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COST-BENEFIT ANALYSIS OF A STANDBY SYSTEM WITH INSPECTION SUBJECT TO DEGRADATION

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ABSTRACT

The purpose of this paper is to develop a cost-benefit analysis for a standby system considering the concepts of inspection and degradation. The system has two identical units – one is operative and the other unit is kept as cold standby. There is single server who visits the system immediately whenever needed. Unit becomes degraded after repair. Server inspects the degraded unit at its failure to examine the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new one. The distribution of failure time of the units follows negative exponential while that of inspection and repair times are taken as arbitrary with different probability density functions (pdf). Regenerative point technique is adopted to carryout the analysis. Numerical results for some reliability and economic measures pertaining to the case when inspection and repair time distributions are exponentially distributed have been obtained.

1. INTRODUCTION

Two unit reparable systems have been widely studied in the literature of reliability due to their frequent and vital use in modern industry. A large number of authors including Chang and Niu (1981), Naidu and Gopalan (1984), Singh (1989), Kadyan *et al.* (2004), Chander (2005) and Chander *et al.* (2007) have analyzed reliability models with different repair techniques under the following common assumptions:

i) Unit works as new after repair.

ii) Repair of the unit is always possible and economical to the system.

In fact, these assumptions can not be imposed on every system because the working capability of the unit after repair more or less depends on quality of the unit and repair mechanism. Therefore, a unit of substandard quality may have increased failure rate in case of being repaired by an ordinary server and thus the unit is declared as degraded. Mokaddis *et al.* (1997) have proposed a reliability model for standby system subject to degradation. Also, sometimes repair of the degraded unit is neither possible nor economical to the system due to excessive use and high cost of maintenance. In such cases, the failed unit may be replaced by new one and this can be examined by inspection.

In view of the above, this paper deals with a reliability model develop for a two-unit cold standby system by introducing the concepts of inspection and degradation. There is a single server who comes immediately to inspect and repair the unit as and when required. Unit does not work as new after repair and so called a degraded unit. Server inspects the degraded unit at its failure to see the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new one so that unnecessary expanses on repair may be avoided. The failure rate of the degraded unit after repair is taken as same. Switches are perfect. All random variables are mutually independent and uncorrelated. The failure time distribution of the unit follows negative exponential whereas the distributions of inspection and repair times are assumed as arbitrary. Some reliability and economic measures such as mean sojourn times, mean time to system failure $(MTSF)$, steady state availability, busy period and expected number of visits are obtained by using semi-Markov process and regenerative point technique to carry out the cost benefit analysis. Later on, a special case when inspection and repair times are exponentially distributed is considered for evaluating numerical results to some interesting reliability and economic measures.

2. NOTATION

E	:	Set of regenerative states
O	:	Unit is operative
CS	:	Cold standby
λ / λ_1	:	Constant failure rate of original unit / degraded unit
p / q	:	Probability that repair is not feasible / feasible
DO	:	Degraded unit is operative
DCO	:	Degraded unit in cold standby
OCS	:	Original unit in cold standby
$FU_r / FUR / FW_r / FWR$	:	Original unit failed and under repair / under repair continuously from previous state / waiting for repair / waiting for repair continuously from previous state
$DFW_i / DFW / DFU_i / DFUI$	:	Degraded unit failed and waiting for inspection / waiting for inspection continuously from previous state / under inspection / under inspection continuously from previous state
$DFU_r / DFUR$	:	Degraded unit failed and under repair / under repair continuously from previous state

