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TRANSSHIPMENT PROBLEM WITH UNCERTAIN DEMAND AND PROHIBITED ROUTES

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ABSTRACT

The present paper analyses a stochastic linear transshipment problem with uncertain demands and some prohibited routes. The objective is to maximize the net expected revenue, i.e. the total expected revenue minus the transportation and transshipment costs. The stochastic transshipment problem is reduced to an equivalent deterministic transportation problem for which a solution algorithm is developed and numerically illustrated.

1. INTRODUCTION

Many a times, instead of shipping directly from source to sink, the goods are sent via other sources and sinks and are transshipped at these intermediate points. Such transshipment problems often occur in the distribution system of national departmental stores chain. Transshipments also occur in the military logistics where direct transportation of goods to destination may not be advisable for security reasons. Orden (1956) showed that a transshipment problem can always be converted into a direct shipment transportation problem and solved by available methods. However the situation becomes complicated if the demands at the destinations are uncertain and must be treated as random variables instead of fixed constants. Complications are further enhanced if some routes are prohibited due to reasons such as security, road construction, weight limits on bridges, unexpected floods, transportation strikes and local traffic ordinances etc. The purpose of this paper is to study a stochastic linear transshipment problem with uncertain demands and some prohibited routes. No doubt, the prohibited routes may sometimes cause infeasibility Currin (1986), but this study is concerned with feasible problems only. For dealing with uncertainty of demands, we have used the technique of Dantzig (1956, 1963) as applied by Javaid *et al.* (1998, 1999).

2. NOTATIONS AND THE FORMULATION OF THE PROBLEM

Consider a transshipment problem with m sources numbered $1, 2, \cdots, m$ and n sinks numbered as $m+1, m+2, \cdots, m+n$. The sequential numbering of sources and sinks is found convenient because in a transshipment problem every source and sink acts both as a shipping point as well as a receiving point of goods.

Let

a_i : the quantity available at source $i = 1, 2, \cdots, m$

b_j : the quantity demanded at sink $j = m+1, m+2, \cdots, m+n$

x_{ij} : the quantity shipped from station i to j $(i, j = 1, 2, \cdots, m+n)$

c_{ij} : the per unit shipment cost from station i to j $(i, j = 1, 2, \cdots, m+n)$

t_i : quantity transshipped at the station i $(i = 1, 2, \cdots, m+n)$

l_i : per unit transshipment cost (including unloading, reloading, and storage etc.) at the station i $(i = 1, 2, \cdots, m+n)$.

The problem is to determine x_{ij} so as to minimize the total cost of transportation and transshipment. It may be mathematically stated as under:

Problem P$_1$: Find x_{ij} so as to

$$\text{Minimize} \quad F = \sum_{i=1}^{m+n}{}^{*} \sum_{j=1}^{m+n}{}^{*} c_{ij} x_{ij} + \sum_{i=1}^{m+n} l_i t_i \tag{2.1}$$

subject to

$$\sum_{j=1}^{m+n}{}^{*} x_{ij} = \begin{cases} a_i + t_i & i = 1, 2, \cdots, m \\ t_i & i = m+1, \cdots, m+n \end{cases} \tag{2.2}$$

$$\sum_{i=1}^{m+n}{}^{*} x_{ij} = \begin{cases} u_j & j = 1, 2, \cdots, m \\ t_j + u_j & j = m+1, \cdots, .m+n \end{cases} \tag{2.3}$$

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j \tag{2.4}$$

$\sum_{j=1}^{m+n}{}^{*}$ indicates that the term $j = i$ is excluded from the sum. The constraints (2.2) implies that the total quantity that leaves the source i $(= 1, 2, \cdots, m)$ is equal to the quantity available plus the quantity transshipped and the total quantity that leaves the sink i $(= m+1, m+2, \cdots, m+n)$ is equal to the quantity transshipped. Similarly constraints (2.3) imply that the total quantity that arrives at a source i $(= 1, 2, \cdots, m)$ is equal to the quantity that source transships and the total arriving at a sink is equal to the demand at that sink plus the quantity that the sink transships. Constraints (2.4) are the usual nonnegative restrictions. Here t_i are unknown, so we impose an upper bound t_o (say), on the amount that can be transshipped at any point, so that

$$t_i = t_o - x_{ii}, \qquad i = 1, 2, \cdots, m, \tag{2.5}$$

where x_{ii} is a nonnegative slack. After substituting (2.5) in (2.1) to (2.3) and on simplifying the original transshipment problem P_1 is reduced to the following genuine transportation type linear programming problem.

