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A NOTE ON UNIVERSALLY OPTIMAL ONE-SIDED CIRCULAR NEIGHBOUR DESIGN

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ABSTRACT

In many agricultural experiments, the nature of the treatments may be such that the treatment received by a plot may influence the responses on the neighbouring plots or the following plot in the same block. For example of the second condition, the tall varieties may affect the other crops grown on the neighbouring plots by their shades. Bailey (2003) developed a type of design concerned with the study of one sided neighbour effects for application under such condition. This paper gives (i) the relationship of such design with Block Neighbour design of Rees' type (1967), Kunert's type design (1983), Kunert and Martin (2000).

1. INTRODUCTION

For an experiment in agriculture or allied areas, where the treatment applied to one experimental plot may influence the response on the only following neighbour plot as well as the plot to which it is applied, Bailey (2003) proposed a particular type of designs that accounts for one-sided neighbour effects. The application of such design lies in the experiment of cereal crops, sunflowers and others, where tall varieties may shade the plot on their North side and influence the response of the plot as well as in pesticide or fungicide experiments where some portion of the treatment applied may spread to the plot immediately down wind and spores from untreated plots may occur. The blocks of such design have been in linear ridges [Welham *et al*. (1996)] i.e. 1-dimensional. The design where the plots are in two-dimension i.e. the design in which a treatment on a plot influences the responses of its two neighbouring plots, have been studied and also many contributions made in the literatures of Azais *et al*. (1993), Smart *et al*. (1994), Langton (1990) and David and Kempton (1996). Bailey (2003) developed such design concerned with the study of one sided neighbour effects. Later on, Bailey and Druilhet (2004) extended the work. taking into account the effect of the treatment on the preceding plot, in addition to the effect of the treatment on the following plot of the concerned plot apart from the effect of the block (if any). Both such 1-dimensional and 2-dimensional designs are under the emblem-name of Circular Neighbour-Balanced designs in their literature. The neighbour type design introduced by Rees (1967) for serological experiment in which blocks are circular plates and each block (circular plate) is divided into areas in such a way that when antigens are applied to area of circular plates each antigen must come as neighbour of any other antigens equally often, are two-sided neighbour effect design. Those designs are either complete or incomplete. In the Rees' neighbour designs, every plot gets two neighbour effects: one from the forward plot and another from the backward plot. Hwang (1973), Hwang and Lin (1977), Kageyama (1979) and Meitei (1998) had contributed some solutions and constructions of such designs. A Block Neighbour design is an arrangement of v treatments in b circular blocks of size k (not necessarily distinct) such that (i) each treatment replicates r times, (ii) every treatment appears as neighbour with every other treatment λ times and (iii) no treatment appears as neighbour of itself. We denote such a design by $BND(v,b,r,k,\lambda)$. When $k<v$, the design becomes Incomplete and it is denoted by $IBND(v,b,r,k,\lambda)$.

One-sided Circular Neighbour-Balanced design is an arrangement of v treatments in b linear blocks of size k (not necessarily distinct) such that (i) each treatment replicates r times, (ii) every pair of distinct treatments has concurrence μ and (iii) every treatment is followed by every other treatment λ times assuming that in every block the last plot is followed by the first plot. It will be denoted by One-sided $CNBD(v,b,r,k,\mu,\lambda)$. Such a design is neighbour-balanced as every treatment is followed by every other treatment λ times and also pair-wise balanced in the sense that every pair of distinct treatments has concurrence μ. Clearly, $vr=bk$, $\mu \geq 2\lambda$ with inequality sign when $k=3$ and the design is binary. These designs become circular, after having recommended to have a border plot before the first plot of each block, assuming that the treatment already applied to the last plot is applied to this border plot. But its response is not measured. It is only to get the neighbour effect of the treatment in the border plot to the first plot. So, in practical point of view, for conducting an experiment based on such designs of block size k, the planning of the design compels blocks to be of size $k+1$. These type of designs are more generalized form of $CNBD$ in the sense that in a block of One-sided $CNBD$, a treatment can be assigned with a maximum of $k-1$ times. On the other side, in a block of $CNBD$, a treatment may occur at most once, Bailey and Druilhet *op. cit.*, Definition 2, page 1652. One-sided $CNBD$ with block size of k may be $k-$ary whereas $CNBD's$ are binary. In spite of $CNBD$ being binary, the Theorem 6, page 1654 of the same literature "Under model $(M1)$ and for $3 \leq k \leq t$, a circular neighbour-balanced design in $\Omega_{(t,b,k)}$ is universally optimal for the total effects among all the designs with no treatment neighbor of itself" reveals indifferently that $CNBD$ may be $n-$ray $(n \geq 2)$. In the search of Universally optimality conditions of the proposed designs in this paper, the repetition of treatment(s) in a block may be invited. So, the readers may prefer to name the designs proposed in this paper as a generalized $CNBD$. One-sided $CNBD$ may be binary or higher-ary and such after the definition of Universally Optimal One-sided $CNBD$, a straight forward condition under which it becomes binary is given. Here below, an example of One-sided $CNBD$ to be ternary, with parameters $v=3$, $b=6$, $r=6$, $k=3$, $\mu=4$, $\lambda=2$ follows, with a notation $[\theta]$ to mean border plot receiving treatment θ.

