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AN ALTERNATIVE ESTIMATOR OF THE PARAMETER OF GEOMETRIC LIFE- TIME MODEL UNDER TYPE-I PROGRESSIVE CENSORED SAMPLING

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ABSTRACT

The main purpose of the present paper is to propose an estimator for the parameter of the geometric lifetime model whose bias and *MSE* are less than that of *MLE* under Type-I progressive censored sampling and it has been shown through numerical comparison.

1. INTRODUCTION

It is well known that the discrete analogue of the exponential distribution is geometric distribution. It can be used as a discrete failure model to investigate the ability of electronic tubes to withstand successive voltage overloads and the performance of electric switches, which are repeatedly turned on and off. In each of these cases failure can occur at $X-$th trial $(X=1,2,\cdots)$ and it is assumed that the probability of failure at $X-$th trial is $(1-p)$ provided that item has not failed prior to that trail is given by the geometric distribution with probability function,

$$f(x)=(1-p)p^{x-1} \text{ for } X=1,2,\cdots;\ 0<p<1$$

Such a discrete life-time failure model is considered by Patel and Gajjar (1990). Patel and Patel (2003) (a), Patel and Patel (2003) (b) have considered the geometric failure model having probability mass function (pmf) as

$$f(x,\theta)=\left(\frac{1}{1+\theta}\right)\left(\frac{\theta}{1+\theta}\right)^{x-1} \qquad \text{for } x=1,2,\cdots \text{ and } \theta>0$$

Here θ is the mean of the distribution. They have considered maximum likelihood estimators of the parameters of the above geometric failure model under $K-$stage Type-I progressive censoring with changing parameters. They have also obtained bias and mean square error (MSE) of the ML estimators.

In this paper we propose an alternative estimator to the ML estimator for the parameters of geometric lifetime model under $K-$stage Type-I progressive censoring. This alternative estimator has less bias and MSE than that of ML estimator. We have also obtained almost unbiased estimator, which is better than alternative, as well as ML estimator. The numerical evaluations of the estimators are made over ML estimator.

2. THE GEOMETRIC DISTRIBUTION AND LIKELIHOOD

Let an item be placed on test. Suppose that X denotes the trial at which an item is failed during the probability function of X is,

$$f(x,\theta)=\left(\frac{1}{1+\theta}\right)\left(\frac{\theta}{1+\theta}\right)^{x-1}, \quad x=1,2,\cdots \text{ and } \theta>0$$

With mean $E(X)=\theta$ and variance $V(X)=\theta(\theta+1)$

Suppose that the time of censoring are N_{i-1}, $i=1,2,\cdots,K$ and the experiment is finally terminated at N_K, $N_{i-1}<N_i$ for $i=1,2,\cdots,K$. Suppose that the parameter θ of the distribution changes at $N_1,N_2,\cdots,N_{K-1}$. Then the composite probability function in case of Type-I progressive censoring with changing parameters can be written as,

$$f(x;\theta_1,\theta_2,\cdots,\theta_K)=\begin{cases} f_1(x)=\left(\dfrac{1}{1+\theta_1}\right)\left(\dfrac{\theta_1}{1+\theta_1}\right)^{x-1}, & x=1,2,\cdots,N_1 \\ f_i(x)=\displaystyle\prod_{j=1}^{i-1}\left(\frac{\theta_j}{1+\theta_j}\right)^{N_j-N_{j-1}}\left(\frac{1}{1+\theta_i}\right)\left(\frac{\theta_i}{1+\theta_i}\right)^{x-N_{i-1}-1} & \end{cases}$$

$$x=N_{i-1}+1,\ N_{i-1}+2,\cdots,N_i;\ i=2,3,\cdots,K$$

And the corresponding distribution function is given by,

$$F(x)=\begin{cases} F_1(x)=1-\left(\dfrac{\theta_1}{1+\theta_1}\right)^{x}, & x=1,2,\cdots,N_1 \\ F_i(x)=1-\displaystyle\prod_{j=1}^{i-1}\left(\frac{\theta_i}{1+\theta_i}\right)^{N_j-N_{j-1}}\left(\frac{\theta_i}{1+\theta_i}\right)^{x-N_{i-1}} & \end{cases}$$

$$x=N_{i-1}+1,\ N_{i-1}+2,\cdots,N_i;\ i=2,3,\cdots,K$$

where $N_K=\infty$.

Suppose n items are placed on a life test without replacement and that n_i be the number of items that fail during discrete time interval $[N_{i-1}+1,N_i]$ in the $i-$th stage and let $N_{i-1}+1\le x_1^{(i)}\le x_2^{(i)}\le\cdots\le x_{n_i}^{(i)}\le N_i$ be the discrete times of failure for $i = 1, 2, \cdots, K$ and $K > 1$. Let r_i be the number of items removed or censored from the test immediately after time N_{i-1}, $i = 1, 2, \cdots, K$.