$g(t)/G(t)$	:	pdf/cdf of repair time of original unit
$g_1(t)/G_1(t)$	:	pdf/cdf of repair time of degraded unit
$h(t)/H(t)$	:	pdf/cdf of inspection time of server to the degraded unit
$q_{ij}(t),\ Q_{ij}(t)$	:	pdf/cdf of direct transition time from a regenerative state i to a regenerative j or a failed state j without visiting any other regenerative state in $(0,t]$
$M_i(t)$	:	Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state
$W_i(t)$	:	Probability that the server is busy in state S_i upto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
m_{ij}	:	Contribution to mean sojourn time in state S_i when the transition is to S_i $(S_i, S_j \in E)$ given by $m_{ij} = \int q_{ij}(t)dt = \int dQ_{ij}(t) = -\left[\frac{d}{ds}(Q_{ij}^*(s))\right]_{s=0}$ and $\mu_i = \sum_j m_{ij}$, where μ_i is the mean sojourn time in state $S_i \in E$
pdf/cdf	:	Probability density function / cumulative distribution function
LST/LT	:	Laplace Stieltjes transform / Laplace transform
~	:	Symbol for LST e.g. $\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st} q_{ij}(t)\,dt$
*	:	Symbol for Laplace transform
′	:	Desh
Ⓢ/©	:	Symbol for Stieltjes convolution/Laplace convolution
$S_i\ (i = 0-17)$	:	The possible transition states

The transition states $S_0 - S_9$ are regenerative while the remaining states are non-regenerative. The possible transition between states along with transition rates for the system model is shown in figure 1.

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt \text{ as}$$

$$p_{01} = 1 = p_{23} = p_{12,3} = p_{68} = p_{45} = p_{15,5} = p_{13,5}, \quad p_{12} = g^*(\lambda),$$

$$p_{1,16} = 1 - g^*(\lambda), \quad p_{13.16} = 1 - g^*(\lambda), \quad p_{34} = g^*(\lambda_1), \quad p_{35.14} = 1 - g^*(\lambda_1),$$

$$p_{3,14} = 1 - g^*(\lambda_1), \quad p_{57} = qh^*(\lambda_1), \quad p_{5,10} = 1 - h^*(\lambda_1), \quad p_{5,6} = ph^*(\lambda_1),$$

$$p_{58,10} = [1 - h^*(\lambda_1)]p, \quad p_{55.10,15} = [1 - h^*(\lambda_1)]q, \quad p_{74} = g_1^*(\lambda_1),$$

$$p_{7,13} = 1 - g_1^*(\lambda_1), \quad p_{75.13} = 1 - g^*(\lambda_1), \quad p_{89} = qh^*(\lambda), \quad p_{8,11} = 1 - h^*(\lambda),$$

$$p_{80} = ph^*(\lambda), \quad p_{81.11} = [1 - h^*(\lambda)]p, \quad p_{83.11,17} = [1 - h^*(\lambda)]q,$$

$$p_{92} = g_1^*(\lambda), \quad p_{9,12} = 1 - g_1^*(\lambda), \quad p_{93.12} = 1 - g_1^*(\lambda), \quad p_{11.1} = p = p_{10,8},$$

$$p_{11,17} = q = p_{10,15}, \quad p_{14,5} = g^*(0) = p_{16,3}, \quad p_{17,3} = g_1^*(0) \tag{3.1}$$

The mean sojourn times μ_i in the state S_i are

$$\mu_0 = \int_0^\infty P(T > t)\,dt = \frac{1}{\lambda} = \mu_2, \quad \mu_1 = \frac{1 - g^*(\lambda)}{\lambda}, \quad \mu_3 = \frac{1 - g^*(\lambda_1)}{\lambda_1},$$

$$\mu_4 = \frac{1}{\lambda_1} = \mu_6, \quad \mu_5 = \frac{1 - h^*(\lambda_1)}{\lambda_1}, \quad \mu_7 = 1 - g_1^*(\lambda_1), \quad \mu_8 = \frac{1 - h^*(\lambda)}{\lambda},$$

$$\mu_9 = \frac{1 - g_1^*(\lambda_1)}{\lambda_1} \tag{3.2}$$

and

$$\mu_1' = m_{12} + m_{13-16}, \quad \mu_3' = m_{35.14} + m_{34}, \quad \mu_5' = m_{57} + m_{56} + m_{58.10} + m_{55.10,15},$$

$$\mu_7' = m_{74} + m_{75.13}, \quad \mu_8' = m_{80} + m_{89} + m_{8,11}, \quad \mu_9' = m_{9,2} + m_{93.12}. \tag{3.3}$$

4. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (*MTSF*)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t),\ \phi_1(t) = Q_{12}(t) Ⓢ \phi_2(t) + Q_{1,16}(t),$$

$$\phi_2(t) = Q_{23}(t) Ⓢ \phi_3(t),\ \phi_3(t) = Q_{34}(t) Ⓢ \phi_4(t) + Q_{3,14}(t),$$

$$\phi_4(t) = Q_{45}(t) Ⓢ \phi_5(t),\ \phi_5(t) = Q_{56}(t) Ⓢ \phi_6(t) + Q_{5,7}(t) Ⓢ \phi_7(t) + Q_{5,10}(t),$$

$$\phi_6(t) = Q_{68}(t) Ⓢ \phi_8(t),\ \phi_7(t) = Q_{74}(t) Ⓢ \phi_4(t) + Q_{7,13}(t),$$

$$\phi_8(t) = Q_{80}(t) Ⓢ \phi_0(t) + Q_{89}(t) Ⓢ \phi_9(t) + Q_{8,11}(t),$$

$$\phi_9(t) = Q_{92}(t) Ⓢ \phi_2(t) + Q_{9,12}(t). \tag{4.1}$$

Taking LST of above relations (3.4) and solving for $\tilde{\phi}_0(s)$.

We have,

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \tag{4.2}$$

The reliability $R(t)$ of the system model can be obtained by taking Laplace inverse transform of (4.2).

The mean time to system failure of the model is given by

$$MTSF = \lim_{s \to 0} R^*(s). \tag{4.3}$$

Thus

$$MTSF = \frac{N_0}{D_0}, \tag{4.4}$$

where

$$\begin{aligned} N_0 = {} & \mu_0[(p_{56}\ p_{7,13} + p_{5,10})(1 + p_{34}\ p_{1,16}) - p_{34}p_{56}\ p_{89}p_{92}] + \mu_1[1 - p_{57}\ p_{74} \\ & - p_{34}p_{56}p_{89}p_{92}] + \mu_2[p_{12}p_{56} + (p_{57}p_{7,13} + p_{5,10})(p_{34} + p_{12}p_{3,14})] \\ & + \mu_3[p_{57}p_{7,13} + p_{5,10} + p_{56}]p_{12} + \mu_4 p_{34}[p_{12}(p_{56} + p_{57}p_{74})] \\ & + p_{57}(1 + p_{7,13}) + p_{12}p_{34}[\mu_5 + p_{56}\mu_6 + p_{57}\mu_7 + p_{56}\mu_8] \end{aligned}$$

and

$$D_0 = 1 - p_{74}p_{57} - p_{34}p_{56}(p_{12}p_{23}p_{80} + p_{23}p_{89}p_{92}).$$

5. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant $'t'$ given that the system entered regenerative state i at $t=0$. The recursive relations for $A_i(t)$ are as follows:

$$A_0(t)=M_0(t)+q_{01}(t) © A_1(t),$$

$$A_1(t)=M_1(t)+q_{12}(t) © A_2(t)+q_{13.16}(t) © A_3(t),$$

$$A_2(t)=M_2(t)+q_{23}(t) © A_3(t),$$

$$A_3(t)=M_3(t)+q_{34}(t) © A_4(t)+q_{35.14}(t) © A_5(t),$$

$$A_4(t)=M_4(t)+q_{45}(t) © A_5(t),$$

$$A_5(t)=M_5(t)+q_{56}(t) © A_6(t)+q_{57}(t) © A_7(t)+q_{58.10}(t) © A_8(t) \\ +q_{55.10,15}(t) © A_5(t),$$

$$A_6(t)=M_6(t)+q_{68}(t) © A_8(t),$$

$$A_7(t)=M_7(t)+q_{74}(t) © A_4(t)+q_{75.13}(t) © A_5(t),$$

$$A_8(t)=M_8(t)+q_{80}(t) © A_0(t)+q_{89}(t) © A_9(t)+q_{81.11}(t) © A_1(t) \\ +q_{83.11,17}(t) © A_3(t),$$

