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CONSTRUCTION OF A DECISION MAKING INDEX FOR INVENTORY SYSTEM USING DEMAND AND LEAD -TIME DISTRIBUTIONS

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ABSTRACT

The aim of the present paper is to construct a decision making index for inventory management system based on demand and lead-time distributions. The proposed index utilizes the joint distribution of demand per unit lead-time. For the statistical application, the probability density function of the above distribution has also been derived. Here the distributions of demand and lead-time are taken to be exponential and gamma respectively. The numerical value of the proposed index lies between 0 and 1. The magnitude of the index indicates the degree of satisfaction or dissatisfaction etc. of customers with the system. The suggested index can be used in taking decisions for better services and also to maintain equilibrium between the inventory and demand of the customers. The application of the proposed index has been illustrated with the help of numerical example.

1. INTRODUCTION

In real life situations, many customers come to the wholesaler to fulfill their requirements. The wholesaler takes some time for fulfilling their demands, which is called lead-time [see Sharma (1997) and Taha (1982)]. The rate of the flow of customers depends on the service facility of the wholesaler. Sometimes there is big crowd of customers arriving to the wholesaler for fulfilling their demands and the customers have to wait a lot or the wholesaler takes more time to fulfill the requirements of the customers. Customer's lead-time depends on his/her demand.

A reasonably large demand per unit lead-time in the system may result in loss of customers. In order to cope up with the situation, the wholesaler specifies the demand per unit lead-time say (R^*) below which all the customers must be served in order to avoid congestion. The proposed index is based on the joint distribution of demand per unit lead-time.

The main objective of the present paper is to construct a decision making index for inventory management system based on the joint distribution of demand per unit lead-time. In inventory theory no mathematical or statistical index is developed so far to check the efficiency of the inventory management system and also to find the level of satisfaction or dissatisfaction of the customers with the system. The development of the index is an effort in this direction.

This objective has been achieved in two dimensions:

Firstly, we have developed the mathematical index. Secondly, the composition of the suggested index has been done on the basis of the parameters of the distributions. For this purpose, the truncated distribution of the number of customers having demand per unit lead-time greater than R^* has been derived.

Section 2 deals with the development of the mathematical index. Section 3 deals with the derivation of the index using the probability distribution of demand per unit lead-time. The application of the index is done in section 4.

2. DEVELOPMENT OF MATHEMATICAL INDEX

Let there be 'n' customers in the system. Now, consider their demands with respective lead-times and obtain the demand per unit lead-time for all customers. Let it be denoted by R. Assume that the wholesaler specifies the demand per unit lead-time be R^* below which all the customers are served. Generally, decisions for improving the system are taken on the basis of mean and variance, and then mean and variance of demand per unit lead-time for all the customers are to be determined. In this regard, the customers whose demand per unit lead-time is less than R^*, has no role to play as these are the customers who are quite satisfied with the system. Therefore, the construction of index based on mean and variance of all the customers cannot be considered as satisfactory criteria for taking decision.

Therefore, we have to focus only on those customers whose demand per unit lead-time is greater than R^*. Since they are the customers who are dissatisfied with the system. Some of them may wait for their demand to be fulfilled with dissatisfaction and some of them may go elsewhere to fulfill their demands. This will result in loss of the business to the wholesaler. Thus, decisions based on these customers will provide a firm basis for the construction of efficient index for making decisions.

Now, we attempt to derive a general decision making index based on demand and lead-time distributions. Suppose a sample survey of 'n' customers is conducted with respect to their demands $D_1, D_2, \cdots, D_n$ and lead-times $L_1, L_2, \cdots, L_n$. We consider the ratio of demand and lead-time for all the customers and for convenience; these ratios are arranged in decreasing order.

$$D : D_1, D_2, \cdots, D_{n_0}, D_{n_0+1}, \cdots, D_n .$$

Let $L : L_1, L_2, \cdots, L_{n_0}, L_{n_0+1}, \cdots, L_n$

$$R : \frac{D_1}{L_1} \geq \frac{D_2}{L_2} \geq \cdots \geq \frac{D_{n_0}}{L_{n_0}} \geq \frac{D_{n_0+1}}{L_{n_0+1}} \geq \cdots \geq \frac{D_n}{L_n} ,$$

where $R_i = \dfrac{D_i}{L_i}$, be the demand per unit lead-time of $i-$th customer, $i = 1, 2, \cdots, n_0, \cdots, n$.

Suppose the wholesaler specifies the demand per unit of lead-time be R^* below which all the customers are served.