Suppose $n^{(i)}$ be the number of items entering the $i-$th stage of censoring $[N_{i-1} + 1, N_i]$ and we shall assume it to be given or known. Here $n^{(1)} = n$ and $n^{(i)} = n^{(i-1)} - n_i - r_i$ for $i = 1, 2, \cdots, K$. The number n_i of failures in the $i-$th stage of censoring is, of course, a random variable following the binomial distribution $B_n\,(n^{(i)},\ p_i)$. Where p_i denotes the probability of failure of an item in the $i-$th stage of censoring.

Under this progressive scheme Patel and Patel (2003 b) have obtained the ML estimator and MSE of the ML estimator θ_i which are as under:

$$T_{1i} = \hat{\theta}_i = \frac{\sum x_j^{(i)} - n_i - n_i N_{i-1} + (n^{(i)} - n_i)(N_i - N_{i-1}}{n_i}$$

$$i = 1, 2, \cdots, K \text{ and } N_0 = 0 \tag{2.1}$$

The bias of T_{1i} for given $n^{(i)}$ as,

$$B(T_{1i}) = E[(T_{1i} - \theta_i) | n^{(i)}] = S_i \left[n^{(i)} E\left(\frac{1}{n_i}\right) - \frac{1}{p_i} \right] \tag{2.2}$$

where

$$S_i = N_i - N_{i-1} \text{ and } p_i = 1 - q_i = 1 - \left(\frac{\theta_i}{1+\theta_i}\right)^{N_i - N_{i-1}} \tag{2.3}$$

and the mean square error (MSE) of T_{1i} can be written as,

$$MSE(T_{1i}) = \left[S_i^2 (n^{(i)})^2 V\left(\frac{1}{n_i}\right) + \left(\frac{S_i^2(1-p_i)}{p_i^2} + \frac{q_i^{1/S_i}}{(1-q_i^{1/S_i})^2}\right) E\left(\frac{1}{n_i}\right) \right]$$

$$+ \left[n^{(i)} E\left(\frac{1}{n_i}\right) - \frac{S_i}{p_i} \right]^2 \tag{2.4}$$

Using the tables of negative moments of positive binomial given by Mendenhall and Lehman (1960) one can numerically evaluate the amount of bias and MSE for given p_i, $n^{(i)}$ and S_i.

3. AN ALTERNATE ESTIMATOR

Let T_{2i} be an alternative estimator of θ_i which is given by,

$$T_{2i} = \frac{\sum x_j^{(i)} - n_i - n_i N_{i-1} + (n^{(i)} - n_i)(N_i - N_{i-1})\left(1 - \frac{1}{n_i}\right)}{n_i} \tag{3.1}$$

$$= T_{1i} - \frac{(n^{(i)} - n_i)(N_i - N_{i-1})}{n_i^{\,2}} \tag{3.2}$$

$$E(T_{2i}) = \theta_i - \frac{(N_i - N_{i-1})(1 - p_i)}{p_i} + \left(\frac{n^{(i)}}{n_i} - 1\right)(N_i - N_{i-1})$$

$$- \frac{1}{n_i^{\,2}}(n^{(i)} - n_i)(N_i - N_{i-1})$$

which reduces to,

$$E(T_{2i}) = \theta_i - \frac{S_i(1 - p_i)}{p_i} + \left(\frac{n^{(i)}}{n_i} - 1\right)S_i - \frac{1}{n_i^{\,2}}(n^{(i)} - n_i)S_i \tag{3.3}$$

Hence bias of T_{2i} for given $n^{(i)}$ can be given as,

$$B(T_{2i}) = B(T_{1i}) - S_i\left[n^{(i)} E\left(\frac{1}{n_i^{\,2}}\right) - E\left(\frac{1}{n_i}\right)\right] \tag{3.4}$$

and the MSE of T_{2i} for given $n^{(i)}$ can be written as,

$$MSE(T_{2i}) = MSE(T_{1i}) + S_i^{\,2}\left[n^{(i)} E\left(\frac{1}{n_i^{\,4}}\right) - (2n^{(i)2} + 2n^{(i)})E\left(\frac{1}{n_i^3}\right)\right.$$

$$\left. + \left(\frac{2n^{(i)}}{p_i} + 2n^{(i)} + 1\right)E\left(\frac{1}{n_i^2}\right) - \frac{2}{p_i}E\left(\frac{1}{n_i}\right)\right] \tag{3.5}$$

