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POOLED TEST STATISTIC OF TREATMENT CONTRAST AND RANDOMIZED BLOCK DESIGN WITH HETEROGENEOUS ENVIRONMENT

A. A. JAMJOOM AND M. A. ALI

ABSTRACT

A test statistic following the work of James (1954) has been discussed for pooled estimation and test of treatment contrasts applied on a group of randomized block design experiments with the same set of treatments in presence of heterogeneous error variances. The distribution of the suggested test statistic has been studied by the Monte Carlo study and it was observed that its distribution did not follow the suggestion of James (1954) and Bhuyan (1986) for both large and small error $d.f.$ of individual experiments. The critical values of the test statistic along with some other distributional characteristics that were found by simulation and numerical illustrations are also included.

1. INTRODUCTION

In agricultural, industrial, scientific and medical investigations, experiments are generally repeated over places or environments or both for formulating any scientific law on the use of treatments. These repeated experiments involve the pooled analysis for the combined estimation and test of treatment contrasts. The pooled analysis creates a problem if experiments are replicated over heterogeneous environments and the problem becomes more serious if the analysis of variance technique is used as a method of statistical inference on treatment contrasts. The problem arises due to the presence of environments (places) and treatment interaction variances. This was observed first by Cochran (1937, 1954). Bhuyan (1986) and Ali, *et al.* (1999) have suggested a similar method of estimation and testing treatment contrasts in the way of pooled analysis with interaction model under heterogeneous error variances based on James (1954). This test statistic is approximately χ^2 and, for large error degrees of freedom; it is exactly χ^2 .

In this paper, assuming error variances are unknown. The combined estimates and test of treatments contrasts of a group of experiments are obtained. Also assume that error variance of a particular experiment is homogeneous and it varies from experiment to experiment. Least square method may be applied to estimate the treatment contrast to the individual experiments. Then weighted pooled estimate of treatment contrast and test statistic are provided based on work of James (1951, 1954). The distribution of the suggested test statistic has been critically studied by the Monte Carlo study and the critical values as well as some other distributional properties are cited. The critical values and distributional properties of the suggested test statistic show nonconformity of James's (1954) and Bhuyan's (1986) suggestions.

2. METHODOLOGY

Let us suppose that $\pi_1, \pi_2, ..., \pi_v$ are the treatment effects of v treatments which are to be investigated. For this, a group of p randomized block design experiments are conducted with these v treatments. The main object of the analysis is to estimate the contrast of π_j's $(j = 1, 2, \cdots, v)$ and to test the hypothesis that the treatment contrasts effects are independent of the locations.

Let us suppose that the yield of $j-$th treatment in $i-$th block of $h-$th place be denoted by y_{hij} and the yield follows the linear model,

$$y_{hij} = \mu_h + \alpha_{hi} + \pi_j + e_{hij}; \quad h = 1, 2, \ldots, p; i = 1, 2, \ldots, b; j = 1, 2, \ldots, v \tag{2.1}$$

Here, μ_h = general mean of $h-$th place

α_{hi} = effect of $i-$th block at $h-$th place

π_j = $j-$th treatment effect, and

e_{hij} = random error.

It is assumed that e_{hij} are normally and independently distributed with mean zero and variance σ_h^2. The restrictions for the model are

$$\sum_{i=1}^{b} \alpha_{hi} = \sum_{j=1}^{v} \pi_j = 0 \tag{2.2}$$

As the places are different, the estimates of treatment effects may differ from place to place. For more details about the model and estimation of parameters can be referred to Sahai and Ageel (2000). Let us denote the intra-block estimates of $j-$th treatment effect at $h-$th place by t_{hj}, whereby the usual least square method t_{hj} is given by

$$t_{hj} = \bar{y}_{h.j} - \bar{y}_{h..} \tag{2.3}$$

The pooled estimate of treatment contrasts and test of the hypothesis is based on the above individual analysis of p experiments. The method of estimation and test is an adaptation of the work of James (1951, 1954) to the problem under consideration.

Let $\theta_1, \theta_2, ..., \theta_p$ be the location-specific vector of v treatment effects at p different places respectively. The problem is to test if the treatment effects are independent of the locations.

That is, we wish to test

$$H_0 : A\theta_1 = A\theta_2 = ... = A\theta_p \tag{2.4}$$

where,

$$A\theta_h = \begin{bmatrix} 1 & -1 & 0 & 0 & ... & 0 \\ 1 & 0 & -1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... \\ 1 & 0 & 0 & 0 & ... & -1 \end{bmatrix}_{(v-1)\times v} \begin{bmatrix} \pi_{h1} \\ \pi_{h2} \\ ... \\ ... \\ \pi_{hv} \end{bmatrix}_{v\times 1}$$

In the above, π_{hj} is the location-specific effect of treatment $'j'$ in the location $'h'$, $h = 1,2,..., p$ and $j = 1,2,..., v$. Let $\hat{\theta}_h$ be any solution from the normal equation

$$Q_h = C_h \hat{\theta}_h ; h = 1,2,..., p$$

Since $A\theta_h$ is estimable, the best linear unbiased estimate of $A\theta_h$ is

$$A\hat{\theta}_h = T_h = \begin{bmatrix} 1 & -1 & 0 & 0 & ... & 0 \\ 1 & 0 & -1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... \\ 1 & 0 & 0 & 0 & ... & -1 \end{bmatrix}_{(v-1)\times v} \begin{bmatrix} t_{h1} \\ t_{h2} \\ ... \\ ... \\ t_{hv} \end{bmatrix}_{v\times 1}$$

where, $\hat{\pi}_{hj} = t_{hj}$ for $j = 1, 2, \cdots, v$.