Problem P_2:

Minimize $$Z = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} + \sum_{i=1}^{m+n} l_i t_o$$

subject to

$$\sum_{j=1}^{m+n} x_{ij} = \begin{cases} a_i + t_o & i = 1, 2, \cdots, m \\ t_o & i = m+1, \cdots, m+n \end{cases} \tag{2.6}$$

$$\sum_{i=1}^{m+n} x_{ij} = \begin{cases} t_o & j = 1, 2, \cdots, m \\ b_j + t_o & j = m+1, \cdots, m+n \end{cases} \tag{2.7}$$

$$x_{ij} \geq 0, \tag{2.8}$$

where $c_{ii} = -l_i$, and the asterisk (*) on the summations has now been disappeared due to the inclusion of x_{ii}.

Here, the upper bound t_o can interpreted as the size of a fictitious stockpile at each source and sink which is large enough to take care of all transshipments. Assume initially a value for t_o which is sufficiently large to ensure that all x_{ii} will be in the optimal basis. Such a value can be easily found as the volume of goods transshipped at any point cannot exceed the total volume of goods produced (or received). Hence, set,

$$t_o = \sum_{i=1}^{m} a_i \tag{2.9}$$

which ensures that t_o is not limiting. The unused stockpile at the station $i = 1, 2, \cdots, m+n$, if any, will be absorbed in the slack x_{ii}.

Thus, the $m \times n$ order transshipment problem P_1 has been converted into a direct shipment transportation problem P_2 of order $(m+n) \times (m+n)$ which can have no more than $2(m \times n) - 1$ variables different from zero. However, $(m+n)$ of these non zero variables are the slack variables x_{ii} representing the unused stockpile and hence there are in fact no more than $(m+n-1)$ variables of interest which are different from zero.

So far we have been silent about the nature of the demands b_j and have treated them as fixed constants. However, in real life the demands are usually uncertain.

So, in order to make our study more realistic, we assume b_j to be independent discrete random variables with known probability distributions as below:

Demand b_j	$b_{1j} <$	$b_{2j} <$	$\cdots$	$b_{H_j i}$
$\Pr(b_j = b_{hj}) = p_{hj}$	p_{1j}	p_{2j}	$\cdots$	$p_{H_j i}$
$\Pr(b_j \geq b_{hj}) = \pi_{hj}$	$\pi_{1j} = \sum_{h=1}^{H_j} p_{hj}$ (1)	$\pi_{2j} = \sum_{h=2}^{H_j} p_{hj}$	$\cdots$	$\pi_{H_j j} = p_{H_j j}$

However, the moment we treat b_j as random variable, a new problem begins to rear its head. The constraints (2.7) fail to make sense. To make the problem meaningful it has to be reformulated into a deterministic equivalent (1956, 1963).

3. EQUIVALENT DETERMINISTIC PROBLEM

To take care of the randomness of demands, instead of minimizing the total cost, we take our objective as the maximization of the net expected revenue (i.e., total expected revenue minus transshipment and transportation costs).

Let s_j be the per unit selling price of the product shipped to the sink j and let $f_j(s_j, Y_j)$ be the yet unknown function that describes the expected revenue from the sink j if a net total of Y_j units are shipped to that sink. So the net expected revenue is:

$$R = \sum_{j=1}^{m+n} f_j(s_j, Y_j) - \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} - \sum_{i=1}^{m+n} l_i t_o$$

Here the third term of the right hand side viz. $\sum_{i=1}^{m+n} l_i t_o$ is a constant that can be ignored but adjusted in the end. Since we have to maximize R (or minimize – R), so, after ignoring the constant term, our objective becomes:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} - \sum_{j=1}^{m+n} f_j(s_j, Y_j) \tag{3.1}$$

To determine the function $f_j(s_j, Y_j)$, note that Y_j, the net quantity shipped to sink j, can be any amount between the lowest value b_{1j} and the highest value $b_{H_j j}$ in the probability distribution of the demand b_j, $j = 1, 2, \cdots, n$.