Let n_0 be the number of customers whose demand per unit lead-time is greater than R^*. Now it is worthwhile to consider the partitioned vector having the values greater than R^*. Since the number of customers below n_0 i.e. whose demand per unit lead-time is less than R^* will be fully satisfied with the system.

i.e. $\underline{R}_{n_0} = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_{n_0} \end{pmatrix}$

Let $\underline{d}^* = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_{n_0} \end{pmatrix}$; where $d_i = (R_i - R^*)$

Now consider $\underline{w}^* = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$ as the weights attached to $\underline{d}'^*_i$ such that $\underline{d}'^* \underline{w}^*$ determines the index.

Therefore, the general form of the index is

$$I = \phi \sum_{i=1}^{n_0} \underline{d}'^* \underline{w}^* ,$$ where ϕ be the normalizing parameter.

Now

$$I = \phi \sum_{i=1}^{n_0} (R_i - R^*) \, w_i \qquad (2.1)$$

Take, $w_i \propto (R_i - R^*)$

$$w_i = k \, (R_i - R^*) , \qquad (2.2)$$

where k is the constant of proportionality, which is determined by the condition, that sum of the weights is equal to 1.

Taking summation on both the side of equation (2.2),

$$\sum_{i=1}^{n_0} w_i = k \sum_{i=1}^{n_0} (R_i - R^*)$$

Since sum of the weights is equal to unity, i.e.

$$1 = k \sum_{i=1}^{n_0} (R_i - R^*)$$

$$k = \frac{1}{\sum_{i=1}^{n_0} (R_i - R^*)}$$

$$w_i = \frac{(R_i - R^*)}{\sum_{i=1}^{n_0} (R_i - R^*)} \qquad \text{(from 2.2)}$$

$$I = \phi \frac{\sum_{i=1}^{n_0} (R_i - R^*)^2}{\sum_{i=1}^{n_0} (R_i - R^*)} \qquad \text{(from 2.1)}$$

Now it remains to find only ϕ. Consider the case $R_i \geq R^{**}$, $i = 1, 2, \cdots, n_0$, where R^{**} is assumed to very large demand per unit lead-time for the wholesaler. No customer will like to wait or to be served when demand per unit lead-time is greater than R^{**}.

Hence, if $R_i \geq R^{**}$, $i = 1, 2, \cdots, n_0$, then the inventory management system fails completely. This is the worst possible situation and practically this situation does not hold. In this situation, we assume the value of the index to be maximum.

Let we take the maximum value of index $I = 1$, (all R_i's are equal to R^{**})

$$1 = \phi (R^{**} - R^*)$$

$$\phi = 1/(R^{**} - R^*)$$

Hence, the index comes to be

$$I = \frac{1}{(R^{**} - R^{*})} \frac{\sum_{i=1}^{n_0} (R_i - R^{*})^2}{\sum_{i=1}^{n_0} (R_i - R^{*})} \tag{2.3}$$

The index has been derived for n_0 customers whose demand per unit lead-time is greater than R^*, so it will take minimum value $I = 0$ when n_0 is zero which is the case when all the customers are served i.e. the system is completely efficient where all customers are fully satisfied with the system. The maximum values of index $I = 1$ indicates that customers are highly dissatisfied with the system. In between 0 and 1, the index indicates different magnitude or level of satisfaction of the customers.

The numerical value of the index is useful for the management to take the decision for improving a particular system.

3. DERIVATION OF THE INDEX USING THE PROBABILITY DISTRIBUTION OF DEMAND PER UNIT LEAD-TIME

The index I can also be decomposed as,

$$I = \frac{1}{(R^{**} - R^{*})} \frac{\sum_{i=1}^{n_0} [(R_i - \overline{R}_{n_0}) + (\overline{R}_{n_0} - R^{*})]^2}{\sum_{i=1}^{n_0} [(R_i - \overline{R}_{n_0}) + (\overline{R}_{n_0} - R^{*})]}$$

$$= \frac{1}{(R^{**} - R^{*})} \frac{\sum_{i=1}^{n_0} (R_i - \overline{R}_{n_0})^2 + n_0 (\overline{R}_{n_0} - R^{*})^2}{n_0 (\overline{R}_{n_0} - R^{*})}$$

$$= \frac{1}{(R^{**} - R^{*})} \frac{S^2_{\underline{R}_{n_0}} + (\overline{R}_{n_0} - R^{*})^2}{(\overline{R}_{n_0} - R^{*})}, \tag{3.1}$$

where $\overline{R}_{n_0}$ and $S^2_{\underline{R}_{n_0}}$ are the means and variance of $\underline{R}_{n_0} = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_{n_0} \end{pmatrix}$