For randomized block design

$$A\hat{\theta}_h = T_h = \begin{bmatrix} 1 & -1 & 0 & 0 & ... & 0 \\ 1 & 0 & -1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... \\ 1 & 0 & 0 & 0 & ... & -1 \end{bmatrix}_{(v-1)\times v} \begin{bmatrix} \bar{y}_{h.1} \\ \bar{y}_{h.2} \\ ... \\ ... \\ \bar{y}_{h.v} \end{bmatrix}_{v\times 1}$$

where, $\bar{y}_{h.j}$ is the $j-$th $(j=1,2,\cdots,v)$ treatment mean of $h-$th $(h=1,2,\cdots,p)$ place.

It is observed that $T_h \sim$ NID $(A\theta_h, W_h\sigma_h^2)$ for $h=1,2,\cdots,p$; where $W_h = AC_h^{-}A'$ is a non-singular $(v-1)$ order square matrix and is unique with respect to any choice of g-inverse C_h^{-} of C_h. In practice

$$W_h = \begin{bmatrix} 2/b & 1/b & \ldots & 1/b \\ 1/b & 2/b & \ldots & 1/b \\ \cdots & \cdots & \cdots & \cdots \\ \ldots & \ldots & \ldots & \ldots \\ 1/b & 1/b & 1/b & 1/b \end{bmatrix}_{(v-1)\times(v-1)}$$

where, b is the number of blocks (replications) of each treatment in experiment. Thus, for known σ_h^2 $(h=1,2,...,p)$, the pooled test statistic for the null hypothesis (2.4) is given by

$$t = \sum_{h=1}^{p}(T_h - T)' W_h^{-1}(T_h - T)/\sigma_h^2 \tag{2.5}$$

This test statistic is distributed as χ^2 with $(p-1)(v-1)$ $d.f.$ Here, T is given by

$$T = \left[\sum_{h=1}^{p}\left(W_h\sigma_h^2\right)^{-1}\right]^{-1} \sum_{h=1}^{p} W_h^{-1}T_h/\sigma_h^2 \tag{2.6}$$

The proof has been discussed by James (1954) who provides a generalization of his work of 1951. When σ_h^2 is unknown, then σ_h^2 can be replaced by its usual unbiased estimate $\hat{\sigma}_h^2$, *i.e.*, mean sum square of the model (2.1) in $h-$th place.

We know $f_h\hat{\sigma}_h^2/\sigma_h^2$ are independently distributed as χ^2 with f_h $(h=1,2,\cdots,p)$ $d.f.$ In our case, $\hat{\sigma}_h^2$ is the error mean sum of squares from $h-$th experiment and all f_h's are equal having value $(b-1)(v-1)$. Let $\hat{t}$ and $\hat{T}$ be the t and T respectively after replacing σ_h^2 by $\hat{\sigma}_h^2$. Then, the test statistic (2.5) can be written as

$$\hat{t} = \sum_{h=1}^{p}(T_h - \hat{T})' W_h^{-1}(T_h - \hat{T})/\hat{\sigma}_h^2 \tag{2.7}$$

Where

$$\hat{T} = \left[\sum_{h=1}^{p}\left(W_h\hat{\sigma}_h^2\right)^{-1}\right]^{-1}\sum_{h=1}^{p}W_h^{-1}T_h/\hat{\sigma}_h^2 \tag{2.8}$$

James (1954) discussed that the statistic (2.7) is also distributed as χ^2 with $(p-1)(v-1)$ $d.f.$ provided that f_h are large. For f_h not large enough, the statistic (2.7) can be compared with

$$h = \chi^2\left[1+\frac{3\chi^2+\{(p-1)(v-1)+2\}}{2(p-1)(v-1)\{(p-1)(v-1)+2\}}\times\sum_{h=1}^{p}\frac{1}{f_h}\left\{1-\left(2\hat{\sigma}_h^2/b\right)^{-1}\left(\sum_{h=1}^{p}\left(2\hat{\sigma}_h^2/b\right)^{-1}\right)^{-1}\right\}^2\right] \tag{2.9}$$

where χ^2 is the α% point of χ^2-variate with $(p-1)(v-1)$ $d.f.$. The test statistic (2.7) can be computed easily which will provide the pooled estimate of treatment contrasts.