[2]	[3]	[2]	[1]	[3]	[1]
1	2	3	2	1	3
2	3	2	1	3	1
2	3	2	1	3	1

In any block the above exemplary design, a treatment occurs either 0, 1 or 2 times. For a design, we denote by $d(i, j)$, the treatment assigned to plot j of block i and by $d(i,0)$ the border plot of block i. For the circularity of block i, $d(i,0)=d(i,k)$. However, the direct effect of $d(i,0)$ on plot j of block i is not counted for analysis purposes. By Y_{ij}, the response on plot j of block i is denoted. All the observations Y_{ij} are assumed to be with common variance and also to be independent. The linear model equations are

$$Y_{ij}=\beta_i+\tau_{d(i,j)}+\lambda_{d(i,j-1)}+\xi_{d(i,j)},$$

where β_i, $\tau_{d(i,j)}$, $\lambda_{d(i,j-1)}$ and $\xi_{d(i,j)}$ are the effect of block i, the effect of treatment $d(i, j)$, the left neighbour effect of treatment $d(i, j-1)$ and the random error of $d(i, j)$ such that

$$E(\beta_i)=\beta_i,\ E(\tau_{d(i,j)})=\tau_{d(i,j)},\ E(\lambda_{d(i,j-1)})=\lambda_{d(i,j-1)}$$

and $E(\xi_{d(i,j)})=0$.

Circular block designs [Bailey and Druilhet (2004)] are universally optimal for estimation of the total effects of direct effect of a treatment, its neighbouring treatment effect and interaction effect between the treatment and its neighbour, if (i) no treatment is ever adjacent to itself (ii) no treatment occurs more than once in a block (iii) every pair of distinct treatments is in the same number of blocks and (iv) every treatment is followed by each other same number of times. In the context of the optimality, Bailey and Druilhet (2004), Proposition 9, have mined out that such design, called One-sided Circular Neighbour-Balanced design in this paper, is universally optimal for estimation of the total effects if

(i) there are only s different types of treatments in every block,

(ii) each of n_1, out of the s different types of treatments, repeat $m(\geq 1)$ times in the block and each of the remaining n_2 occur m_1+1 times where $n_1+n_2=s$ and

(iii) all occurrences of a treatment in a block must be in a serial of adjacent plots i.e. all plots receiving a treatment in a block are one after another (possibly including both the last plot and the first plot).