If $0 \le Y_j \le b_{1j}$, then each of the Y_j units shall be absorbed with probability $\pi_{1j}\ (=1)$.

Hence, the expected revenue is $s_j \pi_{1j} Y_j$.

If $b_{1j} \le Y_j \le b_{2j}$, then each unit upto b_{1j} shall be absorbed with probability π_{1j} and each of the additional units $(Y_j - b_{1j})$ shall be absorbed with probability π_{2j}.

Hence, the expected revenue is $s_j \pi_{1j} b_{1j} + s_j \pi_{2j} (Y_j - b_{1j})$.

So, in general, if $b_{hj} \le Y_j \le b_{h+1j}$, then the expected revenue is

$$s_j \{\pi_{1j} b_{1j} + \pi_{2j} (b_{2j} - b_{1j}) + \cdots + \pi_{2j} (b_{hj} - b_{h-1j}) + \pi_{h+1j} (Y_j - b_{hj})\}.$$

Let us now break y_j into incremental units y_{hj} $(h = 1, 2, \cdots, H_j)$ as:

$$Y_j = y_{1j} + y_{2j} + \cdots + y_{hi} + \cdots + y_{H_j j}, \tag{3.2}$$

where,

$$\left.\begin{array}{ll} 0 \le y_{1j} \le b_{1j} & = R_{1j} \\ 0 \le y_{2j} \le b_{2j} - b_{1j} & = R_{2j} \\ \vdots \quad \vdots \quad \vdots & \vdots \\ 0 \le y_{H_j j} \le b_{H_j j} - b_{H_j - 1, j} & = R_{H_j j} \end{array}\right\} \tag{3.3}$$

Relation (3.2) makes physical sense only if there exists some $h = h_j$ (say) such that all intervals below the $h_j -$th interval are filled to capacity and all intervals above it are empty i.e.

$$\left.\begin{array}{ll} y_{hj} = R_{hj} & (h = 1, 2, \cdots, h_j - 1) \\ y_{hj} \le R_{hj} & (h = h_j) \\ y_{hj} = 0 & (h = h_j + 1, \cdots, H_j) \end{array}\right\} \tag{3.4}$$

Assuming for the time being that the conditions (3.4) hold, the total expected revenue from sink j is:

$$f_j (s_j, y_j) = \sum_{h=1}^{H_j} s_j \pi_{hj} y_{hj}.$$

Substituting the value of $f_j (s_j, y_j)$ in (3.1) and treating both x_{ij} and y_{hj} as decision variables, the deterministic equivalent to Problem P_2, is:

Problem P$_3$:

Minimize $$Z = \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} c_{ij}\, x_{ij} + \sum_{i=1}^{m+n}\sum_{h=1}^{H_j} d_{hj}\, y_{hj}\,, \tag{3.5}$$

where $(d_{hj} - s_j \pi_{hj})$.

subject to

$$\sum_{j=1}^{m+n} x_{ij} = \begin{cases} a_i + t_o & i = 1, 2, \cdots, m \\ t_o & i = m+1, \cdots, .m+n \end{cases} \tag{3.6}$$

$$\sum_{i=1}^{m+n} x_{ij} = t_o\,, \qquad j = 1, 2, \cdots, m \tag{3.7}$$

$$\sum_{i=1}^{m+n} x_{ij} - \sum_{h=1}^{H_j} y_{hj} = t_0\,, \qquad j = m+1, \cdots, m+n \tag{3.8}$$

$$x_{ij}\,,\ y_{hj} \ge 0 \qquad (\forall\, i, j, h) \tag{3.9}$$

$$y_{hj} \le R_{hj} \qquad (\forall\, j, h) \tag{3.10}$$

and subject to the additional stipulation that the constraints (3.4) are also satisfied.

Fortunately, it turns out that (3.4) do not restrict our choice of optimum solution in any way. This we prove in the following theorem.

Theorem 3.1: A feasible solution to Problem P$_3$ can always be improved if it violates any of the constraints (3.4).

Proof: Let (x_{ij}^*, y_{hj}^*) be a feasible solution to Problem P$_3$ obtained on ignoring (3.4). The value of Z at this solution is:

$$Z^* = \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} c_{ij}\, x_{ij}^* + \sum_{i=1}^{m+n}\sum_{h=1}^{H_j} d_{hj}\, y_{hj}^*$$

Suppose that there exists some $h = h^o$ and $j = j^o$ such that $y_{h^o j^o}^* < R_{h^o j^o}$ and $y_{h^o+1, j^o}^* > 0$.