$$A_9(t)=M_9(t)+q_{92}(t) © A_2(t)+q_{93.12}(t) © A_3(t), \tag{5.1}$$

where

$$M_0(t)=e^{-\lambda t}=M_2(t),\ M_1(t)=e^{-\lambda_1 t}\,\overline{G(t)},\ M_3(t)=e^{-\lambda_1 t}\,\overline{G(t)},$$

$$M_4(t)=e^{-\lambda_1 t}=M_6(t),\ M_5(t)=e^{-\lambda_1 t}\,\overline{H(t)},\qquad M_7(t)=e^{-\lambda_1 t}\,\overline{G_1(t)},$$

$$M_8(t)=e^{-\lambda t}\,\overline{H(t)},\ M_9(t)=e^{-\lambda t}\,\overline{G_1(t)}.$$

Taking LT of above relations (5.1) and solving for $A_0^*(s)$. Using this, the steady state availability can be determined as

$$A_0(\infty)=\lim_{s\to 0} s\,A_0^*(s). \tag{5.2}$$

Thus

$$A_0=\frac{N_1}{D_1}, \tag{5.3}$$

where

$$N_1 = (p_{56} + p_{58.10})[(p_{80}\mu_0 + (p_{80} + p_{81,11})\mu_1 + \mu_3 - \{p_{89}(p_{12}\ p_{93.12} - p_{13.16}p_{92}) + p_{12}p_{83.11,17}\}\mu_2 - \mu_8 - p_{89}\mu_9] + (1 + p_{13.16})[p_{34}\{p_{56} + p_{58.10} + p_{57}p_{74}(1 + p_{35.14})\}\mu_4 + \mu_5 + p_{56}\mu_6 + p_{57}\mu_7]$$

and

$$D_1 = (p_{56} + p_{58.10})[\mu_3' + p_{34}\mu_4 + \mu_8' + (p_{81.11} + p_{80})(\mu_1' + \mu_2 p_{12}) + (\mu_9' + p_{92}\mu_2)p_{89}] + p_{56}\mu_6 + p_{57}(\mu_7' + p_{74}\mu_4) + \mu_5'.$$

6. BUSY PERIOD ANALYSIS FOR SERVER

Let $B_i(t)$ be the probability that the server is busy at an instant $'t'$ given that the system entered regenerative state i at $t = 0$. We have the following recursive relations for $B_i(t)$:

$$B_0(t) = q_{01}(t) © B_1(t),$$

$$B_1(t) = W_1(t) + q_{12}(t) © B_2(t) + q_{13.16}(t) © B_3(t),$$

$$B_2(t) = q_{23}(t) © B_3(t),$$

$$B_3(t) = W_3(t) + q_{34}(t) © B_4(t) + q_{35.14}(t) © B_5(t),$$

$$B_4(t) = q_{45}(t) © B_5(t),$$

$$B_5(t) = W_5(t) + q_{56}(t) © B_6(t) + q_{57}(t) © B_7(t) + q_{58.10}(t) © B_8(t) + q_{55.10,15}(t) © B_8(t),$$

$$B_6(t) = q_{68}(t) © B_8(t),$$

$$B_7(t) = W_7(t) + q_{74}(t) © B_4(t) + q_{75.13}(t) © B_5(t),$$

$$B_8(t) = W_8(t) + q_{80}(t) © B_0(t) + q_{89}(t) © B_9(t) + q_{81.11}(t) © B_1(t) + q_{83.11,17}(t) © B_3(t),$$

$$B_9(t) = W_9(t) + q_{92}(t) © B_2(t) + q_{93.12}(t) © B_3(t), \tag{6.1}$$

where

$$W_1(t) = e^{-\lambda t}\,\overline{G(t)},\quad W_3(t) = e^{-\lambda_1 t}\,\overline{G(t)},\ W_5(t) = e^{-\lambda_1 t}\,\overline{H(t)},$$

$$W_7(t) = e^{-\lambda_1 t}\,\overline{G_1(t)},\qquad W_8(t) = e^{-\lambda t}\,\overline{H(t)},\ W_9(t) = e^{-\lambda t}\,\overline{G_1(t)}.$$