The hypothesis (2.4) may be presented in another way. Assume that $A\theta_h = \delta$, $\forall\ h = 1,2,...,p$. Then, the hypothesis can be written as,

$H_{01}:\delta = 0$, against the alternative, $H_{11}:\delta \neq 0$

The test statistic is given by

$$t_1 = T'\left(\sum_{h=1}^{p}W_h^{-1}/\sigma_h^2\right)T \tag{2.10}$$

This $t_1 \sim \chi^2_{v-1}$ if σ_h^2's are known. For unknown σ_h^2's are to be replaced by its estimates and the test statistic is given by

$$\hat{t}_1 = \hat{T}'\left(\sum_{h=1}^{p}W_h^{-1}/\hat{\sigma}_h^2\right)\hat{T}$$

This $\hat{t}_1 \sim \chi^2_{v-1}$ under H_{01}. For f_h not large enough, the statistic $\hat{t}_1$ is to be compared with

$$h_1 = \chi^2\left[1+\frac{3\chi^2+(v+1)}{2(v^2-1)}\sum_{h=1}^{p}\frac{1}{f_h}\left\{1-(2\hat{\sigma}_h^2/b)^{-1}\left(\sum_{h=1}^{p}(2\hat{\sigma}_h^2/b)^{-1}\right)^{-1}\right\}^2\right]$$

where χ^2 is the $\alpha\%$ point of χ^2 – variate with $(v-1))$ $d.f.$. The hypothesis (2.4) implies that all $(v-1)$ contrasts are insignificant. But it is sometimes required to test the insignificancy of any one of the contrasts. For this**,** the test statistic is

$$\hat{t}_2 = \sum_{h=1}^{p}\left(\frac{2\hat{\sigma}_h^2}{b}\right)^{-1} T_{hj}^2 - \left\{\sum_{h=1}^{p}\left(\frac{2\hat{\sigma}_h^2}{b}\right)^{-1} T_{hj}\right\}^2 \left\{\sum_{h=1}^{p}\left(\frac{2\hat{\sigma}_h^2}{b}\right)^{-1}\right\}^{-1}$$

where, $T_{hj} = t_{hj} - t_{hj'}$, $j' \neq j = 1,2,...,v$. The statistic $\hat{t}_2 \sim \chi_{p-1}^2$ provided f_h's are large. For f_h's not large enough, $\hat{t}_2$ is to be compared with

$$h_2 = \chi^2\left[1 + \frac{3\chi^2 + (p+1)}{2(p^2-1)} \sum_{h=1}^{p} \frac{1}{f_h}\left\{1 - (2\hat{\sigma}_h^2/b)^{-1}\left(\sum_{h=1}^{p}(2\hat{\sigma}_h^2/b)^{-1}\right)^{-1}\right\}^2\right]$$

where χ^2 is the $\alpha\%$ point of χ^2-variate with $(p-1)$ $d.f.$

In every case, it is seen that the test statistic is distributed as χ^2 if f_h's are large, but there is no definite indication of the value of large f_h. Thus, it is decided to study the distributional properties of the test statistic (2.7) for both small and moderately large values of f_h using simulation technique. For this, sets of random normal samples are drawn. In order to find the exact critical values of the statistic, a Monte Carlo study is performed.

In this study, attempts are made to find the exact critical values of the distribution of the test statistic under null hypothesis. Critical values are calculated with eliminating outliers and without eliminating outliers from the series of the statistic.

For eliminating lower and upper outliers, the formulae are $F_L \mp 1.5 d_F$, where, F_L = first quartile, F_U = third quartile and $d_F = F_U - F_L$. For a particular value of σ_h and for different values of π_j $(j = 1,2,...,v)$ sets of b $(=5,6,7,...)$ normal observations are generated. A set of b observations for a particular value of π_j is considered as the observations of j – th treatment of b blocks. The set of b observations for different values of π_j are considered the observations of a randomized block design. For different values of σ_h $(h = 1,2,..., p)$, the observations of p randomized block designs (RBD) are generated. The samples are generated to calculate the test statistic (2.7) under the null hypothesis and these processes are repeated 5000 times and obtained

5000 values of test statistic (2.7). From these of the test statistic, critical values corresponding to the nominal size 1%, 2%, 2.5%, 5%, 10%, 20%, 25%, 30%, 50%, 70%, 75%, 80%, 90%, 95%, 97.5%, 98%, 99% are calculated first from the original values and then eliminating outliers. Beside these, some other distributional characteristics such as mean, variance, first quartile $(Q1)$, median, third quartile $(Q3)$, skewness and kurtosis are calculated for different $d.f.$ of the test statistic. In each case, percentile points and other distributional characteristic are studied. A bar diagram along with fitted curve of the distribution of the statistic are also presented to observe the trend of change with the change of $d.f.$ and $error\ d.f.$ For computer programming, MATLAB 7 version is performed [Hanselman and Littlefield, (2001)].

3. THE MONTE CARLO STUDY

According to the methodology discussed in the previous section, two tables are presented in this section. Table 3.1 and Table 3.2 are simulated percentile points and distributional characteristics of the test statistic (2.7) under the null hypothesis with eliminating outliers. The entries of the Tables 3.1 are $\hat{t}_p(v)$,

where, $p = \int_0^{\hat{t}_p(v)} f(\hat{t})d\hat{t} = P[\hat{t} \le \hat{t}_p(v)]$ and $f(\hat{t})$ is the $p.d.f.$ of $\hat{t}$.