In order to further reduce the bias effect we replace the factor $\left(1 - \frac{1}{n_i}\right)$ in T_{2i} by $\left(\frac{\alpha}{n_i} + \frac{\beta}{n_i^2} + \frac{\gamma}{n_i^3}\right)$ in (3.1) as considered by Huang and Chen (1992) and consider the following class of estimators which includes both T_{1i} and T_{2i}

Let T_{3i} be a new alternative estimator for θ_i defined as,

$$\frac{1}{n_i}\sum_{j=1}^{n_i} x_j^{(i)} - n_i - n_i\, N_{i-1} + (n^{(i)} - n_i)(N_i - N_{i-1})$$

$$+\frac{1}{n_i}\left(\frac{\alpha}{n_i} + \frac{\beta}{n_i^2} + \frac{\gamma}{n_i^3}\right)(n^{(i)} - n_i)(N_i - N_{i-1})$$

It can be easily seen that for $\alpha = 0$, $\beta = 0$, $\gamma = 0$ we get $T_{3i} \equiv T_{1i}$ and for $\alpha = -1$, $\beta = 0$, $\gamma = 0$ we get $T_{3i} \equiv T_{2i}$ which are defined respectively in (2.1) and (3.1).

Let $\alpha = -1$, $\beta = 1$, $\gamma = -1$ we can write T_{3i} as,

$$T_{3i} = T_{1i} + \frac{1}{n_i}\left(-\frac{1}{n_i} + \frac{1}{n_i^2} - \frac{1}{n_i^3}\right)(n^{(i)} - n_i)(N_i - N_{i-1}) \tag{3.6}$$

$$= \hat{\theta}_i + \frac{1}{n_i^4}[(-n_i^2 + n_i - 1)(n^{(i)} - n_i)(N_i - N_{i-1})]$$

Now using (2.1) we can write,

$$E(T_{3i} \mid n_i) = \theta_i - \frac{S_i(1-p_i)}{p_i} + \left(\frac{n^{(i)}}{n_i} - 1\right)S_i - \frac{(1 - n_i - n_i^2)(n^{(i)} - n_i)S_i}{n_i^4} \tag{3.7}$$

Thus bias of T_{3i} for given $n^{(i)}$ can be written as,

$$B(T_{3i}) = E[(T_{3i} - \theta_i) \mid n^{(i)}]$$

$$B(T_{1i}) - S_i\left[n^{(i)}E\left(\frac{1}{n_i^4}\right) - (n^{(i)} + 1)E\left(\frac{1}{n_i^3}\right) + (n^{(i)} + 1)E\left(\frac{1}{n_i^2}\right) - E\left(\frac{1}{n_i}\right)\right] \tag{3.8}$$

and the MSE of T_{3i} for given $n^{(i)}$ can be given as,

$$MSE(T_{3i} \mid n_i) = E[(T_{3i} - \theta_i) \mid n^{(i)}]$$

$$= MSE(T_{1i}) + S_i^{\,2}\left[n^{(i)2}E\left(\frac{1}{n_i^8}\right) - (2n^{(i)2} + 2n^{(i)})E\left(\frac{1}{n_i^7}\right)\right]$$

$$+ S_i^2\left[(3n^{(i)2}+4n^{(i)}+1)\,E\left(\frac{1}{n_i^6}\right)-(4n^{(i)2}+6n^{(i)}+2)\,E\left(\frac{1}{n_i^5}\right)\right]$$

$$+ S_i^2\left[\left(3n^{(i)2}+6n^{(i)}+3+\frac{2n^{(i)}}{p_i}\right)E\left(\frac{1}{n_i^4}\right)-\left(2n^{(i)2}+4n^{(i)}+2+\frac{2n^{(i)}}{p_i}+\frac{2}{p_i}\right)E\left(\frac{1}{n_i^3}\right)\right]$$

$$+ S_i^2\left[\left(2n^{(i)}+\frac{2n^{(i)}}{p_i}+\frac{2}{p_i}+1\right)E\left(\frac{1}{n_i^2}\right)-\frac{2}{p_i}E\left(\frac{1}{n_i}\right)\right] \qquad (3.9)$$

Tables of bias: The following tables compare the biases of T_{1i}, T_{2i} and T_{3i} for some fixed values of $n^{(i)} = n$, $p_i = p$ and S_i.