Taking LT of relations (6.1) and solving for $B_0^*(s)$. Using this, the fraction of time for which the server is busy in steady state is given by

$$B_0(\infty) = \lim_{s \to 0} s\, B_0^*(s)\,. \tag{6.2}$$

Thus

$$B_0 = \frac{N_2}{D_1}\,, \tag{6.3}$$

where

$$N_2 = (p_{56} + p_{58.10})[(p_{89}\mu_9 + \mu_8 - \mu_1(p_{89} + (p_{83.11,17})]$$
$$+ (p_{56} + p_{58.10})(\mu_1 + \mu_3) + (1 + p_{13.16})(\mu_5 + p_{57}\mu_7)$$

and D_1 is already defined.

7. EXPECTED NO. OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$, we have the following recursive relations for $N_i(t)$:

$$N_0(t) = Q_{01}(t) \circledS [1 + N_1(t)], \quad N_1(t) = Q_{12}(t) \circledS N_2(t) + Q_{13.16}(t) \circledS N_3(t)\,,$$

$$N_2(t) = Q_{23}(t) \circledS [1 + N_3(t)]\,,$$

$$N_3(t) = Q_{34}(t) \circledS N_4(t) + Q_{35.14}(t) \circledS [1 + N_5(t)]\,,$$

$$N_4(t) = Q_{45}(t) \circledS [1 + N_5(t)], \quad N_5(t) = Q_{68}(t) \circledS [1 + N_8(t)]\,,$$

$$N_7(t) = Q_{74}(t) \circledS N_4(t) + Q_{75.13}(t)\, N_5(t)\,,$$

$$N_8(t) = Q_{80}(t) \circledS N_0(t) + Q_{89}(t) \circledS N_9(t) + Q_{81.11}(t) \circledS N_1(t)$$
$$+ Q_{83.11,17}(t) \circledS N_3(t)\,,$$

$$N_9(t) = Q_{92}(t) \circledS N_2(t) + Q_{93.12}(t) \circledS N_3(t) \tag{7.1}$$

Taking LST of relations (7.1) and solving for $\tilde{N}_0(s)$. Using this, the expected number of visits per unit time can be obtained as

$$N_{i0} = \lim_{s \to 0} s\, \tilde{N}_0(s)\,. \tag{7.2}$$

Thus

$$N_0 = \frac{N_3}{D_1}\,,$$

where

$$N_3 = (p_{56} + p_{58.10})[p_{80} + p_{12}(p_{80} + p_{81.11}) + p_{89}p_{92}] \\ + (1 + p_{13.16})[p_{34}(p_{56} + p_{58.10}) + p_{57}p_{74}p_{34}(1 + p_{35.14})]$$

and D_1 is already defined.

8. COST-BENEFIT ANALYSIS

Profit incurred to the system model in steady state is given by

$$P = K_1 A_0 - K_2 B_0 - K_3 N_0, \tag{8.1}$$

where

$K_1 =$ Revenue per unit up time of the system

$K_2 =$ Cost per unit for which server is busy

$K_3 =$ Cost per unit visit by the server

9. SPECIAL CASE

Let us take $g(t) = \theta e^{-\theta t}$, $h(t) = \theta_1 e^{-\theta_1 t}$, $g_1(t) = \theta_2 e^{-\theta_2 t}$