Table 3.1: Percentile points of $\hat{t}$, *i.e.*, $p = P[\hat{t} \le \hat{t}_p(v)]$

		Probability							
df	f_h	0.010	0.020	0.025	0.100	0.950	0.975	0.980	0.990
1	3	0.0002	0.0007	0.0010	0.0148	3.0534	3.5519	3.6565	3.8716
2	8	0.0185	0.0343	0.0467	0.2054	5.2909	6.0776	6.2580	6.6267
3	15	0.1161	0.1746	0.1993	0.5507	7.1180	7.8865	8.1494	8.6837
4	24	0.2954	0.4160	0.4683	1.0394	8.8485	9.8095	10.0657	10.6649
5	35	0.5297	0.7225	0.8287	1.5976	10.5416	11.6354	11.9142	12.5684
6	48	0.9154	1.1199	1.2294	2.1133	11.8811	13.0640	13.3950	14.2913
7	63	1.1440	1.4590	1.6244	2.6621	13.3832	14.7967	15.0702	15.8014
8	80	1.6345	2.0003	2.2182	3.4245	14.9475	16.3410	16.6744	17.6423
9	99	2.0214	2.4765	2.6195	4.0725	16.2942	17.5870	17.9213	18.8053
10	120	2.4445	3.0209	3.2369	4.7698	17.6927	19.2585	19.6432	20.5033
11	143	2.9712	3.5661	3.7940	5.5410	19.1858	20.7301	21.1041	21.8472
12	168	3.4897	4.0371	4.3070	6.2253	20.3338	22.0780	22.5783	23.5386
13	195	4.0977	4.7301	4.9424	6.7974	21.9184	23.7858	24.2333	25.1221

14	224	4.5279	5.2240	5.4500	7.6133	22.8114	24.4986	24.9420	26.2770
15	255	5.1973	6.0602	6.2904	8.5059	24.4512	26.3290	26.8188	27.9142
16	288	5.6928	6.5641	6.8385	9.2944	25.7339	27.5533	27.9081	29.1325
17	323	6.3386	7.2532	7.6159	10.0875	27.0510	29.2361	29.6873	30.7591
18	360	7.0014	7.7315	8.1483	10.7100	28.0470	30.1675	30.5053	31.9842
19	399	7.5235	8.4114	8.7067	11.4561	29.5170	31.4887	32.0222	33.2490
20	440	8.1453	9.2027	9.6176	12.2198	30.6946	32.7860	33.4829	34.7979
21	483	8.9845	9.8996	10.2693	13.0772	31.8330	34.0763	34.5309	35.6589
22	528	9.2651	10.2021	10.6184	13.8882	33.5175	35.7604	36.2822	37.7331
23	575	9.8712	11.1700	11.5914	14.7903	34.4907	37.2355	37.8253	39.2796
24	624	10.9677	11.8958	12.3491	15.6121	35.3021	37.5402	38.0663	39.3557
25	675	11.2984	12.4925	13.0227	16.2984	36.5424	38.8017	39.5141	41.1523
26	728	11.9662	13.1913	13.5374	17.0648	38.5130	40.5857	41.1209	42.9445
27	783	12.4230	13.5813	14.1680	18.0665	39.4365	42.0337	42.6526	44.2522
28	840	13.6950	14.9047	15.3633	18.8485	40.5727	43.3940	43.8852	45.2216
29	899	13.7279	15.0865	15.5630	19.4969	41.6118	44.1591	44.7453	46.6199
30	960	14.6881	15.9999	16.6515	20.4985	43.1557	45.6574	46.1609	47.6648
31	1023	15.8145	17.0853	17.4021	21.3293	43.7031	46.6948	47.4248	49.1444
32	1088	16.3362	17.7069	18.2946	22.0203	45.2223	47.7630	48.3862	49.9971
33	1155	17.3352	18.5371	19.1545	23.0398	46.5989	49.1926	49.8980	51.9109
34	1224	16.9750	18.9344	19.5887	24.0730	47.7580	50.8067	51.8388	53.2179
35	1295	18.0892	19.6352	20.1277	24.8462	48.7736	51.6847	52.3829	53.9705
36	1368	19.1404	20.9338	21.3895	25.7127	50.2151	53.3244	53.9742	56.2519
37	1443	19.9347	21.4453	21.9925	26.5547	51.7410	54.5432	55.5158	57.4507
38	399	20.5973	22.4953	23.0412	27.1787	52.5723	55.1535	55.9404	58.3806
40	440	22.1743	23.7230	24.3460	28.8199	55.4628	58.1207	59.1096	61.2878
42	483	22.9267	24.6156	25.4367	30.6425	57.4852	60.0645	60.7755	62.8061
44	528	25.0468	26.9266	27.8210	32.5673	59.8413	62.9230	63.9406	66.5374
46	575	26.5129	28.5028	29.3523	34.3868	62.4975	65.4148	66.2378	68.0744
48	624	27.8955	29.6094	30.3625	35.8925	65.0085	68.0769	68.9199	70.8633
50	675	29.8193	31.7648	32.6815	37.6907	66.2744	69.4724	70.2053	73.0290
52	728	31.1232	32.9954	33.8057	39.2798	69.2160	72.1354	73.1565	75.5349
54	783	32.2682	34.4299	35.1456	41.0813	70.8838	74.0554	75.1810	77.4262

