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ON USE OF SIMULATION TECHNIQUE TO AUGMENT THE RELIABILITY OF A SYSTEM

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ABSTRACT

The use of simulation in industry, of late, has become an important technique. It is used in the analysis of a number of complex systems, where analytical methods are either difficult to apply or not possible to apply. The problem discussed here is to increase the reliability of a system, which, depends on a set of components connected in series. The increment can be done, by connecting to each of the components, similar components in parallel under some price and weight restrictions. The number of each of these components, are selected using simulation and the combination giving the maximum reliability can be decided. An algorithm to reach the maximum reliability using simulation, under the price and weight restriction is been discussed. In the last section with some numerical values we find the maximum reliability for a five component series system and reach an important result.

1. INTRODUCTION

A key aspect of product quality is the reliability of the product. A number of specialized techniques have been developed to quantify reliability and to estimate the "life expectancy" of a product. Standard references and textbooks describing these techniques include Lawless (1982), Nelson (1990), Lee (1980). Dodson (1994) gives an excellent overview with many examples of engineering applications.

The reliability of a product or component is an important aspect of product quality. It is however necessary to quantify the product's reliability, so that one can derive estimates of the product's expected life. For example, suppose you are flying a small single engine aircraft. It would be very useful (in fact vital) information to know what the probability of engine failure is at different stages of the engine's "life" (e.g., after 500 hours of operation, 1000 hours of operation, etc.). Given a good estimate of the engine's reliability, and the confidence limits of this estimate, one can then make a rational decision about when to swap or overhaul the engine.

This takes us to statistical reliability or probabilistic reliability as it is often called. Probabilistic reliability theory deals with solution to problems useful in predicting, estimating or optimizing a suitable objective function. The objective function may be mean life, survival probability, etc. For bibliography related to statistical reliability one can see Bain (1978), Barlow *et al.* (1975), Barlow and Proschan (1975) etc.

Let a non-negative random variable T denotes the lifetime of a system. Let the *cdf* of T is given by $F(t)$. Then the reliability of the system is given by

$$R(t) = \overline{F}(t) = 1 - F(t) = \Pr(T > t)$$

i.e. probability that the system survives at least up to time t.

If the system has more than one component, then the reliability depends on the way they are connected. If components are connected in series, then the system survives up to time t, if all the components survive to that time

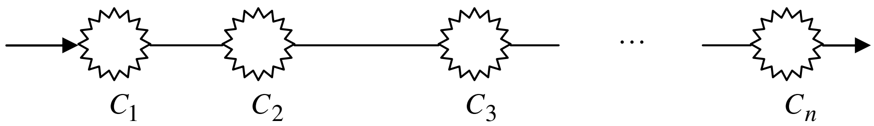

Figure showing components C_i $(i = 1, 2, \cdots, n)$ connected in series

Now, if p_i is the probability that the $i-$th component survives up to time t.

Then probability that the system survives up to time t is given by

$$R(t) = \prod_{i=1}^{n} p_i \tag{1.1}$$

However, if the components of the system are connected in parallel, then, even if one of them fails the other component does its work. The system fails, when the entire component fails.

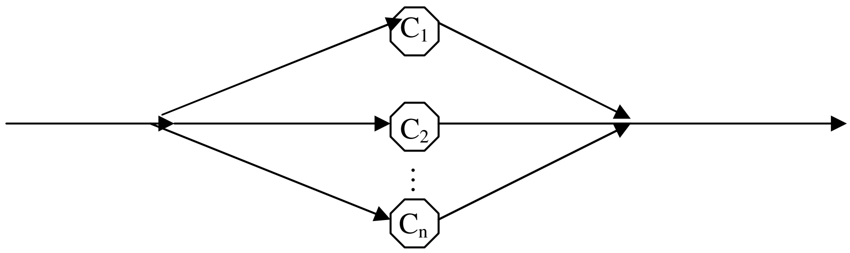

Figure showing components C_i $(i = 1, 2, \cdots, n)$ connected in parallel.

Thus, if p_i is the probability that the $i-$th component survives up to time t, then probability that the $i-$th component fails before time t is $(1 - p_i)$. So, probability that the system fails before time t is $\prod_{i=1}^{n}(1 - p_i)$. Thus

$$R(t) = 1 - \prod_{i=1}^{n}(1 - p_i) \tag{1.2}$$

2. THE PROBLEM

To understand the problem let us take the help of a numerical example. Let us consider a five-component system, where the components are connected in series. Let the probability of survival of each of the components at least up to time t be equal to 0.95, which is very high. Thus using (1.1), we have the system reliability as $R(t) = 0.7737$, not a very reliable system.