(iv) under the above universally optimal condition, a treatment may appear either 0, m, or $m+1$ times in any block of such design. When $n_2 = 0$, the design becomes binary. Thus universally optimal one-sided $CNBD's$ may be either binary or ternary. For the future use in the sequel, universally optimal one-sided $CNBD(v,b,r,k,s,n_1,n_2,\mu,\lambda)$ denotes the design. Clearly, $n_1 = s - n_2$, $n_2 = k - sm$, $bs = v(v-1)\lambda$. The contribution of each block to the sum of the concurrences of all possible pairs of treatments is $\theta/2$, where

$$\theta = n_1(n_1 - 1)m^2 + n_2(n_2 - 1)(m+1)^2 + 2n_1n_2\, m(m+1)$$

$$= sm(m+1) + k(k - 2m - 1).$$

Thus

$$b\theta = v(v-1)\mu \quad \text{and} \quad bs = v(v-1)\lambda.$$

A design d with its information matrix, C_d, completely symmetric, is said to be universally optimal over a class Ω of designs iff $\text{trace}(C_d) = \max_{d\in\Omega} \text{trace}(C_d)$. Two sequences of treatments on a block are equivalent if one sequence can be obtained from the other one by relabeling the treatments. If we denote by ξ the equivalence class of the sequence l on the block u of the design d, the trace of C_{du} is given by [Bailey and Druilhet (2004)]

$$c(\xi) = tr(C_{du}) = \frac{1}{2}\left(k - \frac{2}{k}\sum_{i=1}^{v} g_i^2 + \sum_{i=1}^{v} h_i\right),$$

where g_i is the number of occurrences of treatment i in the sequence l and h_i is the number of times treatment i is on the left hand side of itself in sequence l. If a sequence 1^* in the equivalence class of sequences l, maximizes $c(\xi)$, then the equivalence class of sequences is known as optimal.

Recalling the result of Theorem 10 in Bailey and Druilhet (2004), for finding efficiency factor, we have that if D^* be a class of designs with b blocks of size $k(\geq 3)$ and v treatments and further s^* denotes an optimal equivalence class of sequences, then a design d^* that has each sequence is s^* equally often is universally optimal among all possible designs with the same size in D^*. And

$\text{trace}(C_{d^*}) = bc(s^*)$. Taking use of the efficiency factor of a design D relative to d^* given by $\text{eff}(D) = \text{trace}(C_D)/\text{trace}(C_{d^*})$ it has been further explored that for $k = 3$ or 4, one-sided *CNBD* is universally optimal for total effects among all possible designs of equal size with efficiency factor 1. When k is large, the efficiency factor for One-sided *CNBD* in D^* is approximated with $(k-2)/\{2(k-\sqrt{2k})\}$ which tends to 0.5 for k value ∞.

2. RELATIONSHIP WITH FEW OTHER DESIGNS

In this section, we will discuss the synchronized forms of One-sided *CNBD*, whether universally optimal or not, from the view of Rees' type Neighbour design, Kunert's type design and Kunert and Martin's universally optimal design.

With Rees' type Neighbour design (1967)

Both Rees' type Neighbour design (Block Neighbour design) and One-sided *CNBD* can be in complete block as well as in incomplete block. For both the designs, all the block-contents of a block are not necessarily distinct. In reality, every block of *BN* design is of circular type, whereas every block of One-sided *CNBD* is in the form of a single line with an additional plot known as border plot before the first plot of each block, where the treatment already applied to the last plot of the block, is applied. Though its outer appearance is not circular, it carries the circularity, in sense. Two distinct treatments applied to the neighbour plots of a block are considered as influential to one another. A treatment applied on a plot influences both the responses of the two neighbour plots. In the case of the One-sided *CNBD*, the treatment in a plot influences only the responses of the plot where it is assigned the treatment and the following plot, but not the response of the preceding plot. In the *BN* design, no treatment can be neighbour of itself. However, in the One-sided *CNBD* to be universally optimal, does a treatment occur repeatedly in a block, all of them should be in consecutive plots of the block so as each treatment has no neighbour effect on itself. As far as application is concerned, it can be clarified that in the experiment of sunflowers, tall sunflowers shade shorter varieties but not other sunflowers of the same height. So, the plant growing next to another of the same variety can make use of sunlight equally apart from a plant with shading i.e. of the shorter height, for photosynthesis. The plants of same height have no neighbour effect, but it is not so for the plants of the different height. Every One-sided *CNBD* with $k = s$ is *BN* design, since whenever a treatment α (or β), say, is followed by another treatment, β (or α), say, it contributes one time to λ of α and β in *BN* design. The converse is not true, because the neighbouring of α and β does not mean that α (and β) is followed by β (and α) simultaneously.