56	840	34.7008	36.7885	37.6674	43.0151	73.6660	76.2016	77.1861	79.7275
58	899	35.9780	38.2771	39.0971	44.4283	76.1764	79.7583	80.5672	82.9046
60	960	37.1112	39.4970	40.2713	46.5861	77.8041	81.0009	81.9430	84.1567
62	1023	38.5241	40.9514	41.5508	48.2228	80.3625	83.7524	84.7074	86.7199
64	1088	40.6284	43.0483	43.8781	49.9061	83.7111	86.8912	87.8556	90.5235
66	1155	42.1625	44.2690	45.4324	51.5529	85.4982	88.8426	89.7302	92.0291
68	1224	43.8404	46.3988	47.1084	53.8675	87.7034	90.9611	91.8144	94.3549
70	1295	45.8136	48.3730	49.2469	55.3608	89.2996	93.4328	94.2832	97.1035
72	1368	47.2649	49.5055	50.4387	57.1212	92.1354	95.9823	97.0995	99.5251
74	1443	48.4652	50.9830	51.8212	58.6828	94.2635	98.0063	99.0065	102.0335
76	1520	50.1430	52.3372	53.0688	60.8015	96.3274	99.9073	100.8259	103.1925
78	1599	51.3126	54.0074	55.0140	62.1425	99.1846	103.0285	103.8408	106.4709
80	1680	53.7726	56.9288	57.9406	64.3230	100.8988	104.9405	105.9784	109.4555
82	1763	54.2926	56.9334	57.8567	65.6978	102.6282	106.9045	108.0697	110.3559
84	1848	56.3117	59.0112	60.1803	67.6932	106.0670	110.3235	111.2406	114.0014
86	1935	58.8250	61.4721	62.2901	69.4416	108.0647	111.9194	113.1699	115.6917
88	2024	60.0429	62.4642	63.5182	71.6378	111.1851	115.4196	116.3803	119.4371
90	2115	61.0267	64.0596	64.9963	73.3771	113.1094	117.2141	118.3065	120.7805
92	2208	63.2918	65.7328	66.7527	74.6739	114.6256	119.4627	120.6794	123.4902
94	2303	63.9861	67.8057	69.1042	77.1482	116.8639	120.7506	121.7841	124.3743
96	2400	66.9992	70.4557	71.4875	78.6094	119.8331	123.6712	125.5954	128.8514
98	2499	68.2139	71.4890	72.3762	80.4819	121.3710	125.2523	126.5076	130.4200
100	2600	70.3382	73.6803	74.7184	82.8059	124.0521	128.5356	129.6226	132.6535

Table 3.2: Distributional characteristics of $\hat{t}$

df	f_h	Mean	Median	Variance	Q1	Q2	Skewness	Kurtosis
1	3	0.8192	0.4160	0.9475	0.0918	1.2016	1.4893	4.4619
2	8	1.8543	1.3475	2.6936	0.5591	2.7457	1.1004	3.4874
3	15	2.7701	2.2666	4.2580	1.1312	3.9332	0.9635	3.3003
4	24	3.8774	3.4130	6.2052	1.9228	5.3543	0.7802	2.9835
5	35	4.8739	4.3944	8.0729	2.6613	6.6333	0.7285	2.9539
6	48	5.7736	5.2158	9.8687	3.3153	7.7130	0.7029	2.9076
7	63	6.7482	6.1758	11.7291	4.1751	8.9464	0.6340	2.8458
8	80	7.8726	7.3378	13.7464	5.0465	10.2024	0.5855	2.7953