Table 1: $S_i = 1$

n	p	$B(T_{1i})$	$B(T_{2i})$	$B(T_{3i})$
5	0.05	-15.253997	-18.926696	-18.90868239
	0.1	-5.515195	-8.8574895	-9.63557576
	0.2	-1.052991	-3.7359062	-3.6749183
	0.5	0.3844028	-0.584459	-0.8469102
	0.8	0.0914425	-0.0437115	-0.019919
	0.95	0.014375	-4.0796E-03	-4.234E-04
10	0.05	-11.12339	-18.584562	-18.48687694
	0.1	-2.223206	-8.2468726	-8.081655735
	0.2	0.768192	-2.8340598	-2.6220954
	0.5	0.291094	-0.1338256	-0.0651042
	0.8	0.037175	-5.8335E-3	-1.99E-05
	0.95	6.23842E-3	-7.4558E-4	-1.038E-05
20	0.05	-4.555676	-17.253926	-16.88089096
	0.1	-2.223206	-8.2468726	-5.824451743
	0.2	0.768192	-2.8340598	-0.8538226
	0.5	0.291094	-0.1338256	-6.451E-04
	0.8	0.037175	-5.8335E-3	-5.7E-06
	0.95	6.23842E-3	-7.4558E-4	-7.8E-07
50	0.05	4.886755	-11.09714	-9.866148058
	0.1	2.62186	-1.6657978	-0.195977086
	0.2	0.503795	-0.0789341	-0.0328131
	0.5	0.042635	-1.8623E-3	-6.3E-06
	0.8	6.445E-3	-1.66E-4	2E-07
	0.95	1.1284E-3	-2.64E-5	-1.7E-06
100	0.05	5.51316	-3.6201684	-2.376333433
	0.1	1.1527	-0.191643	-0.02688649
	0.2	0.22099	-0.0129001	-4.54E-05
	0.5	0.02062	-4.338E-4	-0.0104864
	0.8	3.17E-3	-3.83E-5	-5.41E-05
	0.95	5.5842E-4	3.2E-7	-4.8E-07

Table 2: $S_i = 5$

n	p	$B(T_{1i})$	$B(T_{2i})$	$B(T_{3i})$
5	0.05	-76.26998	-94.633475	-94.543406
	0.1	-27.57597	-44.287443	-48.1778738
	0.2	-5.264955	-18.679531	-18.374592
	0.5	1.922014	-2.92295	-4.234551
	0.8	0.4572125	-0.2185575	-0.09595
	0.95	0.071875	-4.0796E-3	-2.117E-03
10	0.05	-55.61669	-92.922805	-92.43437992
	0.1	-11.11603	-41.234363	-40.4082786
	0.2	3.84096	-14.170299	-13.110477
	0.5	1.45547	-0.669128	-0.325521
	0.8	0.185875	-0.0291675	-9.955E-04
	0.95	0.0311921	-7.4558E-4	-5.19E-05
20	0.05	-22.77838	-86.269629	-84.3990347
	0.1	7.69119	-31.519051	-29.1225872
	0.2	6.74982	-5.817639	-4.269113
	0.5	0.59902	-0.447809	-3.22255E-03
	0.8	0.08462	-5.829E-3	-2.85E-05
	0.95	0.0146621	-1.6338E-4	-3.9E-06
50	0.05	24.43377	-55.485705	-49.330745
	0.1	13.1093	-8.328989	-0.97988543
	0.2	2.518975	-0.3946705	-2.4698655
	0.5	0.213175	-9.3115E-3	-3.15E-05
	0.8	0.032225	-8.305E-4	-0.0290015
	0.95	5.642E-03	-1.32E-04	-8.5E-06
100	0.05	27.5658	-18.100842	-11.88166717
	0.1	5.7635	-0.958215	-0.1344324
	0.2	1.10495	-0.0645005	-2.27E-04
	0.5	1.1031	-2.16E-3	-3.2E-05
	0.8	0.01585	-1.915E-4	-2.705E-04
	0.95	2.792E-3	3.2E-7	-2.4E-06

Conclusion: From the above table it can be seen that the bias of new alternative estimator is less than that of the bias of MLE and alternate estimator for all values of $n^{(i)}$ and p. Also, the absolute bias of new alternative estimator decreases with S_i for fixed values of n and p.

Tables of MSE **:** The following tables compare the MSE of T_{1i}, T_{2i} and T_{3i} for some fixed values of $n^{(i)} = n$, p_i and S_i.