With Kunert's type design (1983)

In the class $RMD(t, N, p)$ or repeated measurements designs where t is the number of treatments, N the number of subjects and p the number of periods, a subject is repeatedly exposed to a sequence of different or identical treatments. The preceding (previous) period treatment effect (known as residue effect) on subject is counted as an effect on the same subject in the current period. So, for better analysis so as every subject has equal frequency of indirect effect (influence) from the treatment of the preceding period excepting of the first period, the design demands that in the all sequences of treatments on all subjects, every treatment is preceded by every other treatment an equal number of times. Thus, the influence (residue effect) of a treatment to direct effect of every other treatment is balanced. Suppose that a particular subject, i is exposed to a sequence of treatments, $\{d(i, j)\}$, where $d(i, j)$ denotes the treatment to the $i-$th subject in the $j-$th period $(i = 1, 2, \ldots, N\,;\; j = 1, 2, \ldots, p)$. The direct effect of $d(i,1)$ influences on subject i and no residue effect on subject i influences in the first period. For the precise, every subject in the first period doesn't get any influence from the preceding period. With imagination of layer of subjects corresponding to a subject for different periods as the plots in serial of a block, it can be asserted that the response from a plot is influenced by the treatment allotted to the preceding plot excepting the first plot of the block. The response model, in general, assumed by the authors, viz, Cheng and Wu (1980), Kunert (1983), Hedayat and Zhao (1990) is $y_{ij} = \tau_{d(i,j)} + \rho_{d(i,j-1)} + \pi_j + \beta_i + \xi_{ij}\,;\; 1 \le i \le N\,;\; 1 \le j \le p$, in which $d(i, j)$ denotes as mentioned above; $\tau_{d(i,j)}$ and $\rho_{d(i,j-1)}$ are the treatment and residue (carry over) effects; π_j the period effect and β_i the subject effect. The N independent vectors $\xi_i = (\xi_{ij})\,;\; 1 \le j \le p$, are multivariate normal with mean $\underline{0}$ and $p \times p$ covariance matrix, C. Here they assume that $\rho_{d(i,j-1)} = 0 \;\; \forall \;\; i$ at $j = 1$ i.e. there is no residual effect to all the subjects in the first period. If happened to treat the given model with $\pi_i = 0$ for One-sided $CNBD$, it is necessary to assume that $\rho_{d(i,0)} = \rho_{d(i,p)}$ i.e. the residual effect influencing the first plot of a block is from the $p-$th plot of the same block. While hunting optimality condition, covariance matrix, C, of error components of p periods is unavoidable and importantly dealt along with the sequences of the treatments, in serial, corresponding to a subject, Kushner (1997).

With Kunert and Martin's Universally Optimal design (2000)

For the undefined terms and notations in this subsection, the readers are referred to the original work of Kunert and Martin (2000). The model assumed by these authors for finding optimal repeated measurements design in more generalized form, under an interference model is for a one dimensional layout without guard plots (also known as border plots) and with different left and right neighbour effects. To determine optimal designs for contrasts among direct treatment effects that can be useful for many kinds of interference models, they consider the experiments with t treatments (their effects to be compared) on b blocks of size k with a one-dimensional arrangement of plots in each block. All blocks are assumed to be strip (non-circular). Every first plot and every last plot of a block do not get any influence as left neighbour effect (the effect from the treatment applied in the preceding plot) and as right neighbour effect (the effect from the treatment applied in the following plot) respectively as no guard (border) plot is deployed at all. Denoting $d(i, j)$, $\mu, \tau_{d(i,j)}$, $\lambda_{d(i,j-1)}$, $\rho_{d(i,j+1)}$, β_i and ξ_{ij}, the treatment assigned to the plot (i, j) in the $j-$th position of the block, the general mean, the direct effect of treatment $d(i, j)$, the left neighbour effect from the plot $(i, j-1)$, the right neighbour effect from the plot $(i, j+1)$, the effect of the block and the random error, the linear model is

$$y_{ij} = \mu + \tau_{d(i,j)} + \lambda_{d(i,j-1)} + \rho_{d(i,j-1)} + \beta_i + \xi_{ij}$$