9	99	8.7548	8.2391	15.4824	5.7114	11.1904	0.5577	2.7731
10	120	9.7350	9.2091	17.2271	6.5336	12.3121	0.5599	2.8028
11	143	10.8811	10.4309	19.3241	7.5418	13.7190	0.4734	2.7113
12	168	11.7392	11.2326	21.0338	8.2886	14.6195	0.5146	2.7945
13	195	12.8724	12.3552	23.9317	9.2210	16.0386	0.4545	2.7080
14	224	13.7522	13.2044	24.5346	10.1186	17.0521	0.4079	2.7063
15	255	14.8180	14.2533	27.1446	10.8837	18.2198	0.4395	2.6898
16	288	15.8593	15.3992	28.3792	11.9136	19.3091	0.3925	2.7038
17	323	16.8646	16.3242	30.7061	12.8282	20.5378	0.3966	2.7336
18	360	17.5802	16.9777	31.7528	13.3781	21.1782	0.4375	2.7717
19	399	18.7998	18.3287	34.6654	14.4063	22.6557	0.3490	2.6821
20	440	19.7611	19.2879	36.5020	15.2834	23.7067	0.3665	2.7073
21	483	20.7332	20.2154	36.7832	16.2544	24.6614	0.3470	2.7001
22	528	21.8152	21.2759	41.4984	17.1458	25.9822	0.3448	2.6942
23	575	22.8054	22.2797	41.8603	18.0057	27.0319	0.3550	2.7790
24	624	23.7419	23.3866	41.8020	19.0087	27.8886	0.2820	2.6720
25	675	24.5292	23.9672	44.8549	19.6843	29.0316	0.3285	2.7020
26	728	25.7950	25.3440	48.3286	20.7587	30.3432	0.3119	2.7011
27	783	26.7686	26.2353	50.0256	21.6534	31.3863	0.3073	2.7233
28	840	27.7294	27.1239	51.9498	22.5387	32.4976	0.3372	2.6678
29	899	28.5844	28.0525	52.8262	23.4726	33.4594	0.2759	2.7779
30	960	29.6364	29.0726	54.7531	24.2182	34.4874	0.3205	2.7375
31	1023	30.6174	30.0432	55.0030	25.3327	35.4663	0.3015	2.7696
32	1088	31.5478	30.9974	57.2827	26.1208	36.4931	0.2790	2.7162
33	1155	32.8966	32.3204	60.7936	27.2928	38.1437	0.2730	2.6743
34	1224	33.7333	33.2325	61.7119	28.2199	38.9190	0.2546	2.8392
35	1295	34.8222	34.3670	64.0027	29.0683	40.0755	0.2210	2.7013
36	1368	35.8363	35.3611	65.6411	30.0109	41.1689	0.2641	2.7939
37	1443	36.8223	36.2894	69.0438	30.6697	42.2363	0.3038	2.7396
38	399	37.8621	37.3983	70.7609	31.6878	43.5703	0.2332	2.6689
40	440	39.9957	39.4659	77.0992	33.6894	45.9476	0.2323	2.6813

42	483	41.5441	41.0497	78.1145	35.1701	47.3762	0.2231	2.7186
44	528	43.8972	43.3463	83.5073	37.1970	49.9736	0.2721	2.6939
46	575	46.0190	45.5330	86.0628	39.3093	52.1700	0.2095	2.7114
48	624	47.8523	47.2983	92.2567	41.0066	54.2124	0.2292	2.7422
50	675	49.7612	49.3065	91.8538	42.7814	56.1395	0.2153	2.6776
52	728	51.6880	51.1449	98.5310	44.6059	58.3665	0.2270	2.7098
54	783	53.4147	52.9831	98.9239	46.4241	60.0191	0.1896	2.7609
56	840	55.6710	55.1193	101.8760	48.4167	62.2814	0.2073	2.6889
58	899	57.6496	57.1465	109.3950	50.0640	64.5701	0.2348	2.7102
60	960	59.6802	59.2706	108.9943	52.3067	66.7011	0.1692	2.7018
62	1023	61.7491	61.2882	112.9847	54.4444	68.6706	0.1576	2.7453
64	1088	64.1061	63.5044	124.4162	56.2154	71.7041	0.1675	2.6730
66	1155	65.6463	65.2757	123.1251	57.8650	73.1039	0.1666	2.7754
68	1224	67.9549	67.4163	127.7669	59.8764	75.6259	0.1652	2.7274
70	1295	69.8804	69.3899	128.8327	61.9512	77.6232	0.1492	2.7189
72	1368	71.8760	71.3063	136.2633	63.7376	79.6594	0.1735	2.7396
74	1443	73.7227	73.3082	141.6337	65.2699	81.5706	0.1725	2.7149
76	1520	75.6777	75.3401	137.5297	67.6803	83.1561	0.1229	2.7784
78	1599	77.6355	77.0840	151.1551	68.9318	85.8824	0.1489	2.7282
80	1680	79.8403	79.4058	147.9829	71.3239	88.1669	0.1731	2.7319
82	1763	81.3472	81.1631	150.7146	73.0672	89.5386	0.1008	2.8061
84	1848	83.7359	83.1735	162.0314	74.8138	92.1313	0.1711	2.7359
86	1935	85.5997	85.0609	161.3067	76.4875	94.0453	0.1837	2.7225
88	2024	88.0443	87.6171	173.9543	78.5538	96.9548	0.1668	2.7592
90	2115	89.7316	89.0408	172.8590	80.4685	98.5469	0.1680	2.7705
92	2208	91.7727	91.2428	177.3334	82.6052	100.8112	0.1171	2.7829
94	2303	93.7339	93.3056	171.5761	84.5866	102.3977	0.1204	2.7879
96	2400	96.0188	95.4785	186.4500	86.1370	105.3159	0.1832	2.7068
98	2499	97.6032	96.7866	183.1685	88.1434	106.8953	0.1577	2.7440
100	2600	100.0297	99.2749	187.9171	90.5078	109.1818	0.1715	2.7380

Fig 3.1 – 3.6: Shows Bar Diagram and Curve Fitting of $f\ (\hat{t})$ for Different *df* and f_h.