Table 3:

n	p	$B(T_{1i})$	$B(T_{2i})$	$B(T_{3i})$
5	0.05	-152.53997	-189.26696	-189.0868239
	0.1	-55.15195	-88.574895	-96.353557576
	0.2	-10.52991	-37.359062	-36.749183
	0.5	3.844028	-5.84459	-8.4687272
	0.8	0.914425	-0.437115	-0.1914
	0.95	0.14375	-0.040796	-4.234E-03
10	0.05	-111.2339	-185.84613	-184.8687694
	0.1	-22.23206	-82.468726	-80.816557
	0.2	7.68192	-28.340598	-26.220954
	0.5	2.9104	-1.338796	-0.651582
	0.8	0.37175	-0.058335	-1.1991E-03
	0.95	0.0623842	-7.4558E-3	-1.038E-04
20	0.05	-45.55676	-172.53926	-168.7980696
	0.1	15.49964	-63.038101	-58.244517
	0.2	13.49964	-11.635278	-8.538226
	0.5	1.19804	-0.159998	-6.45E-05
	0.8	0.16924	-0.011658	-5.7E-05
	0.95	0.0293242	-1.6338E-3	-7.8E-06
50	0.05	48.86755	-110.9714	-98.66148058
	0.1	26.2186	-16.657978	-19.597706
	0.2	5.03795	-0.789341	-0.328131
	0.5	0.42635	-0.018623	-6.3E-05
	0.8	0.06445	-1.661E-3	2E-06
	0.95	0.011284	-2.64E-4	-1.7E-05
100	0.05	55.1316	-36.201684	-23.7633433
	0.1	11.527	-1.91643	-0.2688649
	0.2	1.10495	-1.233951	-4.54E-04
	0.5	-0.2062	-4.338E-3	-6.4E-05
	0.8	0.0317	-3.83E-4	-5.4E-04
	0.95	5.5842E-3	3.2E-6	-4.8E-06

Table 4: $S_i = 1$

n	p	$MSE(T_{1i})$	$MSE(T_{2i})$	$MSE(T_{3i})$
5	0.05	954.6618	1079.6613	1079.0068
	0.1	192.95316	239.9918	239.4305
	0.2	34.46756	45.650039	45.262072
	0.5	3.5986048	2.3272702	2.2384102
	0.8	0.393787	0.2107155	0.22600326
	0.95	0.0446271	0.033333	0.032819236
10	0.05	803.04869	1012.301	1017.3893
	0.1	152.45986	208.85216	206.60039
	0.2	32.181549	31.790126	32.181544
	0.5	2.4240757	1.274445	1.3530183
	0.8	0.1422703	0.1183461	0.12160195
	0.95	0.019281	0.0174372	0.017667637
20	0.05	669.40281	919.90593	909.37639
	0.1	141.45856	147.14364	143.08167
	0.2	30.820003	15.388343	17.553712
	0.5	0.7806338	0.6270307	0.6366784
	0.8	0.0635108	0.0593631	0.059631355
	0.95	9.14619E-3	8.793363E-3	8.653E-03
50	0.05	606.8893	521.72095	684.42162
	0.1	120.00613	55.046328	57.320068
	0.2	9.0053454	6.3971125	6.210184
	0.5	0.2604865	0.2465827	0.24765210
	0.8	0.030549	0.0300815	0.030103566
	0.95	3.5589E-3	3.7146579E-3	3.24436E-3
100	0.05	512.21995	235.29927	244.13374
	0.1	42.607577	28.930116	30.569266
	0.2	3.4882326	3.1570546	3.1808256
	0.5	0.1247263	0.1211741	0.1197738
	0.8	0.0150499	0.016833	0.01418836
	0.95	1.7642E-3	4.2400574E-3	2.78E-3

Table 5: $S_i = 5$

n	p	$MSE(T_{1i})$	$MSE(T_{2i})$	$MSE(T_{3i})$
5	0.05	23868.444	26993.432	26977.7
	0.1	7106.925	8282.8929	8268.8585
	0.2	863.26383	1142.8258	1133.1266
	0.5	90.89549	59.112125	58.140625
	0.8	10.315793	5.7390055	6.1211995
	0.95	1.1015441	0.8191938	0.806347502
10	0.05	20077.992	25309.3	25436.508
	0.1	4183.7495	5593.5571	5537.2628
	0.2	805.68897	795.9034	796.8212
	0.5	57.755813	29.015046	30.379378
	0.8	3.7827915	3.1846878	3.2660827
	0.95	0.4750145	0.4289209	0.434677928
20	0.05	15964.561	22227.139	21963.901
	0.1	3715.0803	3857.2073	3755.658
	0.2	771.13557	385.34406	389.4783
	0.5	19.722629	15.882552	16.123744
	0.8	1.6994505	1.595758	1.602464388
	0.95	0.2251604	0.2163397	0.2128344
50	0.05	15173.24	13044.032	17111.548
	0.1	3000.6569	1376.6618	1433.5054
	0.2	225.35304	160.14722	155.4741
	0.5	7.9223906	7.5747956	7.6015306
	0.8	0.647768	0.6360817	0.636632162
	0.95	0.0931894	0.0970833	0.0853259
100	0.05	12806.009	5882.992	6103.8537
	0.1	1065.4123	723.47577	764.4545
	0.2	87.309915	79.030465	79.62474
	0.5	3.1575813	3.0687763	3.0337688
	0.8	0.3192569	0.3638356	0.297718412
	0.95	0.0461923	0.1080884	0.071773