The following results are obtained

$$MTSF\,(T) = \frac{N_0}{D_0}, \tag{9.1}$$

where

$$N_0 = (\lambda_1 + \theta)[p\theta\{\theta_1\lambda(\lambda + \theta_2)R_1 - q\theta_1^2\theta_2\lambda_1(2\lambda + \theta)\} + \lambda\lambda_1(\lambda + \theta)z_1R_2 \\ + \lambda_1 z_1 z_2\{(\lambda_1 + \theta) + \theta_2(\lambda_1 + p\theta_1)(\lambda + \lambda_1 + \theta) - \theta(\lambda_1 + \theta)\}],$$

$$D_0 = \lambda\lambda_1(\lambda_1 + \theta)[(z_2 - q\theta_1\theta_2)z_1z_3 - p\theta\theta_1\{\theta_1\lambda(\lambda + \theta_2) \\ + q\theta_1\theta_2(\lambda + \theta)\}(\lambda_1 + \theta_2)],$$

$$R_1 = \lambda_1\theta[(2\lambda_1 + \lambda)(q\theta_1 + \lambda_1 + \theta_2) - (q\theta_1 + \lambda_1)],$$

$$R_2 = (\lambda + \theta_1)(\lambda_1 + 2\lambda) + \lambda(2\lambda + \theta_1),$$

$$z_1 = (\lambda + \theta_1)(\lambda + \theta_2), \quad z_2 = (\lambda_1 + \theta_1)(\lambda_1 + \theta_2), \quad z_3 = (\lambda_1 + \theta)(\lambda + \theta).$$

$$\text{Availability } (A_0) = \frac{N_1}{(\lambda_1 + \theta)D_1}, \tag{9.2}$$

where

$$N_1 = \theta\theta_1\theta_2[\{(\lambda_1 + \theta)z_2 p k_4 + q\theta\theta_1\theta_2(2\lambda_1 + \theta)(2\lambda + \theta)\lambda + z_3 z_4\lambda(2\lambda + \theta)$$

$$(\lambda_1+\theta_1(p+q\lambda_1))z_1\}+p\{(\lambda+\theta_1 q)(\lambda+\theta)(\lambda+\theta_2)+q\lambda z_1$$
$$+q\lambda\theta_1(\lambda+\theta_2)+q\lambda^2(\lambda+\theta_2)\}\lambda_1(\lambda_1+\theta)^2 z_2],$$

$$D_1=\theta\theta_1\theta_2[\lambda(\lambda+\theta_2)(\lambda_1+\theta_2)\{p z_3 k_1+p\theta_2(\lambda+\theta_1)k_2+\theta z_5(\lambda+\theta)k_3\}$$
$$+\theta_1\theta_2(\lambda_1+\theta)(\lambda+\theta)\{pq\theta_1^2\theta_2\lambda_1 z_2+q\theta_1^2\theta_2\lambda z_1+\lambda\lambda_1 z_1(\lambda_1+\theta_2)$$
$$+q\lambda_1^2\lambda z_1(\lambda_1+\theta_2)\}],$$

$$k_1=\lambda_1\theta_2(\lambda+\theta+\theta_1)+q\theta\theta_1,\quad k_2=\theta(\lambda+\theta)+p(\lambda+2\theta)(\lambda_1+\theta),$$

$$k_3=p\theta_1^2\theta_2+\lambda(\lambda_1+\theta_1)(q\theta_1+\theta_2),$$

$$k_4=\lambda_1(\lambda+\theta)(\lambda+\lambda_1+\theta)+2\lambda\lambda_1(\lambda_1+\theta)+\theta\lambda(2\lambda+\theta),$$

$$z_4=(\lambda_1+\theta)(\lambda_1+\theta_2),\ z_5=(\lambda_1+\theta)(\lambda+\theta_1).$$

Busy Period $(B_0)=\dfrac{N_2}{D_1}$, (9.3)

where

$$N_2=\lambda\lambda_1\theta\theta_1\theta_2(\lambda+\theta)z_2 p[z_3(q\theta_1+\lambda+\theta_2)+z_1(\lambda+\theta+p\lambda_1+p\theta)]$$
$$+\lambda\lambda_1\theta\theta_1\theta_2(\lambda+\theta)(2\lambda+\theta)(\lambda+\theta_2)z_5(\lambda_1+\theta_2+q\theta_1).$$

Expected Number of Visits $(N_0)=\dfrac{N_3}{(\lambda+\theta)(\lambda_1+\theta)D_1}$, (9.4)

where

$$N_3=p(\lambda_1+\theta)^2\lambda_1\theta\theta_1\theta_2 z_2[\theta_1(\lambda+\theta)(p+\lambda+\theta_2)+p\lambda z_1]$$
$$+\theta^2\lambda\theta_1\theta_2 z_1[(2\lambda+\theta)p z_1 z_2+q\theta_1\theta_2(2\lambda_1+\theta)(\lambda+\theta)]$$

and D_1, z_1, z_2, z_3, z_5 are already defined.