It is clearly a violation of the constraints (3.4).

Now, we increase $y^*_{h^o j^o}$ and decrease $y^*_{h^o+1, j^o}$ by equal amounts $\theta(>0)$ such that the feasibility of the solution is not disturbed. The new value of the objective function becomes: $Z^o = Z^* + \theta(d^*_{h^o j^o} - d^*_{h^o+1, j^o})$.

But $(d^*_{h^o j^o} - d^*_{h^o+1, j^o}) \le 0$, as $\pi_{hj} \ge \pi_{h+1, j}$ for all h and j.

Hence it follows that $Z^o \le Z^*$.

This result shows that if an optimum solution to Problem P_3 is obtained after ignoring (3.4), it shall on its own satisfy (3.4). Thus, to solve problem P_3, we may simply ignore the constraints (3.4).

4. PRELIMINARIES TO THE SOLUTION OF PROBLEM P_3

i) It is assumed that the set of all feasible solutions of Problem P_3 is regular (i.e. non- empty and bounded).

ii) Problem P_3 is a transportation type linear programming problem with upper bound restrictions on some variables. So, its global minimum exists at a basic feasible solution of its constraints.

iii) We shall, hereinafter, call the constraints (3.5) through (3.8) as the original system and the constraints (3.5) through (3.9) as the capacitated system. As none of the constraints in the original system is redundant, a basic feasible solution to the original system shall contain $2(m+n)$ basic variables. For the capacitated system also, a basic feasible solution shall contain $2(m+n)$ basic variables and the same may be found by working on the original system provided that some of the non basic variables are allowed to take their upper bound values (1963).

iv) The special structure of Problem P_4 permits us to arrange it into an array as shown in table 1.

In the table below, there are $(m+n)$ rows in columns $j = 1,2,\cdots,m$ and $(m+n+H)$ rows in columns $j = m+1,\cdots,m+n$. Here $H = \max.H_j$, so that there shall be some empty boxes near the bottom of the table in columns $j = m+1,\cdots,m+n$. These empty boxes shall be crossed out.

Absence of the row totals for y_{hj}'s in the table indicates that there are no row equations for y_{hj} variables. Besides, to obtain the column equations (3.8), each y_{hj} has to be multiplied by (-1). We have omitted (-1) from y_{hj} boxes for convenience.

Table 1:

x_{11} c_{11}	…	x_{1m} c_{1m}	x_{1m+1} c_{1m+1}	…	x_{1m+n} c_{1m+n}	$a_1 + t_o$
… …	… …	… …	… …	… …	… …	… …
x_{m1} c_{m1}	…	x_{mm} c_{mm}	$x_{m\,m+1}$ $c_{m\,m+1}$	…	$x_{m\,m+n}$ $c_{m\,m+n}$	$a_m + t_o$
x_{m+11} c_{m+11}	…	x_{m+1m} c_{m+1m}	x_{m+1m+1} c_{m+1m+1}	…	x_{m+1m+n} c_{m+1m+n}	t_o
… …	… …	… …	… …	… …	… …	… …
x_{m+n1} c_{m+n1}	…	x_{m+nm} c_{m+nm}	$x_{m+n\,m+1}$ $c_{m+n\,m+1}$	…	$x_{m+n\,m+n}$ $c_{m+n\,m+n}$	t_o
t_o	…	t_o	y_{1m+1} R_{1m+1} d_{1m+1}	…	y_{1m+n} R_{1m+n} d_{1m+n}	
			… …	… …	… …	
			$Y_{H\,m+1}$ $R_{H\,m+1}$ $d_{H\,m+1}$	…	$Y_{H\,m+n}$ $R_{H\,m+n}$ $d_{H\,m+n}$	
			t_o	…	t_o	

5. INITIAL BASIC FEASIBLE SOLUTION

To start with, we fix the demands b_j's approximately equal to their expected values such that

$$\sum_{j=m+1}^{m+n} b_j = \sum_{i=1}^{m} a_i$$

and also such that for all j except $j = j^*$, each b_j falls at the upper end of one of the intervals y_{hj} into which b_j has been divided, i.e.

$$b_j = \sum_{h=1}^{h'_j} R_{hj}$$

for some $h'_j \le H_j$ and for all j except $j = j^*$ (the b_j can always be so chosen that it is done).