Table 6: $S_i = 10$

n	p	$MSE(T_{1i})$	$MSE(T_{2i})$	$MSE(T_{3i})$
5	0.05	95474.013	107973.93	107908.52
	0.1	19302.712	24006.584	23950.446
	0.2	3453.2527	4571.5006	4532.7039
	0.5	363.70108	236.56762	232.68162
	0.8	41.329773	23.022623	24.551399
	0.95	5.6406271	4.511226	4.459840709
10	0.05	80312.191	101237.42	101746.26
	0.1	14574.254	20213.484	19988.307
	0.2	3222.9025	3183.7602	3187.4314
	0.5	231.07734	116.11427	123.9716
	0.8	15.163098	12.770683	13.09626315
	0.95	2.5125798	2.3282056	2.351233513
20	0.05	63858.435	88908.747	87855.793
	0.1	14150.613	14719.121	14312.924
	0.2	3084.6129	1541.4469	1557.9838
	0.5	79.25211	63.89379	64.85657
	0.8	6.813522	6.398762	6.42560877
	0.95	1.2059449	1.1706623	1.1566409
50	0.05	60693.085	52176.253	68446.317
	0.1	12002.691	5506.7108	5734.0848
	0.2	901.43994	640.61665	621.92381
	0.5	26.377664	24.987284	25.094326
	0.8	2.5973218	2.5505768	2.55277845
	0.95	0.4722296	0.4878053	0.4407758
100	0.05	51224.099	23532.031	24415.478
	0.1	4261.6767	2893.9306	3057.8456
	0.2	345.59011	312.47231	314.84941
	0.5	12.63537	12.28015	12.14012
	0.8	1.2801399	1.458459	1.19398595
	0.95	0.2345551	0.4821398	0.3368779

Conclusion: The above tables present that MSE of T_{3i} is less than that of T_{1i} and T_{2i} when $n > 20$ and $0.2 \le p < 0.95$. Also, for small sample size (less than 20) MSE of T_{3i} is smaller than that of T_{1i} and T_{2i} for $0.5 \le p \le 0.95$ and for fixed values of n and p MSE of T_{3i} decreases with S_i. Thus in this situation new alternative estimator is better than the usual ML estimator.

4. ALMOST UNBIASED ESTIMATOR

Regal Ronald (1980) defined an almost unbiased estimator as an estimator whose bias tends to zero more rapidly than any negative power of n. Using Regal's results; the almost unbiased estimator can be defined as follows:

$$T_{4i} = \hat{\theta}_{iAu} = \bar{x}^{(i)} - 1 - N_{i-1} + \frac{(n^{(i)} - n_i)(N_i - N_{i-1})}{n_i + 1} \tag{4.1}$$

From (2.1), we can write

$$T_{4i} = T_{1i} - \left(\frac{n^{(i)} - n_i}{n_i}\right) S_i + \left(\frac{n^{(i)} - n_i}{n_i + 1}\right) S_i \tag{4.2}$$

$$E(T_{4i}) = \theta_i - \frac{S_i \left(\frac{\theta_i}{1+\theta_i}\right)^{S_i}}{1 - \left(\frac{\theta_i}{1+\theta_i}\right)^{S_i}} + \left[(n^{(i)} + 1) E\left(\frac{1}{n_i + 1}\right) - 1\right] S_i \tag{4.3}$$

Hence using (4.3), the bias of T_{4i} can be given as

$$B(T_{4i}) = E(T_{4i} - \theta_i) = \left[(n^{(i)} + 1) E\left(\frac{1}{n_i + 1}\right) - 1\right] S_i - \frac{S_i \left(\frac{\theta_i}{1+\theta_i}\right)^{S_i}}{1 - \left(\frac{\theta_i}{1+\theta_i}\right)^{S_i}} \tag{4.4}$$

Using (2.3), we can write the bias of T_{4i} for given $n^{(i)}$ as

$$B(T_{4i}) = S_i \left[(n^{(i)} + 1) E\left(\frac{1}{n_i + 1}\right) - \frac{1}{p_i}\right] \tag{4.5}$$

and the MSE of T_{4i} for given n_i using (4.2) can be written as,

$$MSE(T_{4i}) = E(T_{4i} - \theta_i)^2 = E\left[T_{1i} - \left(\frac{n^{(i)} - n_i}{n_i}\right) S_i + \left(\frac{n^{(i)} - n_i}{n_i + 1}\right) S_i - \theta_i\right]^2$$