10. NUMERICAL RESULTS

To this particular case, the following numerical results for *MTSF* and profit are obtained as shown in tables 1 and 2 respectively.

Table 1: for *MTSF*

λ	$\theta=1.1$, $\theta_1=10.0$, $\theta_2=1.1$, $p=0.6$, $q=0.4$, $\lambda_1=0.4$	$\theta=1.3$, $\theta_1=10.0$, $\theta_2=1.1$, $p=0.6$, $q=0.4$, $\lambda_1=0.4$	$\theta=1.1$, $\theta_1=15.0$, $\theta_2=1.1$, $p=0.6$, $q=0.4$, $\lambda_1=0.4$	$\theta=1.1$, $\theta_1=10.0$, $\theta_2=1.3$, $p=0.6$, $q=0.4$, $\lambda_1=0.4$
0.1	7.953908	8.092638	11.1848	6.088222
0.11	7.728308	7.902712	10.86775	5.920355
0.12	7.508788	7.71677	10.55944	5.756757
0.13	7.295035	7.53465	10.25941	5.597216

0.14	7.08676	7.356199	9.967271	5.441529
0.15	6.883689	7.181272	9.682624	5.289512
0.16	6.685569	7.009731	9.405109	5.140988
0.17	6.492161	6.841446	9.13439	4.995794
0.18	6.303242	6.676292	8.870147	4.853774
0.19	6.118602	6.51415	8.612083	4.714783

Table 2: for profit $K_1 = 7000,\ K_2 = 500,\ K_3 = 100$

λ	$\theta = 1.1$, $\theta_1 = 10.0$, $\theta_2 = 1.1$, $p = 0.6$, $q = 0.4$, $\lambda_1 = 0.4$	$\theta = 1.3$, $\theta_1 = 10.0$, $\theta_2 = 1.1$, $p = 0.6$, $q = 0.4$, $\lambda_1 = 0.4$	$\theta = 1.1$, $\theta_1 = 15.0$, $\theta_2 = 1.1$, $p = 0.6$, $q = 0.4$, $\lambda_1 = 0.4$	$\theta = 1.1$, $\theta_1 = 10.0$, $\theta_2 = 1.3$, $p = 0.6$, $q = 0.4$, $\lambda_1 = 0.4$
0.1	444.5778	483.0933	299.5043	401.4137
0.11	426.7863	464.5198	287.6028	384.4722
0.12	411.5072	448.5603	277.3866	369.9968
0.13	398.2864	434.7401	268.5508	357.5278
0.14	386.7723	422.6923	260.8595	346.7124
0.15	376.6881	412.1284	254.1272	337.2754
0.16	367.8135	402.8189	248.2059	328.9989
0.17	359.9705	394.5787	242.9764	321.7083
0.18	353.0141	387.257	238.3414	315.2618
0.19	346.8246	380.7297	234.2206	309.5434

11. CONCLUDING REMARKS

Table 1 clearly indicates that mean time to system failure ($MTSF$) decreases with the increase of different failure rates for fixed values of other parameters. And, it increases as repair rate (θ) of original unit and inspection rate (θ_1) of degraded unit increase. From table 2, it is observed that profit of the system model decreases with the increase of failure rates. However, system model becomes more profitable when repair rate (θ) increases. On the basis of the results obtained for a particular case, it is concluded the concept of inspection to examine the feasibility of repair of the degraded unit is not much economically beneficial.

State Transition Diagram

Figure 1

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