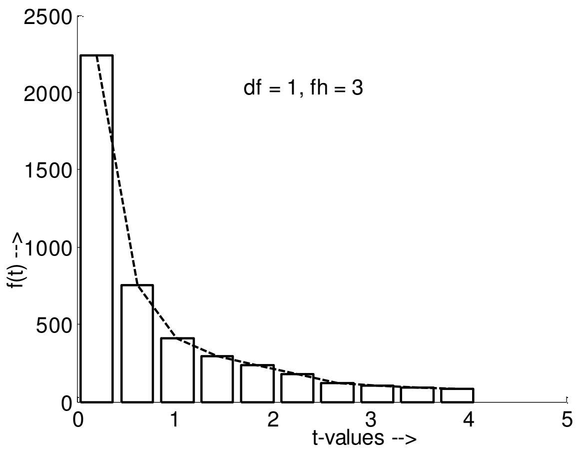


Fig 3.1

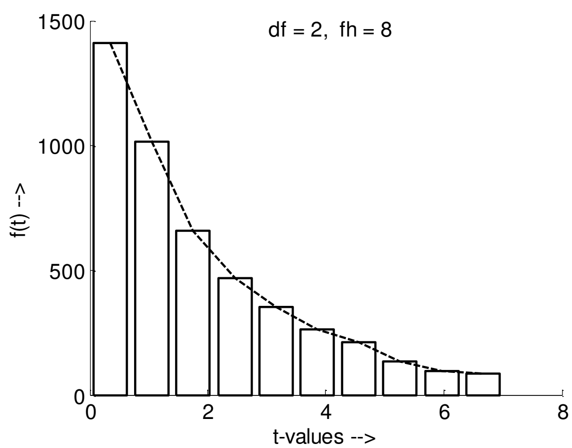


Fig 3.2

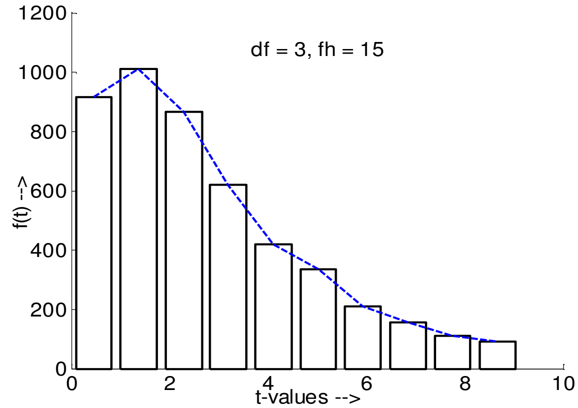
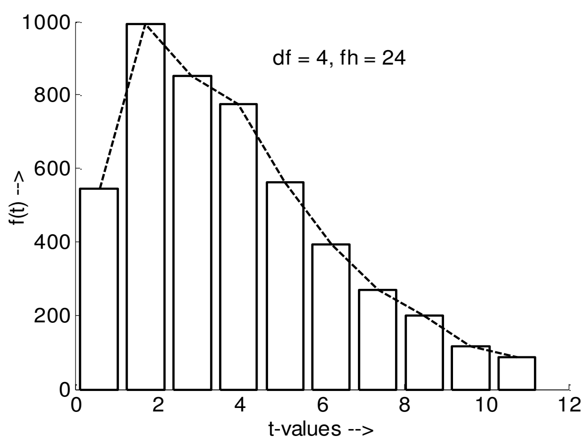

df = 3, fh = 15
f(t) -->
t-values -->
1200
1000
800
600
400
200
0
0
2
4
6
8
10

Fig 3.3


df = 4, fh = 24
f(t) -->
t-values -->
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800
600
400
200
0
0
2
4
6
8
10
12

Fig 3.4

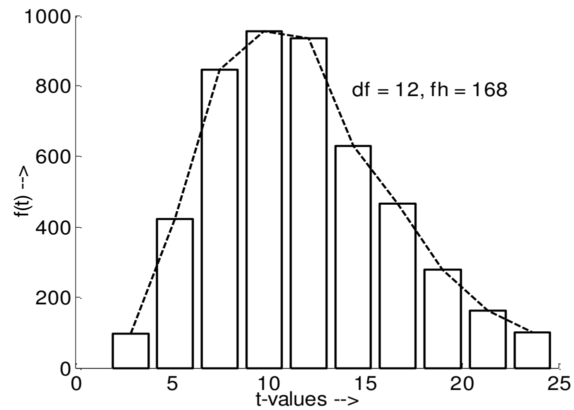

Fig 3.5

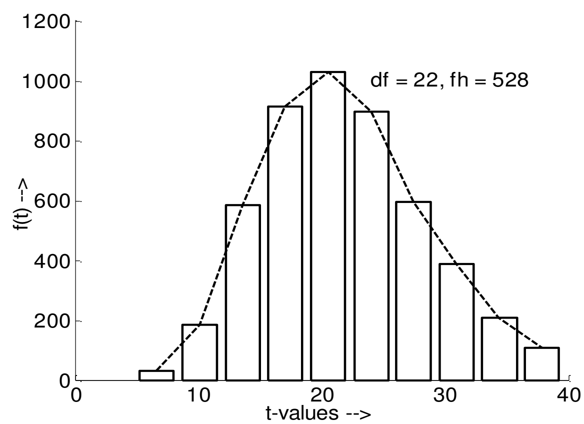

Fig 3.6

It can be noted that according to Bhuyan's (1986) and James's (1954) suggestions, the statistic (2.7) will be distributed as χ^2 when f_h are large and, is exact χ^2, when f_h's are small. The simulated study and figures show that neither critical values nor characteristics of the statistic follow χ^2 distribution, even for large or small values of f_h.