On simplification we get

$$MSE(T_{4i}) = MSE(T_{1i}) + S_i^2 \left[\frac{2n^{(i)}}{p_i} E\left(\frac{1}{n_i}\right) - n^{(i)^2} E\left(\frac{1}{n_i^2}\right)\right.$$

$$-\frac{(n^{(i)}+1)}{p_i}E\left(\frac{1}{n_i+1}\right)+(n^{(i)}+1)^2E\left(\frac{1}{(n_i+1)^2}\right)\Bigg]\Bigg] \quad (4.6)$$

The following are tables of bias and MSE of T_{4i} for a fixed values of $n^{(i)}$, p_i and S_i. Which has been compared with that of T_{3i}.

Table 7: $S_i = 1$

n	p	$B(T_{3i})$	$B(T_{4i})$
5	0.05	-18.90868239	-17.1047
	0.1	-9.63557576	-7.209714
	0.2	-3.6749183	-2.436935
	0.5	-0.8469102	-0.1612904
	0.8	-0.019919	0.0
	0.95	-4.234E-04	0.0
10	0.05	-18.48687694	-10.921308
	0.1	-8.081655735	-5.3534
	0.2	-2.6220954	-1.202903
	0.5	-0.0651042	-0.00976
	0.8	-1.99E-05	0.0
	0.95	-1.038E-05	0.0
20	0.05	-16.88089096	-11.176245
	0.1	-5.824451743	-2.768066
	0.2	-0.8538226	-0.233275
	0.5	-6.451E-04	0.0
	0.8	-5.7E-06	0.0
	0.95	-7.8E-07	0.0
50	0.05	-9.866148058	-4.167953
	0.1	-0.195977086	-0.259026
	0.2	-0.0328131	0.0
	0.5	-6.3E-06	0.0
	0.8	2E-07	0.0
	0.95	-1.7E-06	0.0
100	0.05	-2.376333433	-0.59559
	0.1	-0.02688649	0.0
	0.2	-4.54E-05	0.0
	0.5	-0.0104864	0.0
	0.8	-5.41E-05	0.0
	0.95	-4.8E-07	0.0

Table 8: $S_i = 5$

n	p	$B(T_{3i})$	$B(T_{4i})$
5	0.05	-94.543406	-85.5235
	0.1	-48.1778738	-36.04857
	0.2	-18.374592	-12.184675
	0.5	-4.234551	-0.80645
	0.8	-0.09595	0.0
	0.95	-2.117E-03	0.0
10	0.05	-92.43437992	-74.6054
	0.1	-40.4082786	-26.6767
	0.2	-13.110477	-6.014515
	0.5	-0.325521	0.0
	0.8	-9.955E-04	0.0
	0.95	-5.19E-05	0.0
20	0.05	-84.3990347	-55.88123
	0.1	-29.1225872	-13.84033
	0.2	-4.269113	-1.16638
	0.5	-3.22255E-03	0.0
	0.8	-2.85E-05	0.0
	0.95	-3.9E-06	0.0
50	0.05	-49.330745	-20.8397
	0.1	-0.97988543	-0.29513
	0.2	-2.4698655	0.0
	0.5	-3.15E-05	0.0
	0.8	-0.0290015	0.0
	0.95	-8.5E-06	0.0
100	0.05	-11.88166717	-2.9779
	0.1	-0.1344324	0.0
	0.2	-2.27E-04	0.0
	0.5	-3.2E-05	0.0
	0.8	-2.705E-04	0.0
	0.95	-2.4E-06	0.0

Table 9: $S_i = 10$

n	p	$B(T_{3i})$	$B(T_{4i})$
5	0.05	-189.0868239	-171.047
	0.1	-96.353557576	-72.09714
	0.2	-36.749183	-24.36935
	0.5	-8.4687272	-1.612904
	0.8	-0.1914	0.0
	0.95	-4.234E-03	0.0
10	0.05	-184.8687694	-149.21308
	0.1	-80.816557	-53.534
	0.2	-26.220954	12.02903
	0.5	-0.651582	-0.0976
	0.8	-1.1991E-03	0.0
	0.95	-1.038E-04	0.0
20	0.05	-168.7980696	-111.76245
	0.1	-58.244517	-27.68066
	0.2	-8.538226	-2.33275
	0.5	-6.45E-05	0.0
	0.8	-5.7E-05	0.0
	0.95	-7.8E-06	0.0
50	0.05	-98.66148058	-41.67953
	0.1	-19.597706	-2.59026
	0.2	-0.328131	0.0
	0.5	-6.3E-05	0.0
	0.8	2E-06	0.0
	0.95	-1.7E-05	0.0
100	0.05	-23.7633433	-105.9559
	0.1	-0.2688649	-0.0266
	0.2	-4.54E-04	0.0
	0.5	-6.4E-05	0.0
	0.8	-5.4E-04	0.0
	0.95	-4.8E-06	0.0

Conclusion: From the above tables of bias one can observe that bias of T_{4i} is less than that of T_{3i} for all the values of p and S_i as the sample size increases.