With these fixed demands the upper portion of the Table 5.1 resembles a $(m+n)\times(m+n)$ standard transportation problem for which an initial basic feasible solution with $\{2(m+n)-1\}$ basic variables is obtained by any of the several available methods. Now, in each of the columns $j = m+1,\cdots,m+n$, the values of the non basic y_{hj}'s are entered at their upper bounds in turn $h = 1, 2,\ldots$ until we have entered enough non basic y_{hj}'s so that their sum over h is equal to b_j. Obviously, we shall never have to enter y_{hj} below its upper bound except in column $j = j^*$, where the last nonzero entry will be $y_{hj^*} \le R_{hj^*}$. This last entry and the $\{2(m+n)-1\}$ basic x_{ij}'s found earlier constitute the required initial basic feasible solution with $2(m+n)$ basic variables. In case the last non zero entry in column j^* is also at its upper bound and then we take the last y_{hj} entry of any column as our $2(m+n)-$th basic variable.

6. Optimality Criteria

Let the simplex multipliers corresponding to the objective function Z (Problem P_3) be u_i and v_j $(\forall\, i, j = 1,2,...,m+n)$.

These are determined by solving the following equations.

$$\left.\begin{aligned} c_{ij} + u_i + v_j &= 0 \quad \text{for basic } x_{ij} \\ d_{hj} - v_j &= 0 \quad \text{for basic } y_{hj} \end{aligned}\right\} \tag{6.1}$$

These are $2(m+n)$ linear equations in as many unknowns u_i and v_j and can be easily solved. Let the relative cost coefficients corresponding to the variables x_{ij} and y_{hj} be C'_{ij} and d'_{hj}. These are determined by solving the following equations.

$$\left.\begin{aligned} C'_{ij} &= c_{ij} + u_i + v_j \quad \text{for non basic } x_{ij} \\ d'_{hj} &= d_{hj} - v_j \quad \text{for non basic } y_{hj} \end{aligned}\right\} \tag{6.2}$$

It can be easily shown that for a given basic feasible solution (x_{ij}, y_{hj}) of the Problem P_3, the value of the objective function Z is:

$$Z = \sum_{i=1}^{m+n}\sum_{j=1}^{m+n} C'_{ij} x_{ij} + \sum_{j=1}^{m+n}\sum_{h=1}^{H_j} d'_{hj} y_{hj} - \left\{ \sum_{i=1}^{m} u_i\,(a_i + t_o) + \sum_{i=m+1}^{m+n} u_i t_o + \sum_{j=1}^{m+n} v_j t_o \right\} \tag{6.3}$$

Here, $C'_{ij} = 0$ for all basic x_{ij} and also the values of the non basic x_{ij} are zero. So, the first term on the right hand side of (6.3) vanishes. Similarly $d'_{hj} = 0$ for the basic y_{hj}, but as regards the values of non basic $y_{hj}'s$ - some are zero and the others are at their upper bounds. Hence,

$$Z = \sum_{j=1}^{m+n}{}^{*}\sum_{h=1}^{H_j}{}^{*} d'_{hj} R_{hj} - \left\{ \sum_{i=1}^{m} u_i\,(a_i + t_o) + \sum_{i=m+1}^{m+n} u_i t_o + \sum_{j=1}^{m+n} v_j t_o \right\}, \qquad (6.4)$$

where $\sum^{*}$ indicates the sum over those non basic y_{hj} which are at their upper bounds. Now if the value of any one of the non basic variables x_{st} or y_{rt} is changed to:

$$\hat{x}_{st} = (x_{st} + \theta) \quad \text{or} \quad \hat{y}_{rt} = (y_{rt} + \theta),$$

with the other non basic variables remaining unaltered and the basic variables adjusted to maintain feasibility of the solution, then the improved value of Z shall be:

$$\hat{Z} = Z + \theta C'_{st} \quad \text{or} \quad \hat{Z} = Z + \theta d'_{rt},$$

as the case may be.