The above example shows that though the components were very reliable the system is not so as the components were connected in series. However for the same number of components in parallel we would have $R(t) = 0.99999$ using (1.2). This implies that the system reliability increases for components in parallel.

Now for components in series, if we can connect to each of the component, similar components in parallel then the reliability of each of the component will increase and it will in turn increase the system reliability. Let $C_{11}, C_{21}, \cdots, C_{n1}$ be n components connected in series with probability of survival upto time t is given by probabilities, $p_1, p_2, \cdots, p_n$. To the component C_{11} we connect similar other components $C_{12}, C_{13}, \cdots, C_{1k_1}$. So, the reliability of the first component goes up from p_1 to $1-(1-p_1)^{k_1}$, putting $p_i = p_1$ in (1.2). The same treatment is given to each of the remaining components $C_{21}, \cdots, C_{n1}$.

This is an example of components connected in series where each of the component is connected with similar other components in parallel. Thus we can call it a mixed system.

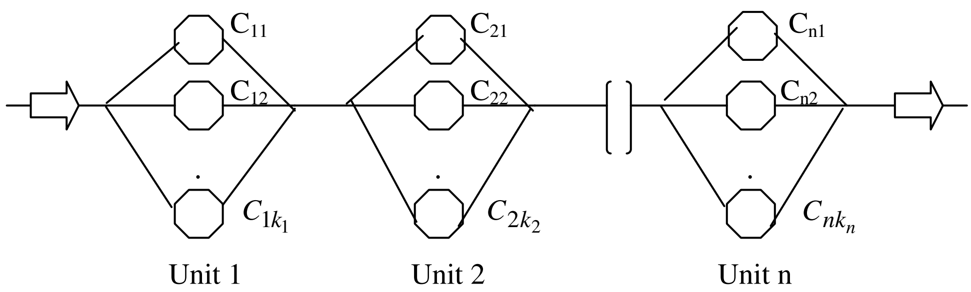

Figure showing components of a mixed system (connected in series and also in parallel)

Writing the same in tabular form we have:

Table 1: Reliability of various components of the system

Components in series C_{i1}	Number of components connected in parallel to C_{i1}	Components connected in parallel to C_{i1}	Reliability of the component C_{i1}	Reliability of the component C_i (after connecting all C_i's in parallel)
C_{11}	k_1	$C_{12}, C_{13}, \cdots, C_{1k_1}$	p_1	$1-(1-p_1)^{k_1}$
C_{21}	k_2	$C_{22}, C_{23}, \cdots, C_{2k_2}$	p_2	$1-(1-p_2)^{k_2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
C_{n1}	k_n	$C_{n2}, C_{n3}, \cdots, C_{nk_n}$	p_n	$1-(1-p_n)^{k_n}$

The reliability of this system is thus

$$R(t) = \prod_{i=1}^{n} [1-(1-p_i)^{k_i}] \tag{2.1}$$

using (1.1) and reliability of components from table 1.

Let us call all the components lying in parallel as components belonging to one unit. In the above example we have n such units. The reliability of the system will increase if the reliability of the units increase i.e. if the value of $1-(1-p_i)^{k_i}$ increases. This function is an increasing function of k_i if p_i remains same. Thus, as the value of k_i $(i = 1, 2, \cdots, n)$ keeps on increasing $R(t)$ also increases.

But increasing k_i $(i = 1, 2, \cdots, n)$ infinitely is not possible for many regions. The basic reasons may be the cost and weight considerations. Let α_i be the price of a component in the $i-$th unit $(i = 1, 2, \cdots, n)$. So, the expenditure of the system is $k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n$. The producer definitely has a maximum expenditure in mind beyond which he is not ready to spend. Let that amount be α. So, that we should have $k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n \le \alpha$. Similarly, let w_i be the weight of the $i-$th unit $(i = 1, 2, \cdots, n)$. So, the weight of the system is $k_1w_1 + k_2w_2 + \cdots + k_nw_n$. The system should not be very heavy, the maximum tolerable weight be w. So, we should have $k_1w_1 + k_2w_2 + \cdots + k_nw_n \le w$. Thus, the final shape of the problem is as follows:

Maximize $R(t) = \prod_{i=1}^{n} [1-(1-p_i)^{k_i}]$ with respect to k_i subject to the conditions

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n \le \alpha \tag{2.2}$$

$$k_1 w_1 + k_2 w_2 + \cdots + k_n w_n \le w \tag{2.3}$$

3. SOLUTION TO THE PROBLEM

There are a number of ways in which solution to the problem can be obtained. This is a problem of integer programming and methods like Branch and Bound method, which was originally devised by Land and Doig (1960), Lagrange's multiplier technique etc can be used to solve it.