Let the demand be exponentially distributed with probability density function,

$$f(D) = \theta e^{-D\theta}, \qquad D \geq 0.$$

And the lead-time be distributed as Gamma distribution with probability density function,

$$f(L) = \frac{e^{-L} L^{\lambda-1}}{\Gamma\lambda}, \quad L \geq 0.$$

Then by applying the transformation of the random variables [see Feller (1970)] we have derived the probability density function of demand per unit lead-time as

$$f(R) = \frac{\theta\lambda}{(1+R\theta)^{\lambda+1}}, \qquad 0 \leq R \leq \infty,$$

where θ be the demand rate and λ be the average lead-time.

Since in the construction of the index we are discarding the values below R^* therefore, the truncated probability density function of R at the point $R = R^*$ is given by [see Rohatgi (1976)]

$$g(R) = \frac{f(R)}{P(R > R^*)}, \quad R > R^*$$

$$\Pr(R > R^*) = \int_{R^*}^{\infty} \frac{\theta\lambda}{(1+R\theta)^{\lambda+1}} dR = \frac{1}{(1+R^*\theta)^{\lambda}}$$

Hence,

$$g(R) = \frac{\theta\lambda(1+R^*\theta)^{\lambda}}{(1+R\theta)^{\lambda+1}}, \qquad R > R^*.$$

The moment generating function of random variable R is given by,

$$M_R(t) = E(e^{tR}) = \int_{R^*}^{\infty} e^{tR} \frac{\theta\lambda(1+R^*\theta)^{\lambda}}{(1+R\theta)^{\lambda+1}} dR$$

$$= \theta\lambda(1+R^*\theta)^{\lambda} \int_{R^*}^{\infty} \frac{1}{(1+R\theta)^{\lambda+1}} \left(1 + \frac{tR}{1!} + \frac{t^2R^2}{2!} + \cdots\right) dR$$

$$= 1 + t\left\{R^* + \frac{(1+R^*\theta)}{\theta(\lambda-1)}\right\} + \frac{t^2}{2!}\left\{R^{*2} + \frac{2R^*(1+R^*\theta)}{\theta(\lambda-1)} + \frac{2(1+R^*\theta)^2}{\theta^2(\lambda-1)(\lambda-2)}\right\} + \cdots$$

$$\bar{R} = E(R) = \text{Coefficient of } t = R^* + \frac{(1+R^*\theta)}{\theta(\lambda-1)}$$

$$E(R^2) = \text{Coefficient of } \frac{t^2}{2!} = R^{*2} + \frac{2R^*(1+R^*\theta)}{\theta(\lambda-1)} + \frac{2(1+R^*\theta)^2}{\theta^2(\lambda-1)(\lambda-2)}$$

$$V(R) = E(R^2) - [E(R)]^2 = \frac{(1+R^*\theta)^2\lambda}{\theta^2(\lambda-1)^2(\lambda-2)}, \ \lambda > 2.$$

Now

$$(\bar{R}-R^*)^2 = \bar{R}^2 + R^{*2} - 2\bar{R}R^* = \frac{(1+R^*\theta)^2}{\theta^2(\lambda-1)^2}$$

Hence, from (3.1) we get,

$$I = \frac{2(1+R^*\theta)}{(R^{**}-R^*)\theta(\lambda-2)}.$$

4. DATA AND APPLICATION

To illustrate the application of the index, a numerical example with the following data is considered.

Demand	:	225	300	315	275	280	275	350	330	200	400
Lead-time	:	9	10	9	11	8	11	10	11	8	10
Demand/Unit lead-time	:	25	30	35	25	35	25	35	30	25	40

From the above data, we have obtained the estimated value of the parameters θ and λ by using the *method of moments*, which are as follows;

$$\hat{\theta} = 0.003, \quad \hat{\lambda} = 9.7 \approx 10.$$

Let the wholesaler specifies the demand per unit lead-time $R^* = 30$ units, below which all the customers are served. And suppose $R^{**} = 150$, which is assumed to be very large demand per unit lead-time for the wholesaler and $\Pr(R > R^{**}) = 0$.

Therefore the numerical value of the index comes out to be $I = 0.75$.

For the above data set, the numerical value (0.75) of the index indicates that the customers are highly dissatisfied with the system. This may result in loss of business to the wholesaler. Therefore, the management needs to take decision for improving the particular inventory system.

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