Note that we take plus sign if $y_{rt} = 0$ and minus sign if $y_{rt} = R_{rt}$.

The objective function will improve iff $\hat{Z} - Z < 0$, i.e.

$$(Z + \theta C'_{st}) - Z < 0 \quad \text{or} \quad (Z + \theta d'_{rt}) - Z < 0$$

$$\Rightarrow \quad \theta C'_{st} < 0 \quad \text{or} \quad \pm\theta d'_{rt} < 0$$

$$\Rightarrow \quad C'_{st} < 0 \quad \text{or} \quad \pm d'_{rt} < 0$$

(Since, in the non degenerate case $\theta > 0$ and in degenerate case $\theta = 0$.

$\Rightarrow \quad \hat{Z} = Z$.

Thus, the current solution is optimum iff

$$\left.\begin{array}{ll} C'_{ij} \geq 0 & (\forall \text{ non basic } x_{ij}) \\ d'_{hj} \geq 0 & (\forall \text{ non basic } y_{hj} \text{ at zero level}) \\ d'_{hj} \leq 0 & (\forall \text{ non basic } y_{hj} \text{ at upper bound}) \end{array}\right\} \qquad (6.5)$$

If any of the optimality criteria (6.5) is violated, the current solution can be improved. The non basic variable which violates (6.5) most severely is selected to enter the basis. The values of the new basic variables are found by applying the usual $\theta-$adjustments. It should, however, be kept in mind that the coefficient of each y_{hj} in column equations (3.8) is (-1). The variable to leave the basis is the one that becomes either zero or equal to its upper bound. If two or more basic variables reach zero or their upper bounds simultaneously then only one of them becomes non basic. Should it happen that the entering variable itself attains upper or lower bound (zero) without simultaneously making any of the basic variables zero or equal to its upper bounds, the set of basic variables remains unaltered; only their values are changed to allow the so-called entering variable to be fixed at its upper or lower bound.

7. PROHIBITED ROUTES

So far all routes from any source or sink to other sources or sinks have been treated as open and usable but in reality some routes may be prohibited due to traffic regulations or other practical considerations. The prohibited routes may sometimes lead to infeasibility even if the total supply exceeds or equals the maximum total demand (1986). If so, one has no option but to adjust the supplies or the demands in order to obtain a feasible solution.

However, in a transshipment problem with a few prohibited routes, infeasibility rarely arises because there are generally several alternative routes between any two points. So, our problem is only to modify the algorithm in such a way that the prohibited routes are eliminated from entering our solution as we move towards optimality. This is easily achieved by assigning a very high cost (say 'M') to each of the prohibited routes.

8. NUMERICAL EXAMPLE

Consider the following transshipment Problem with three origins named S_1, S_2, S_3 and two destinations named S_4, S_5. Table 2 gives the per unit transportation cost between the various origins and destinations, the per unit selling prices s_j of the product at the two destinations and the supplies a_i available at the origins. Shaded cells indicate the prohibited routes.

Table 2:

	S_1	S_2	S_3	S_4	S_5	Supplies a_i
S_1	0	4	6	3		**10**
S_2	3	0	8	2	2	**5**
S_3	6	8	0	0.5	1	**6**
S_4	3	2	0.5	0	5	
S_5		2	1	5	0	
Demands b_j				b_4	b_5	
s_j				**10**	**5**	

For simplicity, we take the transshipment costs $l_i = 0$, $\forall \quad i = 1,2,3,4,5$.

The probability distributions of the random demands b_j, $j = 4,5$ are as given in the following table 3 along with the computed values of π_{hj}, d_{hj} and R_{hj}.

Table 3:

j	b_{hj}	p_{hj}	π_{hj}	$d_{hj} = -s_j \pi_{hj}$	R_{hj}
4	9 12 17	0.3 0.5 0.2	1.0 0.7 0.2	-10 -7 -2	9 3 5
5	7 10	0.2 0.8	1.0 0.8	-5 -4	7 3

Assigning a very high cost M to each of the prohibited routes, and assuming an additional fictitious stockpile equal to total supply of $21(=10+5+6)$ units at each of the 5 shipping and 5 receiving points, the deterministic version of the problem is as given in table 4 below. The last cell of column 5 is empty and hence shaded.

