL EXAMPLE

Consider the stochastic Transportation problem involving Transshipment with 3 origins and 2 destinations in which shipping charges, i.e.,the costs from origin to origin, destination to destination and origin to destinations are denoted by d_{ij} and c_{ij} and are given in each boxes in Table (8.1), Table (8.2) to Table (8.3) respectively. The probability distributions of the random demands b_j $(j = 1,2)$ are given in Table (8.5) along with the computed values of F_{hj} and M_{hj}: The initial basic feasible solution by the North-West Corner is given in Table (8.4).

Table (8.1): Transportation from source to sink.

	A	B	a_i
I	8 9	3 4	10
II	4 6	6 2	5
III	3 8	6 3	6
s_j	10	5	

Table (8.2): Transshipment from sink to sink.

	A	B
A	21 0	6 5
B	6 5	6 2

Table (8.3): Transshipment from source to source.

	I	II	III
I	21 0	3 8	4 3
II	3 8	21 0	6 5
III	4 3	6 5	21 0

Table (8.4): IBF Solution by North-West Corner Rule

	A	B	a_i
I	10 9	 4	10
II	2 6	3 2	5
III	 8	6 3	6
b_j	12	9	

Table (8.5):Assumed distribution of b_j

j	b_j	p_{hj}	t_{hj}	$F_{hj} = s_j t_{hj}$	M_{hj}
	9	0.2	1.0	-10	9
1	12	0.6	0.8	-7	3
	16	0.2	0.2	-2	5
2	7	0.2	1.0	-5	7
	10	0.8	0.8	-4	3

Iteration-1

Step 1. In order to obtain initial basic feasible solution we fix the demands at $b_1 = 12$ and $b_2 = 9$, and then determine a starting basic feasible solution to the (3×2) standard transportation problem represented above the shaded region in Table (8.4), by the North-West corner Rule.

Then a standard transshipment problem can be formed (ignoring the upper bounds). We get,

$x_{11} = 21$, $x_{14} = 10$, $x_{15} = 2$ $x_{22} = 21$,

$x_{24} = 4$, $x_{25} = 1$, $x_{33} = 21$, $x_{35} = 6$,

$x_{44} = 21$, $x_{55} = 21$

To obtain the initial basic feasible solution to the deterministic equivalent transportation problem, we assign y_{hj} entries at their upper bounds (as far as possible) so that the column equations are satisfied. This solution violates the upper bound restrictions as $x_{14} \le 8$

To obtain a basic feasible solution to the deterministic capacitated transshipment problem, we temporarily treat all x_{ij}'s, except the infeasible variable x_{24}, as upper bounded and apply the usual transportation routine to minimize the sum of infeasible variable, i.e., to minimize x_{24}, till the infeasibility of x_{24} is removed. The solution so obtained is:

$x_{11} = 21$, $x_{14} = 8$, $x_{15} = 2$, $x_{22} = 21$,

$x_{24} = 4$, $x_{25} = 1$, $x_{33} = 21$, $x_{35} = 6$,

$x_{44} = 21$, $x_{55} = 21$

For the capacitated transshipment problem $x_{24} = 4$ is non basic variable at its upper bound.

We get $y_{11} = 9$, $y_{21} = 3$, $y_{12} = 7$ and $y_{22} = 2$ $(< M_{22})$.

This provides the required initial basic feasible solution with x_{11}, x_{21}, x_{22}, x_{32} and y_{22} as the basic variables.

Step 2. The simplex multipliers (u_i, v_j) and the relative cost coefficients are determined. These values are also recorded in the following working Table (8.6).

Step 3. The values of Z are obtained as under

$$Z = \sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{m+n}{}^{*} F'_{hj} M_{hj} - \sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{m+n}{}^{*} C'_{ij} d_{ij} - \left\{ \sum_{i=1}^{m+n} u_i (a_i + t_o) + \sum_{j=1}^{m+n} v_j t_o \right\}$$

$$= \{7(9) + 1(7) + (-2)3 + 0(4)\} - \{[31(0) + 26(-2) + 27(-1) + 21(-9) + 21(-4)] + 21(0) + 21(2) + 21(1) + 21(9) + 21(4)\}$$

$$= 26$$

Step 4 .For the non-basic variables, the values of C'_{ij} and F'_{hj} are calculated and found that F'_{21} violates the optimality criteria.

$$F'_{21} = 7 - 9 = -2$$

Obviously the current solution is not optimum and may further improved.

Step 5. Adding θ to y_{24}, we are led to the θ – adjustments as shown in Table (8.6). Here, the maximum possible value of θ is $\theta^* = 1$.

Table (8.6) Deterministic Version of Problem P$_4$

					a_i
[20] 0	 2 4	 4 6	[9] 0 5	 2 3	31
  2 4	[20] 0 0	 3 2	[3] 0 4	[2] 1 2	26
  4 6	 3 2	[20] 0 0	 1 7	[6] 0 1	27
  0 5	 4 1	 1 7	[20] 0 0	 5 4	21
  2 3	 1 2	 0 1	 5 4	[20] 0 0	21
21	21	21	//////	//////	
			[9] -5 -10	[6] -1 -5	
			[3] -3 -7	2 -4	
			-2	//////	
			21	21	

After the ***Third iteration*** the optimal solution has been attained as:

$Z_{opt.} = 28$

$x_{11} = 21$, $x_{14} = 7$, $x_{15} = 3$ $x_{22} = 21$,

$x_{24} = 4$, $x_{25} = 1$, $x_{33} = 21$, $x_{35} = 6$,

$x_{44} = 21$, $x_{55} = 21$

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Shakeel Javaid and M.M. Khalid
Department of Statistics and Operations Research,
Aligarh Muslim University,
Aligarh-202002, India.

email: shakeel.jd@rediffmail.com