4. EXAMPLE

The hypothetical data given below were generated randomly by computer simulation technique for two sets (RBD) of five treatments.

	Data Set 1 (1st place)					Data Set 2 (2nd place)				
	Treatments					Treatments				
	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{21}	t_{22}	t_{23}	t_{24}	t_{25}
	15.10	25.20	36.13	20.39	29.91	16.71	25.47	35.64	16.88	32.74
	15.09	25.71	35.55	19.43	29.45	21.91	32.38	45.17	7.53	28.61
	16.04	25.68	34.74	21.39	28.88	14.34	28.53	29.36	10.92	27.67
	15.47	25.91	35.86	19.42	28.53	09.44	39.57	30.01	18.82	30.00
	15.10	23.74	33.34	18.44	29.64	15.28	23.26	33.25	11.98	37.56
	14.98	25.40	33.74	18.51	28.57	16.15	29.75	26.64	17.24	24.26
	15.25	25.90	34.40	20.19	29.83	18.94	23.23	32.02	21.77	31.06
Mean	15.29	25.36	34.82	19.68	29.26	16.11	28.88	33.16	15.02	30.27
	$\hat{\sigma}_1^2 = 0.48980692303533$					$\hat{\sigma}_2^2 = 28.25689105210884$				

The objective is to test the significance of treatment contrasts for two places and the hypothesis is given by

$$H_0 : A\theta_1 = A\theta_2$$

where,

$$A\theta_1 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}_{4\times 5} \begin{bmatrix} \pi_{11} \\ \pi_{12} \\ \pi_{13} \\ \pi_{14} \\ \pi_{15} \end{bmatrix}_{5\times 1}$$

and

$$A\theta_1 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}_{4\times 5} \begin{bmatrix} \pi_{21} \\ \pi_{22} \\ \pi_{23} \\ \pi_{24} \\ \pi_{25} \end{bmatrix}_{5\times 1}$$

The estimates of treatment effects and the contrasts of the type $t_{h1} - t_{hj}\ (h = 1,2; j = 2,3,...,5)$ were obtained for two places. Using the estimates of $\hat{\sigma}_h^2 (h = 1,2)$ Bartlett's χ^2 – test/ F – test was performed and it was observed that error variances were homogeneous. Thus, to test the significance of the treatment contrasts, the test statistic (2.7) was computed as $\hat{t} = 9.2501$. According to James (1954), the statistic $\hat{t}$ is to be compared with either the tabulated value of χ^2 with $(p-1)(v-1) = 4\ d.f.$ or with the critical value given by the statistic (2.9). The tabulated value of χ^2 at 5% level of significance with $4\ d.f.$ is 9.4877 and the value of h from (2.9) is 9.7621. The tabulated value based on the simulated distribution of $\hat{t}$ is 8.8485. From the analysis, it was observed that the treatment contrasts were insignificant but on the basis of the exact simulated percentile point of the statistic $\hat{t}$, the contrasts were observed as significant. In this case, the suggestions by James (1954), Bhuyan (1986) and Ali, *et al.* (1999) distorted the conclusion on the treatment contrast effects.

5. CONCLUSION

In this paper, an attempt was made to find out the exact critical values and some other distributional characteristics of the statistic (2.7) using the Monte Carlo Study for different parametric conditions. This was done with and without eliminating outliers. It was observed that simulated critical values and other distributional characteristics were less fluctuated in elimination of outliers. It was also observed that the distribution of the statistic (2.7) was not distributed as χ^2 even for large enough error degrees of freedom. The percentile points of the simulated distribution of the statistic differed significantly from those of χ^2 and from the James (1954) suggested approximate critical values h (2.9) for both large and small error degrees of freedom. The curve fitting (Fig 3.1-3.6) of the distribution also illustrated the same.

It is very likely that the investigators would wrongly reject or accept the null hypothesis if they took a decision on the basis of James's (1954) and Bhuyan's (1986) suggestions. It reflects itself clearly in the example provided. However, for valid inference using the statistic $\hat{t}$ (2.7), one may be more careful to consult the simulated critical values of the distribution embodied in this paper.

Finally, the statistic (2.7) may be applied to test the treatment contrasts for a group of experiments by considering any other design of the same sets of treatments with heterogeneous environments.

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A. A. Jamjoom
Department of Mathematics, Girls College of Education, King Abdulaziz University, Jeddah, Saudi Arabia.

M. A. Ali
Department of Statistics, King Abdulaziz University, P.O. Box. 80203, Jeddah 21589, Saudi Arabia and *on leave from* Jahangirnagar University, Dhaka, Bangladesh. Email: almech935@yahoo.com