Here we use a simulation technique to solve the problem. We are to decide the number of components to be used for each of the units. We generate a random integer for each of the units. Thus for an $n-$component system we generate n random integers. The random number represents the number of components to be used for that particular unit. The reliability of each of the component is known. Putting these values we can find the system reliability using (2.1). However before calculating the system reliability it is essential to see weather the combination satisfy the cost and weight considerations given by equation (2.2) and (2.3). A large set of such random integers are selected and the system reliability for each of the set are calculated but only after checking the conditions laid in (2.2) and (2.3).

Then the combination giving the maximum reliability is selected from among all the selected combinations. An algorithm given in the next section will make things more clear.

4. ALGORITHM

The algorithm to the problem is as follows

Step 1) Enter the total number of units (n), price of each component $(\alpha_i,\ i=1,2,\cdots,n)$ and weight of each component $(w_i,\ i=1,2,\cdots,n)$.

Enter the maximum permissible weight and the maximum permissible cost i.e. the values of α and w.

Enter the values of probability of survival up to time t for each of the components i.e. the values of p_i $(i=1,2,\cdots,n)$.

Step 2) Randomly select a set of n integers i.e. the values of k_i $(i=1,2,\cdots,n)$.

Calculate the value of $k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n$ and check if it is less than the maximum permissible cost i.e. α. Similarly, calculate the value of $k_1 w_1 + k_2 w_2 + \cdots + k_n w_n$ and check if its value is less than the maximum permissible weight i.e. w. If both the conditions are satisfied then we can use this set of integers for the next step or we reject it.

Step 3) In this step we calculate individual reliability for each of the units and combine them to find the system reliability using the formula given in (2.1).

Step 4) Steps 2 and 3 are repeated a number of times, the more number of times it is repeated the better it is.

Step 5) In this step we find out the combination of $k_i's$ which gives the maximum reliability. Hence this combination may be used.

5. A NUMERICAL PROBLEM

Here we take a system having five components connected in series. So to each of these components we connect a number of similar components in parallel to get 5 units. Let the total cost be Rs. 2500 and the total weight of the system is 6000 grams. Thus we have $n = 5$, $\alpha =$ Rs. 2500 and $w = 6000$ gms. The other values are given in the table 2 below:

Table 2: Data, Calculation and Results

Weights of the components (w_i) in gms	Price of the components (α_i) in Rs	Probability of survival for each component upto time t (p_i)	Value of maximum reliability $R(t)$	Combination which gives the maximum reliability $(k_i's)$
250, 255, 260, 237, 240	95,110, 75, 140, 112	0.92, 0.89, 0.83, 0.93, 0.9	0.999460	4, 4, 7, 3, 6
250, 260, 277, 305, 225	150, 111, 37, 152, 94	0.96, 0.81, 0.79, 0.98, 0.8	0.999122	3, 7, 6, 2, 5
255, 265, 210, 320, 240	120, 230, 45, 71, 75	0.94, 0.84, 0.79, 0.9, 0.8	0.995342	2, 4, 6, 6, 5
215, 195, 320, 340, 300	63, 93, 100, 111, 115	0.92, 0.87, 0.81, 0.89, 0.86	0.998755	3, 4, 5, 4, 5
222, 324, 357, 189, 110	200, 162, 43, 37, 212	0.9, 0.88, 0.83, 0.88, 0.91	0.997833	3, 4, 6, 4, 3

We study five such cases where we found out the maximum reliability by varying the weights, prices etc of the various components.

In the next step we calculate the correlation and partial correlation of w_i, α_i and p_i, with k_i to find if the number of components to be used is related to it's weight, price or survival probability of the components. Here in the table below we present the mean of partial and simple correlation for the factors mentioned above.

6. CONCLUSION

Table 3: Correlation of the number of components with other factors

Number of each component k_i vs	Average value of correlation	Average value of partial correlation
Weight (w_i)	0.0042	0.1346
Price (α_i)	0.3914	0.017
Probability of survival (p_i)	-0.7012	-0.933

On testing the significance at 5% level of significance for all these correlations and partial correlation, we find that both correlations and partial correlation between the number of components (k_i) and probability of survival (p_i) are significant and negative. However, the correlations of the number of components (k_i) with weight (w_i) or price (α_i) are insignificant.

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