Consequently,

$$\lambda_{d(i,0)} = \rho_{d(i,k+1)} = 0 .$$

As far as Universally Optimal One-sided *CNBD* is concerned, it has only one sort of neighbour effect, viz, left neighbour effect and a treatment allotted to the consecutive plots of a block, do not contribute left neighbour effect to these plots except the following plot to the last one of those consecutive plots in the block. That treatment influences the plot next to the last plot of those consecutive plots receiving the same treatment as the left neighbour effect of the treatment. Whereas in the Kunert and Martin's Universally Optimal design, two consecutive plots of a block, receiving same treatment are influenced by their neighbour effects vice-versa, in the form of, either left neighbour effect or right neighbour effect.

As they have studied the universally optimality of a design for interference effect, it has been shown in the Theorem 1, Kunert and Martin (2000), that the trace of information matrix of a such design, d, with $k=3$ and $t \geq 2$, is less than or equal to $(7t-8)b/6(t-1)$ and d is universally optimal over $\Omega_{t,b,3}$ if half of its blocks is with treatment sequences equivalent to $[A, A, B]$ and another half equivalent to $[A, B, B]$ and if C_{d11}, C_{d12}, C_{d13}, C_{d22}, C_{d23} and C_{d33} are completely symmetric. And also the Theorem 2 of the same literature states that the upper bound of the trace of the information matrix of a design, d, with $t=2$ and $k=4$, is $2b$ and d is universally optimal over $\Omega_{2,b,4}$ if each of $b/4-$th blocks with treatment sequences equivalent to each sequence

$[A, A, B, B]$, $[B, B, A, A]$, $[A, B, B, A]$, $[B, A, A, B]$. Further, for design $d \in \Omega_{3,b,4}$, the Theorem 3, Kunert and Martin *op. cit.* shows that upper bound of the trace the information matrix is $(257/104)b$ and if $b/2$ blocks with treatment sequences are equivalent to each of $[A, A, B, C]$ and $[A, B, C, C]$ and if C_{d11}, C_{d12}, C_{d13}, C_{d22}, C_{d23} and C_{d33} are completely symmetric, then d is universally optimal over $\Omega_{3,b,4}$. In the Theorem 4 of the same literature, it is revealed that a design $d \in \Omega_{t\geq 4,b,4}$, has information matrix C_d whose trace is less than or equal to $b[(135-23\sqrt{17})t-(42-10\sqrt{17})]/16t$. And d is universally optimal if for some α lying between 0 and 1, $(1-\alpha)(1-\delta)$, $\alpha(1-\pi)/2$, and $\{\alpha\pi+(1-\alpha)\delta\}$ of the blocks have treatment sequences equivalent to $[A, A, B, B]$, $[A, A, B, C]$, $[A, B, C, C]$ and $[A, B, C, D]$ respectively and C_{d11}, C_{d12}, C_{d13}, C_{d22}, C_{d23} and C_{d33} are completely symmetric, where $\pi = [(23-5\sqrt{17})t-(10-2\sqrt{17})]/2\sqrt{17}t$ and $\delta = [(23-3\sqrt{17})t-(10-2\sqrt{17})]/4\sqrt{17}t$. From the above Theorem 2 of Kunert and Martin (2000), it is seen that repetition of a treatment in a block may not be in consecutive plots of that block as the block structure permits $[A, B, B, A]$, $[B, A, A, B]$ for universally optimality, unlike the block structure of Universally Optimal One-sided $CNBD$.

In One-sided $CNBD$, all inner plots, in serial, of a block receive a sequence of treatments (either different or identical) and are influenced by the effect of the treatment allotted on it and the effect of the treatment on the preceding plot. Bailly and Druilhet *op. cit.*, page 1657, Proposition 9, denoting equivalence class of sequence l on block u, by s and defining $c(s) = \text{trace}(C_{du})$ where readers are referred to their original paper for notation, characterize the sequence l on block u for which $c(s)$ is to be at maximum. So under the satisfaction of maximization condition of $c(s)$ overall block of a One-sided $CNBD$, the design itself is universally optimal.

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