Table 10: $S_i = 1$

n	p	$MSE(T_{3i})$	$MSE(T_{4i})$
5	0.05	1079.0068	1013.9398
	0.1	239.4305	213.615145
	0.2	45.262072	37.83250246
	0.5	2.2384102	2.284945862
	0.8	0.22600326	0.25042135
	0.95	0.032819236	0.035544
10	0.05	1017.3893	897.94325
	0.1	206.60039	169.7997274
	0.2	32.181544	26.04703898
	0.5	1.3530183	1.49026464
	0.8	0.12160195	0.115143125
	0.95	0.017667637	0.017631
20	0.05	909.37639	747.0550439
	0.1	143.08167	117.3442212
	0.2	17.553712	16.57018317
	0.5	0.6366784	0.6492445732
	0.8	0.059631355	0.05982315
	0.95	8.653E-03	8.69401E-3
50	0.05	684.42162	430.567569
	0.1	57.320068	63.0185933
	0.2	6.210184	6.82716711
	0.5	0.24765210	0.24699686
	0.8	0.030103566	0.030613242
	0.95	3.24436E-3	2.32759E-3
100	0.05	244.13374	266.5672
	0.1	30.569266	32.22612499
	0.2	3.1808256	3.42927185
	0.5	0.1197738	0.12083278
	0.8	0.01418836	0.019439
	0.95	2.78E-3	5.04E-03

Table 11: $S_i = 5$

n	p	$MSE(T_{3i})$	$MSE(T_{4i})$
5	0.05	26977.7	25351.468
	0.1	8268.8585	7623.47458
	0.2	1133.1266	947.387391
	0.5	58.140625	58.054016
	0.8	6.1211995	6.73157175
	0.95	0.806347502	0.8744739
10	0.05	25436.508	22450.35619
	0.1	5537.2628	4617.246184
	0.2	796.8212	652.32621
	0.5	30.379378	34.4090821
	0.8	3.2660827	3.104612125
	0.95	0.434677928	0.43376404
20	0.05	21963.901	17905.8168
	0.1	3755.658	3112.22183
	0.2	389.4783	414.890074
	0.5	16.123744	16.437927
	0.8	1.602464388	1.60725925
	0.95	0.2128344	0.2138484
50	0.05	17111.548	10765.19673
	0.1	1433.5054	1575.96484
	0.2	155.4741	170.89858
	0.5	7.6015306	7.5851496
	0.8	0.636632162	0.649374
	0.95	0.0853259	0.0624066
100	0.05	6103.8537	6664.6906
	0.1	764.4545	805.8759
	0.2	79.62474	85.835896
	0.5	3.0337688	3.060243
	0.8	0.297718412	0.428985
	0.95	0.071773	0.128157

Table 12: $S_i = 10$

n	p	$MSE(T_{3i})$	$MSE(T_{4i})$
5	0.05	107908.52	101406.1122
	0.1	23950.446	21368.9103
	0.2	4532.7039	3789.74694
	0.5	232.68162	232.33518
	0.8	24.551399	26.993208
	0.95	4.459840709	4.732346
10	0.05	101746.26	89801.6477
	0.1	19988.307	16308.2407
	0.2	3187.4314	2609.4514
	0.5	123.9716	137.6904
	0.8	13.09626315	12.45038
	0.95	2.351233513	2.34757
20	0.05	87855.793	71623.458
	0.1	14312.924	
	0.2	1557.9838	1659.63091
	0.5	64.85657	66.1133032
	0.8	6.42560877	6.44476
	0.95	1.1566409	1.16069
50	0.05	68446.317	43060.91198
	0.1	5734.0848	6303.93733
	0.2	621.92381	683.62211
	0.5	25.094326	25.0287
	0.8	2.55277845	2.60374605
	0.95	0.4407758	0.349098758
100	0.05	24415.478	26658.825
	0.1	3057.8456	3223.53148
	0.2	314.84941	339.69403
	0.5	12.14012	12.246018
	0.8	1.19398595	1.719055
	0.95	0.3368779	0.56241425

Conclusion: From the above tables we can see that MSE of T_{4i} is less than that of T_{1i} for almost all values of n and p. Thus, we can suggest that almost unbiased estimator is better than the other estimators.

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