Table 4:

x_{11} 0	x_{12} 4	x_{13} 6	x_{14} 3	x_{15} M	31
x_{21} 3	x_{22} 0	x_{23} 8	x_{24} 2	x_{25} 2	31
x_{31} 6	x_{32} 8	x_{33} 0	x_{34} 0.5	x_{35} 1	27
x_{41} 3	x_{42} 2	x_{43} 0.5	x_{44} 0	x_{45} 5	21
x_{51} M	x_{52} 2	x_{53} 1	x_{54} 5	x_{55} 0	21
21	21	21	33	30	
			Y_{11} -5	Y_{12} -10	
			y_{21} -7	y_{22} -4	
			y_{31} -2		
			21	21	

Working tables for finding the optimum solution are prepared likewise. To avoid confusion, the non basic variables at zero level are omitted and the non basic variables at upper bound are encircled. Besides, C'_{ij} and d'_{hj} which violate the optimality criteria (6.5) are also entered in the top right corner of the non basic cells.

Iteration 1:

Step 1) To obtain initial basic feasible solution we fix the demands at $b_1 = 12$ and $b_2 = 9$, and then determine an initial basic feasible solution to the (5×5) standard transportation problem above the double line in Table 8.3, by the North-West corner Rule. We get,

$x_{11} = 21,\quad x_{14} = 10,\quad x_{22} = 21,\quad x_{24} = 2,\quad x_{25} = 3,\quad x_{33} = 21,$

$x_{35} = 6,\quad x_{44} = 21,\quad x_{55} = 21.$

In order to obtain the *IBFS* to the deterministic equivalent transportation problem of Table 4, we assign y_{hj} entries at their upper bounds (as far as possible) so that the column equations (3.8) are satisfied, we get

$y_{11} = 9,\ y_{21} = 3,\ y_{12} = 7$ and $y_{22} = 2\quad (< R_{22} = 3)$.

This provides the required initial basic feasible solution with x_{11}, x_{14}, x_{21}, x_{22}, x_{24}, x_{25}, x_{33}, x_{35}, x_{44}, x_{55} and y_{22} as the basic variables.

Step 2) The simplex multipliers (u_i, v_j) and relative cost coefficients (C'_{ij}, d'_{hj}), are determined from equations (6.1) and (6.2). These are also recorded in the working Table 8.4.

Step 3) For the current solution, the value of $Z = -108$ is obtained from (6.4).

Step) For the non-basic variables, the values of C'_{ij} and d'_{hj} are calculated and it is found that only $C'_{34} = 0.5 + 3 - 4 = -0.5$ violates the optimality criterion, obviously the current solution is not optimum and may further be improved.

Step 5) Adding θ to x_{34}, we are led to the θ–adjustments as shown in table 8.4. The maximum possible value of θ is $\theta^* = 2$. The new solution is recorded in table 5.

Iteration 2:

Repeating steps 2 to 4, the new solution is found to be optimum. So, after the second iteration the optimal solution obtained is:

$x_{11} = 21,\quad x_{14} = 10,\quad x_{22} = 21,\quad x_{25} = 5,\quad x_{33} = 21,\quad x_{34} = 2,$

$x_{35} = 4,\quad x_{44} = 21,\quad x_{55} = 21$ and $Z_{opt} = -109.$

Table 5:

					a_i	u_i
[21] 0	4	6	[10] 3	M	31	1
3	[21] 0	8	[2] $-\theta$ 2	[3] $+\theta$ 2	26	2
6	8	[21] 0	θ ***-0.5*** 0.5	[6] $-\theta$ 1	27	3
3	2	0.5	[21] 0	5	21	4
M	2	1	5	[21] 0	21	4
			(9) -10	(7) -5		
			(3) -7	[2] -4		
			-2			
v_1= -1	v_2= -2	v_3= -3	v_4= -4	v_5= -4		

Table 6:

					a_i	u_i
[21] 0	4	6	[10] 3	M	31	1
3	[21] 0	8	2	[5] 2	26	2
6	8	[21] 0	[2] 0.5	[4] 1	27	3
3	2	0.5	[21] 0	5	21	4
M	2	1	5	[21] 0	21	4
			(9) -10	(7) -5		
			(3) -7	[2] -4		
			-2			
v_1= -1	v_2= -2	v_3= -3	v_4= -4	v_5= -4		

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